Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel International GCSE

Centre Number	Candidate Number

Thursday 20 June 2019

Morning (Time: 2 hours) | Paper Reference **4PM1/02R**

Further Pure Mathematics

Paper 2R

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page. Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r}$ $\quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule
In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$ $\quad$ $\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$ $\quad$ $\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$ $\quad$ $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

Logarithms

$\log_a x = \frac{\log_b x}{\log_b a}$

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Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 $$f(x) = (x - 3)[x^2 + (p - 2)x + q]$$

Given that $f(0) = -12$

(a) find the value of q.

(2)

(b) Find the range of values of p for which the cubic equation $f(x) = 0$ has only one real root.

(5)

(Total for Question 1 is 7 marks)

2

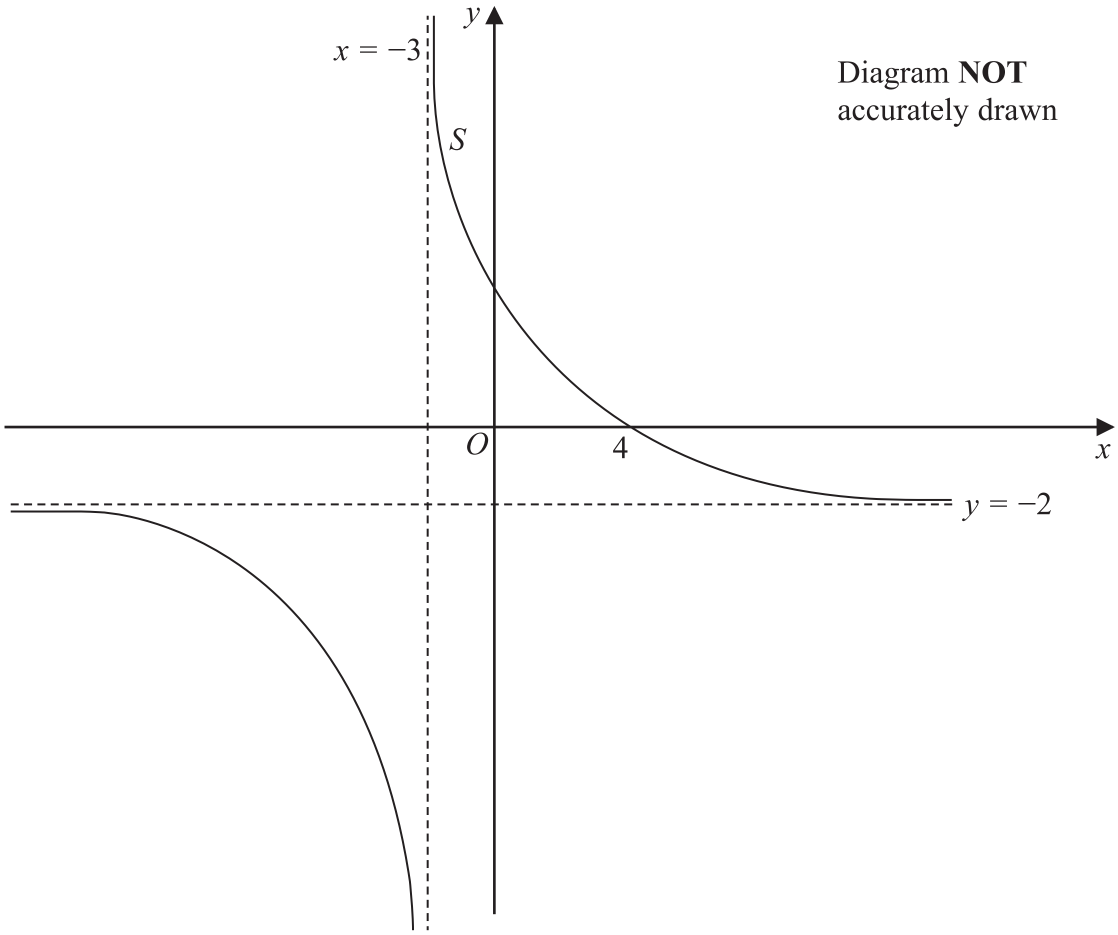

Figure 1

Figure 1 shows part of the curve S with equation $y = \dfrac{ax+b}{x+c}$ where a, b and c are integers.

