Examiners' Report
Principal Examiner Feedback

Summer 2019

Pearson Edexcel International GCSE

In Mathematics (4PM1) Paper 01R and 02R

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Further Pure Mathematics **4PM1 01R and 4PM1 02R** **June 2019**

Principal Examiner's Report

The overall response to the papers was good. Low total scores were usually an accumulation of fragments from a range of questions, rather than an inability to access anything on the more difficult topics. At the other extreme, there were some impressive scripts that showed a deep understanding of the theory involved, often using sophisticated mathematical language to communicate concise answers clearly. Most students followed the rubric and attempted to show sufficient working to justify their answers but the quality of notation and detail varied considerably. Some very long attempts were seen, which were often wrong, so students should be cautious about spending a disproportionate amount of time on any one question.

4PM1 01R

Question 1

This was a straight forward introduction to the paper with many students scoring full marks. Those who lost marks usually muddled the use of π in their formulae, converted to degrees, or neglected to find the proportion of the circle required (finding instead, the whole area).

Question 2

In part (a), the majority of students scored the first two marks out of the four as they correctly used the cosine rule and ended up with the correct expression. Since this was a show that question, use of the trigonometric identity, Pythagoras or equivalent was required. Often students lost the final two marks because of their use of arccos and finding the angle in decimal form.

Part (b), was done more successfully as many students scored full marks. However, errors include the incorrect use of $\sin\frac{3\sqrt{15}}{16}$ in their area formula or the use of the wrong sides in the area formulae.

Question 3

In part (a), virtually all students scored this mark as they were able to state the value of $\log_3 9$.

In part (b), many different approaches to the solution of this problem were offered, some succinct and elegant others rather more convoluted, but this question was a good source of marks for most students as eventually most could be seen to use some of the required laws of logarithms in some part of their solution. The more able students who had a good grasp of the rules of logarithms and that were well prepared usually manipulated the logarithms correctly and generally reached $\sqrt{12}$ and then most simplified this correctly. However, a few students simplified incorrectly or did not simplify at all. Less able students found this question challenging, especially with the $\log_9\left(\frac{12}{t}\right)^2$ and too many students took $\log_3 9$ on the LHS as the coefficient of *t* and so simplified the LHS incorrectly to 2*t*. A few students seemed unfamiliar with the basic rules of logarithms and so scored 0 in this part of the question. Basic algebraic errors also saw some lose marks.

Question 4

In part (a), most students recognised that they needed to differentiate using the product rule and in most cases were generally successful. Most of the errors were seen in the application of the differentiation of a function of a function; e.g. factors 3 for e^{3x} and 2 for $(1+2x)$.Students who were able to apply the product rule successfully were usually able to use algebra to obtain the required expression.

In part (b), most understood the process for finding the equation of a straight line and were able to use the derivative in (a) to find a gradient and understood its connection to the gradient of the normal. Less able students struggled to find a point on the curve to use in their equation of a line and many still did not find the normal but the tangent instead. A number of students lost marks through not giving the answer in the prescribed form.

Question 5

This question proved challenging with only the most able students being able to attempt this question. Very few were able to gain full marks. It appeared that students seemed not to realise this topic was included in the syllabus, perhaps students had not been prepared or had much practice for a question on incremental changes and had never seen, or used, δA or δr before. The product of derivatives was seen very often, as it was assumed that this was a question involving, or similar to, connected rates of change. As the radius of the circle was 3*r* and the incremental change was given as a percentage, rather than an actual increment less able students generally struggled with many differentiating *A* wrt *r*. A mark of M1B0M0M0A0 was not unusual for this question.

Question 6

In part (a), most students gained full marks. Less able students did not realise the summation represented that of an AP and subsequently did not use the correct formula for the summation. Generally, students seemed familiar with the notation.

In part (b), most students managed to form an equation or inequality and proceed to a 3TQ and were usually successful in solving it. However, quite a number then used the non-integer solution 22.6 as the value of n or chose n = 22, rather than n= 23. As the question asked for the least value of n a small minority of students gave the negative solution to the 3TQ (–22(.11)) as their answer without realising that n could not be negative.

