Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel International GCSE

Centre Number	Candidate Number

Monday 17 June 2019

Afternoon (Time: 2 hours)	Paper Reference **4PM1/01**

Further Pure Mathematics
Paper 1

Calculators may be used.	Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page. Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over

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Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration
Surface area of sphere $= 4\pi r^2$ **Curved surface area of cone** $= \pi r \times$ slant height **Volume of sphere** $= \frac{4}{3}\pi r^3$
Series
Arithmetic series Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$ **Geometric series** Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$ Sum to infinity, $S_\infty = \frac{a}{1-r} \quad \lvert r \rvert < 1$ **Binomial series** $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots$ for $\lvert x \rvert < 1, n \in \mathbb{Q}$
Calculus
Quotient rule (differentiation) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$
Trigonometry
Cosine rule In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$ $\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\sin(A+B) = \sin A\cos B + \cos A\sin B$ $\sin(A-B) = \sin A\cos B - \cos A\sin B$ $\cos(A+B) = \cos A\cos B - \sin A\sin B$ $\cos(A-B) = \cos A\cos B + \sin A\sin B$ $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$ $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$
Logarithms
$\log_a x = \frac{\log_b x}{\log_b a}$

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Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 $$f(x) = x^3 + 2x^2 - 5x - 6$$

(a) Factorise $x^2 - x - 2$

(1)

(b) Hence, or otherwise, show that $(x^2 - x - 2)$ is a factor of $f(x)$.

(3)

(Total for Question 1 is 4 marks)

2 Given that $\dfrac{4+2\sqrt{3}}{5-2\sqrt{3}}$ can be written in the form $\dfrac{a+b\sqrt{3}}{c}$ where a and b are integers and c is prime, find the value of a, the value of b and the value of c.

Show your working clearly.

(3)

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Question 2 continued

(Total for Question 2 is 3 marks)

3 In triangle ABC, $AC = 7$ cm, $BC = 10$ cm and angle $BAC = 65°$

(a) Find, to the nearest 0.1°, the size of angle ABC.

(3)

(b) Find, in cm^2 to 3 significant figures, the area of triangle ABC.

(3)

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Question 3 continued

(Total for Question 3 is 6 marks)

4

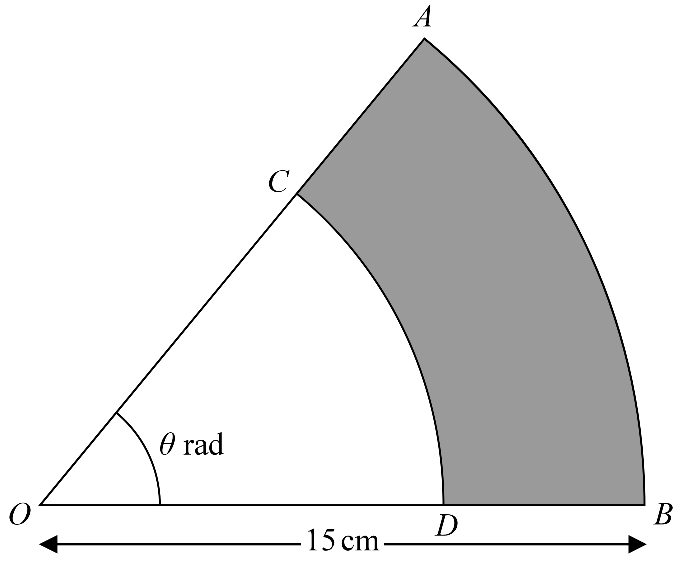

Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows a sector OAB of a circle where angle $AOB = \theta$ radians. The circle has centre O and radius 15 cm. The point C divides OA in the ratio 2 : 1 and the point D divides OB in the ratio 2 : 1

The area of the region $ABDC$, shown shaded in Figure 1, is 100 cm^2

Find

(a) the value of θ, **(3)**

(b) the perimeter of the region $ABDC$. **(3)**

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Question 4 continued

(Total for Question 4 is 6 marks)

5 $$f(x) = 3x^2 - 9x + 5$$

Given that $f(x)$ can be written in the form $a(x - b)^2 + c$, where a, b and c are constants,

find

(a) the value of a, the value of b and the value of c.

(3)

(b) Hence write down

(i) the minimum value of $f(x)$,

(ii) the value of x at which this minimum occurs.