The asymptote to S that is parallel to the x-axis has equation $y = -2$
The asymptote to S that is parallel to the y-axis has equation $x = -3$
The curve crosses the x-axis at the point with coordinates (4, 0)
The curve crosses the y-axis at the point with coordinates (0, p) where p is a rational number.

Find

(i) the value of a,

(ii) the value of b,

(iii) the value of c,

(iv) the value of p.

(4)

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Question 2 continued

(Total for Question 2 is 4 marks)

3

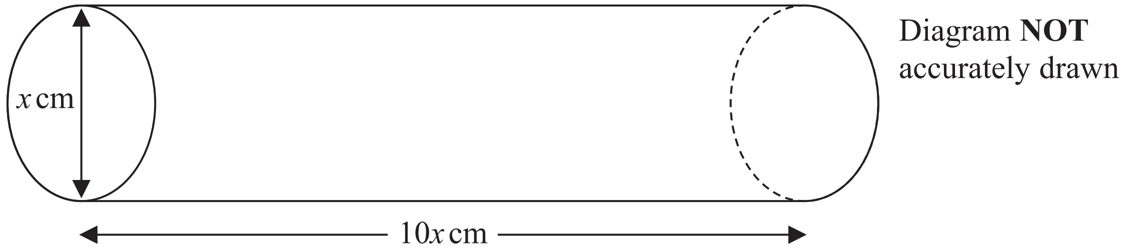

Figure 2

Figure 2 shows a solid right circular cylindrical metal rod.

The diameter of the rod is x cm and the length of the rod is $10x$ cm.

The rod is being heated so that the length of the rod is increasing at a rate of 0.005 cm/s.

Find the rate of increase, in cm^3/s to 2 significant figures, of the volume of the rod when $x = 3$

(6)

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Question 3 continued

(Total for Question 3 is 6 marks)

4 A particle P moves along the x-axis. At time t seconds ($t \geqslant 0$) the acceleration, a m/s^2, of P is given by $a = 6t - 12$

When $t = 0$, P is at rest at the origin.

(a) Find the velocity of P when $t = 2$

(3)

At time T seconds, $T > 0$, P is instantaneously at rest.

(b) Find the value of T.

(2)

(c) Find the distance travelled by P in the first 8 seconds of its motion.

(3)

Question 4 continued

(Total for Question 4 is 8 marks)

5 (a) On the grid opposite, draw the graphs of the lines with equations

$$2x + 3y = 24 \qquad y = 2x \qquad 3y = 2x - 12$$

(3)

(b) Show, by shading on the grid, the region R defined by the inequalities

$$2x + 3y \leqslant 24 \qquad y \leqslant 2x \qquad 3y \geqslant 2x - 12 \qquad y \geqslant 0$$

(1)

For all points in R, with coordinates (x, y)

$$F = 2x + 5y$$

(c) Find the greatest value of F.

(3)

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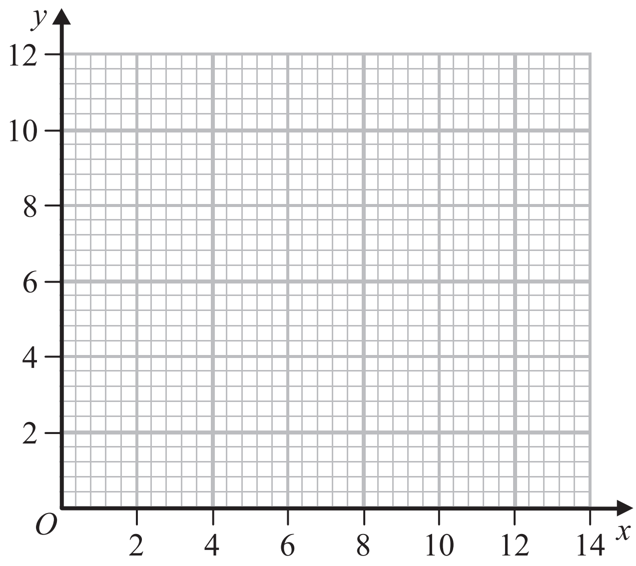
Question 5 continued



Turn over for a spare grid if you need to redraw your graphs.