Part (c) of this question was not done as well as the previous parts of this question and many found this part a challenge, possibly due to lack of familiarity with the notation. The less able students seemed to struggle to know how to approach this type of problem. Quite a number used the formulae for a GP. The most common error was the wrong substitution of (n + 7) and (n + 4) into the given equation.

Question 7

In part (a), most students were able to get full marks here. Only a small minority of made mistakes in the subtraction of the direction vectors.

In part (b), many students scored full marks but some did not understand what a unit vector was or miscalculated the magnitude of BC. Column vectors were seen in a number of responses.

In part (c), students generally knew what was required here and were successful in obtaining a relationship between the vectors $\overrightarrow{MN}$, $\overrightarrow{NC}$ or $\overrightarrow{MC}$ using vector addition or subtraction. Many students were able to state the conclusion as a relationship between two vectors. A few students found the gradient of $\overrightarrow{MN}$, $\overrightarrow{NC}$ and $\overrightarrow{MC}$ and showed that these were equal. The concept of collinearity seemed well understood.

Question 8

Part (a) of this question was answered well by the vast majority of students. The majority successfully found the required values for the table but a small minority made arithmetic errors.

In part (b), the plotting of the points was generally accurate. Only occasionally, the points were plotted incorrectly but the curve was generally accurate and smooth.

In part (c), only the less able students were unable to obtain the required line $y = 3x – 2$. Most students who successfully deduced the correct line went on to easily find the required value of *x*, but a significant number gave a value that was 'too accurate' (the question specified one decimal place) - perhaps suggesting they had found the value on their graphic calculators, and thus lost the final A mark unless the correct rounded value was seen.

Part (d) of this question proved to be more challenging and a number of students were not successful in transforming the exponential equation to reveal the appropriate linear equation through obtaining $\ln(2x+1)+2$. Many students made a start and often got to $6-x=\ln(2x+1)^2$ and then either stopped or decided to that the square root of both side. Again, those who found the required line, plotted and drew it accurately to obtain the required estimate. Again, occasionally final estimates were not given to the required degree of accuracy.

Question 9

In part (a), almost all students arrived at the required expression for the surface area without any errors. They used the correct formula for the volume to make *h* the subject and then used the correct expression for the surface area.

In part (b), most students set their derivative equal to zero and ended up with *x* = 4.93 with some exceptions where either the derivative was incorrect or they failed to solve their equation correctly. Once obtaining *x* = 4.93 some students then failed to show that it is a minimum, some forgot to do that part, others thought that since *x* = 4.93 > 0 then that gives minimum and others found an incorrect expression for the 2nd derivative which cost them 2 marks.

Part (c) was done correctly by almost all, but surprisingly, even with a correct value *x* = 4.93, after substituting into the surface area they ended up with a wrong value for the minimum.

Question 10

Overall, the majority of students did this question successfully and got full marks with a few exceptions.

In part (a), some lost 1 mark as they wrote $-(x+3)^2-9$ or $-(x-3)^2-9$ instead of $-(x-3)^2+9$

In part (b), some students used differentiation to find the maximum value and even students that in part (a) did not get full marks, often got both marks in this part.

Almost all students got full marks in part (c). Occasionally after arriving at $x = 1$ and $x = 4$ students failed to find the corresponding y coordinates suggesting that they had failed to realise that the question asked for the coordinates of the two points.

The majority of students got full marks in part (d). There were only a few exceptions where students used the wrong limits, or lost the last mark due to a numerical error when substituting the limits. However, the majority correctly integrated the difference of the two functions and ended up with 9 or $|-9| = 9$

Question 11

This question proved challenging even for the most able and very few students scored full marks. However, many were able to access parts (a) and (b) and often scored the first 2 marks in part (c). A score of M1A1 M1A1 B1M1M0M0M0A0 M0M0A0 M0A0M0A0 was not unusual.

In part (a) most students scored both marks as it was straight forward and they knew how to find the equation of a line though two given points.

In part (b) the majority of students scored both marks. There was a variety of method used to find the coordinates of *P*, but generally sufficient working was shown to obtain the given coordinates.