(2)

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Question 5 continued

(Total for Question 5 is 5 marks)

6

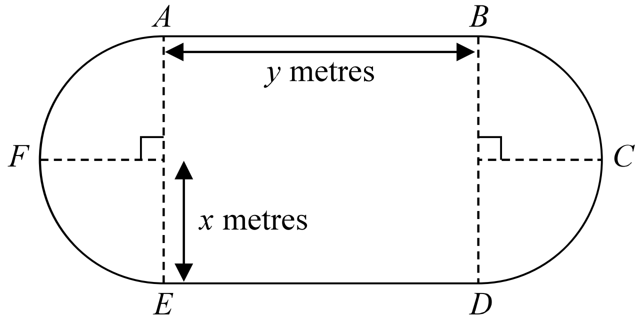

Diagram **NOT** accurately drawn

Figure 2

Figure 2 shows a lawn $ABCDEF$, where $ABDE$ is a rectangle of length y metres and width $2x$ metres. Each end of the lawn is a semicircle of radius x metres. The lawn has perimeter 90 m and area S m^2

(a) Show that S can be written in the form

$$S = kx - \pi x^2$$

where k is a constant.

State the value of k.

(4)

(b) Use calculus to find, to 4 significant figures, the value of x for which S is a maximum, justifying that this value of x gives a maximum value of S.

(5)

(c) Find, to the nearest whole number, the maximum value of S.

(2)

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Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 11 marks)

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7 (a) Solve, in degrees to one decimal place,

$$(3\cos\theta + 5)(5\sin\theta - 3) = 0 \qquad \text{for } 0 \leqslant \theta < 180°$$

(2)

(b) Show that the equation

$$8\sin(x - \alpha) = 3\sin(x + \alpha)$$

can be written in the form

$$5\tan x = 11\tan\alpha$$

(5)

(c) Hence solve, to one decimal place,

$$8\sin(2y - 30°) = 3\sin(2y + 30°) \qquad \text{for } 0 \leqslant y < 180°$$

(5)

Question 7 continued

Question 7 continued

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Question 7 continued

(Total for Question 7 is 12 marks)

8 (a) Solve $5p^2 - 9p + 4 = 0$

(2)

(b) Hence solve $5^{2x+1} - 9(5^x) + 4 = 0$

Give your answers to 3 significant figures where appropriate.

(4)

The curve with equation $y = 5^{2x+1} + 5^x$ intersects the curve with equation $y = 2(5^{x+1}) - 4$ at two points.

(c) Find the coordinates of each of these two points.
Give your answers to 3 significant figures where appropriate.

(4)

P 5 8 2 8 6 A 0 2 0 3 6

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 10 marks)

9 (a) Solve the equation $2\log_p 9 + 3\log_3 p = 8$

(6)

Given that $\log_2 3 = \log_4 3^k$

(b) find the value of k

(2)

(c) Show that

$$6x\log_4 x - 3x\log_2 3 - 5\log_4 x + 10\log_2 3 = \log_4\left(\frac{x^{6x-5}}{3^{6x-20}}\right)$$

(4)

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Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 12 marks)

10 (a) Expand $\left(1+2x^2\right)^{-\frac{1}{3}}$ in ascending powers of x up to and including the term in x^6, expressing each coefficient as an exact fraction in its lowest terms.

(3)

(b) State the range of values of x for which your expansion is valid.

(1)

$$\mathrm{f}(x) = \frac{2 + kx^2}{(1+2x^2)^{\frac{1}{3}}} \qquad \text{where } k \neq 0$$

(c) Obtain a series expansion for $\mathrm{f}(x)$ in ascending powers of x up to and including the term in x^6

Give each coefficient in terms of k where appropriate.

(3)

Given that the coefficient of x^4 in the series expansion of $\mathrm{f}(x)$ is zero

(d) find the value of k.

(2)

(e) Hence use algebraic integration to obtain an estimate, to 4 decimal places, of

$$\int_0^{0.5} \mathrm{f}(x)\,\mathrm{d}x$$

(5)

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 14 marks)

11 The curve C has equation $3y = x^2 + 2$

The point P lies on C and has x coordinate 4

The line k is the tangent to C at P.

(a) Find an equation for k, giving your answer in the form $ay = bx + c$ where a, b and c are integers.

(6)

The line l is the normal to C at P.

(b) Find an equation for l, giving your answer in the form $dy = ex + f$ where d, e and f are integers.

(2)

(c) Find the area of the triangle bounded by the line k, the line l and the x-axis.

(3)

The finite region bounded by C, the line l, the x-axis and the y-axis is rotated through 360° about the x-axis.

(d) Use algebraic integration to find, to the nearest whole number, the volume of the solid generated.

(6)

Question 11 continued

Question 11 continued

Question 11 continued

Question 11 continued

(Total for Question 11 is 17 marks)

TOTAL FOR PAPER IS 100 MARKS