Question 5 continued

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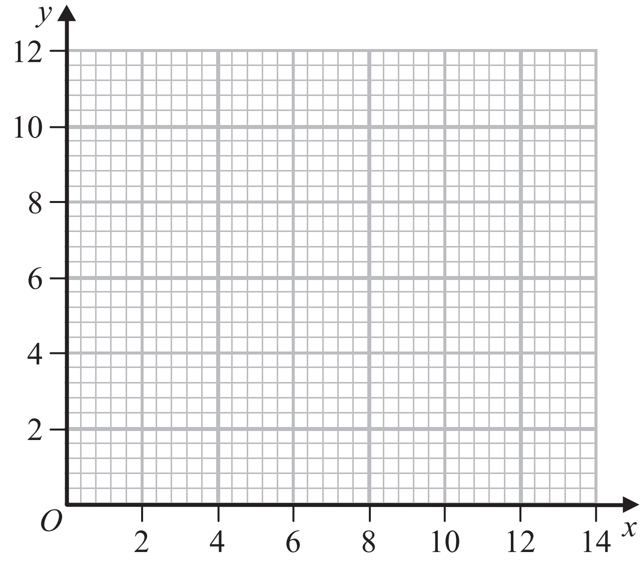
Question 5 continued

Only use this grid if you need to redraw your graphs.



(Total for Question 5 is 7 marks)

6 Given that $\sqrt{9-x}$ can be expressed in the form $p(1+qx)^{\frac{1}{2}}$ where p and q are constants

(a) find the value of p and the value of q.

(2)

(b) Hence expand $\sqrt{9-x}$ in ascending powers of x up to and including the term in x^3 expressing each coefficient as an exact fraction in its lowest terms.

(3)

Using the expansion you found in part (b) with a suitable value of x,

(c) find an estimate to 5 decimal places for the value of $\sqrt{\frac{31}{4}}$

(3)

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Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 8 marks)

7 The nth term of a geometric series G is u_n

The first term of G is a and the common ratio of G is r, where $r > 0$

Given that $u_3 = 4$ and that $u_7 = 16$

(a) (i) show that $r = \sqrt{2}$

(ii) find the value of a.

(3)

(b) Find the least value of n for which $u_n > 500$

(4)

The sum of the first n terms of G is S_n

(c) Find S_{20}

Give your answer in the form $p\left(1+\sqrt{2}\right)$ where p is an integer.

(4)

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Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 11 marks)