In part (c), many scored the first two marks as they could find the linear equation for m and n using the gradient = –2. It was the next part that caused the most problems. After finding $11 = 2m - n$ oe, the vast majority of students failed to realise that they had to use $AC = 10$ and the common error seen used $AC = 5$ which led to a wrong answer.

For many students after struggling with part (c) it appeared that they gave up on this question and left parts (d) and (e) blank.

To get a correct answer in part (d) you needed a correct answer in part(c). The more able students scored 1 mark as they were able to obtain a linear equation using the given information. Often this was for using the point (–1, 3) and a gradient of –2 and therefore arriving at $y = -2x + 1$. Only the most able students were able to obtain a second linear equation from the given information and solve simultaneously. Therefore, many students did not get full marks.

Only the most able scored marks in part (e). Some students were able to score 1 mark for using Pythagoras to find either the length of *AB* or *AD*, but only the most able made an attempt to evaluate the area. A few students used $\text{Area} = \frac{1}{2}\begin{vmatrix} a & c & e & g & a \\ b & d & f & h & b \end{vmatrix}$ to find the area of the trapezium and often this scored 2 out of 4 marks.

4PM1 02R

Question 1

The value of q was usually found correctly. Those who recognised that the quadratic factor must have no real roots were usually successful in using the discriminant to find critical values, though there were plenty of errors in trying to write down a range of values, especially giving the outside region. Many formed an equation with the discriminant, instead of an inequality, and they usually gave the two critical values as their answer, making no attempt to find a range of values. It was often thought necessary to remove brackets before inventing a method to solve the cubic equation. Some tried to apply the idea of a discriminant to the expanded cubic. This tended to develop into long working which was of no value.

Question 2

The vertical asymptote was usually used correctly to give the value of c but there was less understanding of how to use the horizontal asymptote to find the value of a. The mark in part (ii) was frequently awarded, either for the correct value of 8 or following through from an incorrect value for a. The substitution of $x = 0$ was usually made in part (iv) but there were a surprising number of incorrectly calculated fractions, especially $8-2\times 0=6$ in the numerator. Follow through marking also proved helpful in this part. A very small proportion of students were unable to reach numerical solutions and offered various algebraic rearrangements.

Question 3

This question caused considerable difficulty, though marks for the volume and differentiation were often scored. Many students realised that calculus was involved and some attempt to use the chain rule was often seen. Notation was very disordered, however, with much inconsistency in the variables. Problems started with the belief that $\frac{\mathrm{d}x}{\mathrm{d}t}=0.005$ which, if followed through correctly with an appropriate statement of the chain rule, led to the common wrong answer of 1.1 cm^3/s. Students who realised that $\frac{\mathrm{d}l}{\mathrm{d}t}$ was the correct derivative for 0.005 tended to use the label $\frac{\mathrm{d}V}{\mathrm{d}l}$ when differentiating V with respect to x, choosing a label that would fit their statement of the chain rule. Some managed to express the volume in terms of l and they often found a correctly labelled derivative, which generally led to the required answer.

Other problems included treating *x* as the radius and incorrect formulae for the volume of the cylinder. Mixed variables were sometimes left in the expression for the volume, conveniently giving an incorrect derivative, $\frac{\mathrm{d}V}{\mathrm{d}l} = \pi r^2$ from $V = \pi r^2 l$.

Question 4

Students were well prepared to answer the first two parts of this question. They usually understood that variable acceleration required the use of integration to find the velocity, though some did attempt to use $v = at$. The integration was completed well and used to find $v = -12$. The accuracy mark was sometimes lost by following this with $v = 12$ in the belief that the velocity had to be positive. For a complete solution, it was necessary to show that a constant of integration had been considered.

The value of *T* was answered well. A few students wrongly worked with the acceleration to find *T* = 2.

Finding the distance proved difficult for the majority of students. Some used $s = vt$ but most understood that they needed to integrate their velocity, which they did successfully. Definite integrals typically used limits of 0 and 8. Indefinite integrals either omitted the constant or showed that it was 0 before evaluating at *t* = 8 to give an answer of 128. The time when the velocity was zero was found in part (b) but very few students understood the significance of this, that the particle had changed direction, so very few correct answers were seen.

Question 5

The three straight lines were drawn reliably and the region *R* was usually identified clearly. The method that was invariably used to find the greatest value of *F* was to evaluate *F* at various points. The choice of points sometimes appeared to be random, usually either within *R* or on the line $2x + 3y = 24$, but most correctly chose vertices of the region. Coordinates were not always read accurately from the graph and there were also plenty of mistakes in the calculations. Despite this, many fully correct solutions were seen. Some students did not make their method clear, nor was it always obvious which points they were trying to use.

Question 6

Able students quickly found correct values for *a* and for *b* but others struggled with this part of the question. Some formed the equation $\sqrt{9-x} = p(1+qx)^{\frac{1}{2}}$ and then tried to square both sides, but mistakes in this sort of working were common. It was common to see $p = 9$ as well as mistakes such as $q = \frac{1}{9}$ or $q = -\frac{1}{3}$

The binomial expansion was applied well to gain the method mark in part (b) and the simplification of coefficients was done carefully. Those who started with the correct expression frequently scored both accurate marks. The instruction to use the result from part (a) was sometimes ignored, with attempts to expand $(9-x)^{\frac{1}{2}}$ directly.

The final part was a step too far for many, some simply substituting $\sqrt{\frac{31}{4}}$ into their expansion, but a reasonable number of solutions did identify the correct *x* value needed in part (c) and proceeded to use it correctly. Only a minority of students negotiated the whole question without any errors and they usually gave their answer correct to 5 decimal places, as required. Some students used a calculator to evaluate $\sqrt{\frac{31}{4}}$, sometimes using it wisely to check their approximation, but occasionally offering the more precise value as their answer.

Question 7

Most students scored some marks for this question. They usually scored the first mark for writing down expressions for the third and seventh terms of the geometric progression. These were usually processed correctly, occasionally finding the value of *a* before confirming the given value of *r*. There were mixed fortunes in part (b). Looking to the formulae sheet for inspiration, perhaps, some students tried to start with the summation formula for either a geometric or an arithmetic progression. There were also a few instances of using $l = ar^n$ for the n^{th} term. A small proportion of those who started with an appropriate equation or inequality made no further progress, not realising that logarithms were needed. There were many mistakes in manipulating the logarithms and inequalities, such as following $2\left(\sqrt{2}\right)^{n-1} > 500$ with $n-1 > \lg_{2\sqrt{2}} 500$, and also in evaluating the logarithms, but a respectable number of solutions did obtain n = 17 from correct working. There were a few instances where the answer was left as n > 16.9 or given as n = 16.

Greater success was achieved in part (c) with good use being made of calculators. The summation formula was used well and there was a widespread awareness of the method to rationalise the denominator. There were a few cases of answers being spoiled by lost negative signs.

Question 8

The original diagram looked quite friendly so most students started confidently. Working to find *BN* was usually correct but sometimes lacked the full detail needed to show a given result. Some methods were longer than necessary, such as finding *FN* and then using Pythagoras to complete the solution. The effort was not wasted since *FN* was needed in part (b), where answers correct to 3 significant figures were common.

The greatest difficulty in part (c) was identifying the angle to find. Those who did this correctly usually found the angle reliably, but it was not unusual to see attempts to find the angle between *ABCD* and the line joining *A* to the midpoint of *FN*. Another common mistake was 22.6° from $\tan^{-1}\left(\frac{5}{12}\right)$.

Some students gave up at this stage. Many of those who continued became confused by working that was not always labelled very clearly and by a diagram that was now complicated by the addition of many extra lines. Separate diagrams for each stage of the working were very helpful both for the students and the marker, especially when labelling was clear. A common mistake was to regard angle *AYC* as a right angle, but there were many other varied errors as students struggled to interpret the three dimensional diagram and develop a strategy for this multi-stage problem. The most likely marks were for *FY* and *EY*. More able students did manage to demonstrate correct use of the cosine rule in triangle *AYE*, even if there were errors in *AY* or *EY*. It was a strong achievement to find an answer correct to 1 decimal place.