8

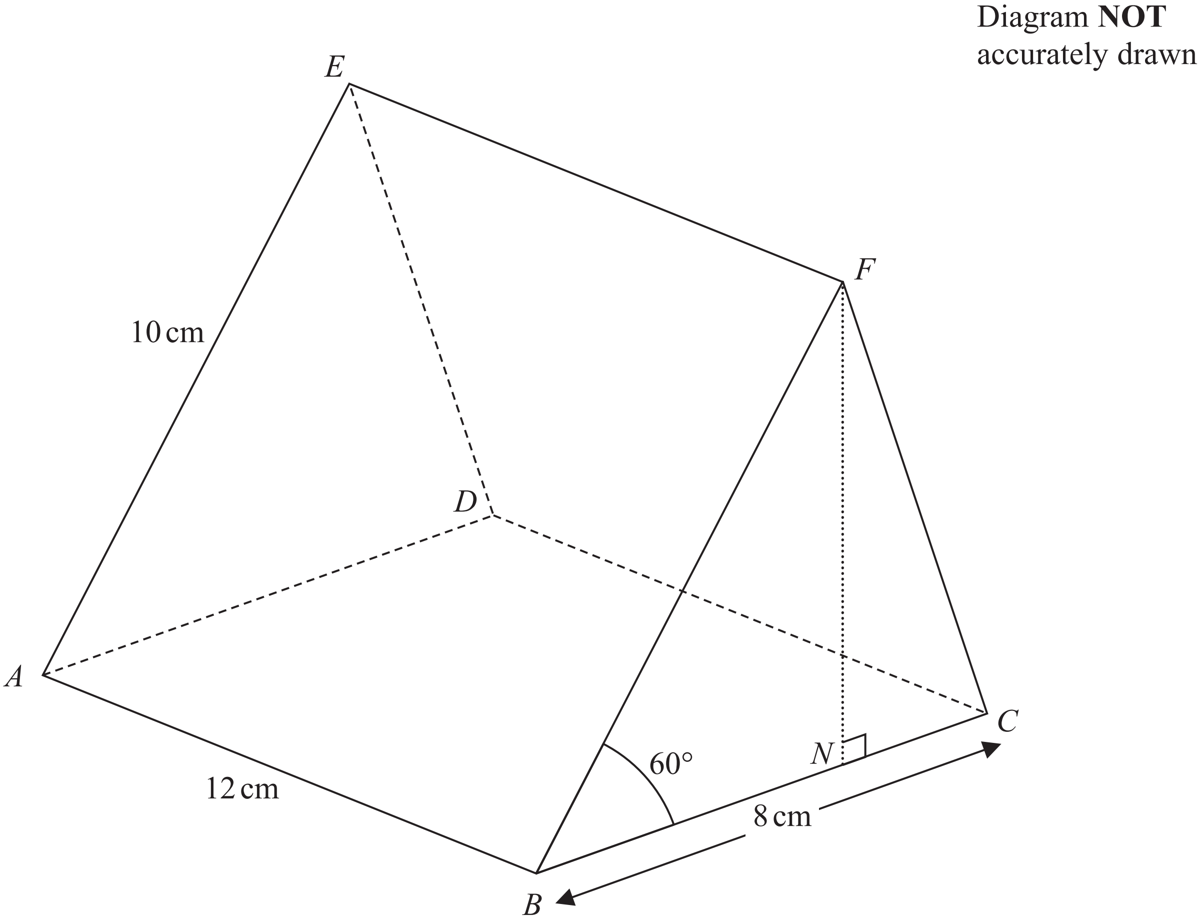

Figure 3

Figure 3 shows a right prism *ABCDEF*. The cross section *BCF* of the prism is a triangle.

$AB = DC = 12\text{ cm}$ $\quad BC = AD = 8\text{ cm}$ $\quad BF = AE = 10\text{ cm}$ $\quad \angle FBC = \angle EAD = 60°$

The point *N* lies on *BC* such that *FN* is perpendicular to *BC*.

(a) Show that $BN = 5\text{ cm}$.

(2)

(b) Find, in cm to 3 significant figures, the length of *EN*.

(3)

The midpoint of *BF* is *X* and the midpoint of *FC* is *Y*.

(c) Find, in degrees to one decimal place, the size of the angle between the plane *ABCD* and the plane *AXYD*.

(2)

(d) Find, in degrees to one decimal place, the size of the angle *AYE*.

(6)

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Question 8 continued

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Turn over

Question 8 continued

Question 8 continued

(Total for Question 8 is 13 marks)

9 The finite region R enclosed by the y-axis, the straight line with equation $y + 2x = 13$ and the curve with equation $y = x^2 - 2$, is defined for points with coordinates (x, y) with $x \geqslant 0$

The region R is rotated through 360° about the y-axis.

Use algebraic integration to find the volume of the solid generated.
Give your answer in terms of π.

(9)

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Question 9 continued

Question 9 continued

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Question 9 continued

(Total for Question 9 is 9 marks)

10 (a) Use the formula for $\cos(A + B)$ to show that $\cos 2A = 2\cos^2 A - 1$

(2)

(b) Show that $\cos 4A = 8\cos^4 A - 8\cos^2 A + 1$

(4)

(c) Solve the equation $\cos^2\left(\frac{\theta}{4} + \frac{\pi}{24}\right)\left[\cos^2\left(\frac{\theta}{4} + \frac{\pi}{24}\right) - 1\right] = -\frac{1}{16}$ $\quad 0 \leqslant \theta < 2\pi$

Give your answers in terms of π.

(5)

$$f(A) = 4\cos^4 A - 4\cos^2 A + 1$$

(d) Using calculus, find the exact value of $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} f(A)\,dA$

Give your answer in the form $a\pi - b\sqrt{c}$ where a and b are fractions in their lowest terms and c is a prime number.

(4)

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Question 10 continued

Turn over

Question 10 continued

Question 10 continued

(Total for Question 10 is 15 marks)

11 The quadratic equation $x^2 - px + q = 0$ where $p > 0$, has roots α and β.

Given that $2\alpha\beta = 3$ and that $4(\alpha^2 + \beta^2) = k^2 - 6k - 3$ where $k > 3$

(a) (i) write down the value of q,

(ii) find an expression, in terms of k, for p.

(5)

Given also that $7\alpha\beta = 3(\alpha + \beta)$

(b) find the value of k.

(2)

(c) Hence form an equation, with integer coefficients, which has roots

$$\frac{\alpha}{\alpha + \beta} \text{ and } \frac{\beta}{\alpha + \beta}$$

(5)

P 6 1 8 8 3 A 0 3 4 3 6

Question 11 continued

Question 11 continued

(Total for Question 11 is 12 marks)

TOTAL FOR PAPER IS 100 MARKS