Question 9

It was surprising how many students tried to answer this question without drawing a sketch to help them see what was required and what values would be useful. Even those who did use a diagram often failed to identify the required volume and split it into two parts. The general idea of volumes of revolution was well known; details were not. Much time was wasted working through the incorrect integrals that arose from attempts to rotate about the x axis. Correct limits were rarely used even when correct integrals were identified.

Many started by finding the intersection of the curve with the line. They found the value $x = 3$ and often used this as a limit, not realising that it was the y value that they really needed. Very few took the shorter route of treating the top section of the volume as a cone. The correct volume of this cone was rarely found by integration. Marks were frequently scored for integrating $y + 2$, but incorrect limits were common, usually ignoring the need to use -2 as the lower limit. There were many attempts to combine two functions into one integration, as is often useful in questions of this kind, but this failed to appreciate that different limits were needed for the two parts of the integration.

Question 10

Part (a) was a standard question that most students were familiar with. In Part (b), surprisingly few students took the hint and used the double angle formula in the form shown in part (a), which leads quickly to the given identity. The more common approach was to write $\cos 4A = \cos^2 2A - \sin^2 2A$ and then use the double angle formula for $\sin 2A$, eventually replacing $\sin^2 A$ by $1-\cos^2 A$. This method was longer and more prone to errors, but the success rate was still quite good.

The trigonometric equation was too difficult for less able students, though many did make muddled attempts. Those who tried to use the result from part (b) had mixed success, often making mistakes with signs and constants. The more able students usually obtained an equation in the form $\cos 4A = \frac{1}{2}$ and went on to find both values of θ. A more common approach was to treat the equation as a quadratic in $\cos^2\left(\frac{\theta}{4}+\frac{\pi}{24}\right)$. This was solved very well, usually giving angles in radians as decimals at first, before contriving to express them as multiples of π.The main problem with this approach was losing sight of the fact that $\frac{2\pm\sqrt{3}}{4}$ was the value of $\cos^2 A$ and not $\cos A$. An occasional mistake with both methods was to find the first value of θ before looking for other values, then assuming that $2\pi-\theta$ was a second value.

Some students made fruitless attempts to integrate f(*A*) in the given form. Others recognised the similarity of f(*A*) with the $\cos 4A$ formula established in part (b) and tried to use it. Errors were not uncommon in signs and constants, but there was reasonable success in transforming the integral and then completing it correctly. For anyone who got this far, the limits did not pose a problem and an answer was given in the required form.

Question 11

The final question was answered well. The value of *q* provided a straightforward first mark, though a few thought the value was negative. Students were generally familiar with the technique of replacing $\alpha^2+\beta^2$ with $(\alpha+\beta)^2-2\alpha\beta$ so they were usually able to obtain an equation involving k and p^2 or $(\alpha+\beta)^2$. Mistakes were frequent in processing this to find an expression for *p*. These included incorrect signs, losing the power of *p*, and taking square roots term by term $\left(\sqrt{\frac{k^2}{4}-\frac{3k}{2}+\frac{9}{4}}=\frac{k}{2}-\sqrt{\frac{3k}{2}}+\frac{3}{2}\right)$. Very few students fully simplified their expression. Though they still scored full marks in part (a), this did tend to complicate working in part (b), where it was not unusual to see a correct method but an incorrect value for *k*, or a failure to exclude $k=-4$ when this was also found from solving a quadratic equation.

Even those who struggled in the first two parts tended to make progress in part (c), though earlier mistakes sometimes made working more difficult. The product of roots was often successful, but some forgot to square the value of $(\alpha+\beta)$ in the denominator, whilst others tried to evaluate it as $(\alpha^2+\beta^2)$. The sum of roots was often less successful. Many students failed to use the existing common denominator to add the fractions, preferring to make it $(\alpha+\beta)^2$ and then getting lost with the algebra. Whatever values were found for the sum and product of roots, they were usually used correctly to give an expression, and most remembered to try to clear the fractions and give the answer with integer coefficients, not forgetting to complete the equation by putting their expression equal to zero.

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