Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel International GCSE

Centre Number | Candidate Number

Monday 17 June 2019

Afternoon (Time: 2 hours) | Paper Reference **4PM1/01R**

Further Pure Mathematics

Paper 1R

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page. Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

1/1/1/1/1/

Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere $= 4\pi r^2$

Curved surface area of cone $= \pi r \times$ slant height

Volume of sphere $= \frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

Logarithms

$\log_a x = \frac{\log_b x}{\log_b a}$

Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1

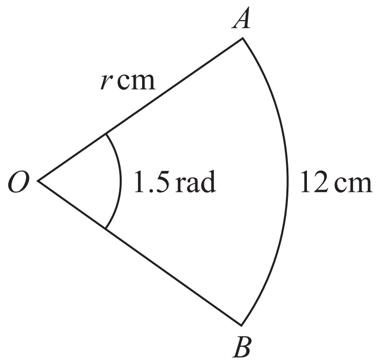

Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows sector *AOB* of a circle with centre *O* and radius *r* cm. The angle *AOB* is 1.5 radians and the length of arc *AB* is 12 cm.

Calculate

(a) the value of *r*,

(1)

(b) the area of the sector *AOB*.

(2)

(Total for Question 1 is 3 marks)

2

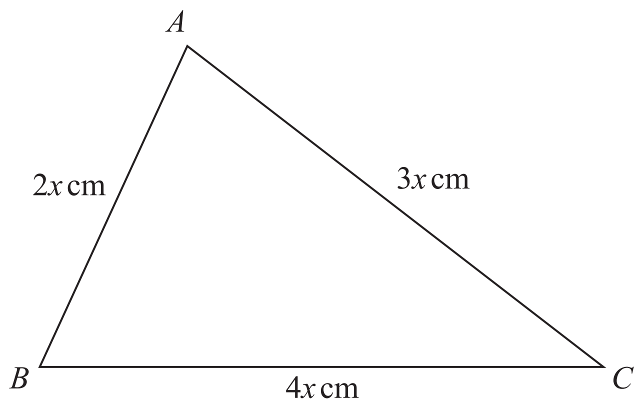

Diagram **NOT** accurately drawn

Figure 2

Figure 2 shows triangle ABC in which

$$AB = 2x \text{ cm} \qquad AC = 3x \text{ cm} \qquad BC = 4x \text{ cm}$$

(a) Show that $\sin ABC = \dfrac{3\sqrt{15}}{16}$

(4)

Given that the area of triangle ABC is $\dfrac{75\sqrt{15}}{64}$ cm^2

(b) find the value of x.

(2)

Question 2 continued

(Total for Question 2 is 6 marks)

3 (a) Write down the value of $\log_3 9$

(1)

(b) Solve the equation $\log_3 9t = \log_9 \left(\frac{12}{t}\right)^2 + 2$ where $t > 0$

Give your answer in the form $a\sqrt{b}$ where a and b are prime numbers.

(6)

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Question 3 continued

(Total for Question 3 is 7 marks)

P 6 1 8 8 2 A 0 7 3 6

4

$$f(x) = e^{3x}\sqrt{1+2x}$$

(a) Show that $$f'(x) = \frac{2e^{3x}(2+3x)}{\sqrt{1+2x}}$$

(4)

(b) Find an equation of the normal to the curve with equation $y = f(x)$ at the point on the curve where $x = 0$

Give your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(6)

Question 4 continued

(Total for Question 4 is 10 marks)

5 A circle has radius $3r$ cm and area A cm^2

Given that the value of r increases by 0.05%

use calculus to find an estimate for the percentage increase in the value of A.

(5)

Question 5 continued

(Total for Question 5 is 5 marks)

6 (a) Show that $\sum_{r=1}^{n}(4r-3) = n(2n-1)$

(3)

(b) Hence, or otherwise, find the least value of n such that $\sum_{r=1}^{n}(4r-3) > 1000$

(3)

Given that $S_n = n(2n-1)$, $t_n = (4n-3)$ and that $18 + 3t_{n+7} = S_{n+4}$

(c) find the value of n.

(4)

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Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 10 marks)

7 O, A, B and C are fixed points such that

$$\overrightarrow{OA} = 8\mathbf{i} - 6\mathbf{j} \qquad \overrightarrow{OB} = 15\mathbf{i} - 6\mathbf{j} \qquad \overrightarrow{OC} = 8\mathbf{i} + \mathbf{j}$$

(a) Find $\overrightarrow{BC}$ as a simplified expression in terms of **i** and **j**

(2)

(b) Find a unit vector parallel to $\overrightarrow{BC}$

(2)

The point M is the midpoint of OA and the point N lies on OB such that $ON:NB = 1:2$

(c) Show that the points M, N and C are collinear.

(4)

Question 7 continued

Question 7 continued

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Question 7 continued

(Total for Question 7 is 8 marks)

8 (a) Complete the table of values for $y = 2 + \ln(2x + 1)$ giving your answers to 2 decimal places.

(2)

x	0	0.25	0.5	1	1.5	2	3
y	2			3.10	3.39	3.61	

(b) On the grid opposite, draw the graph of $y = 2 + \ln(2x + 1)$ for $0 \leqslant x \leqslant 3$

(2)

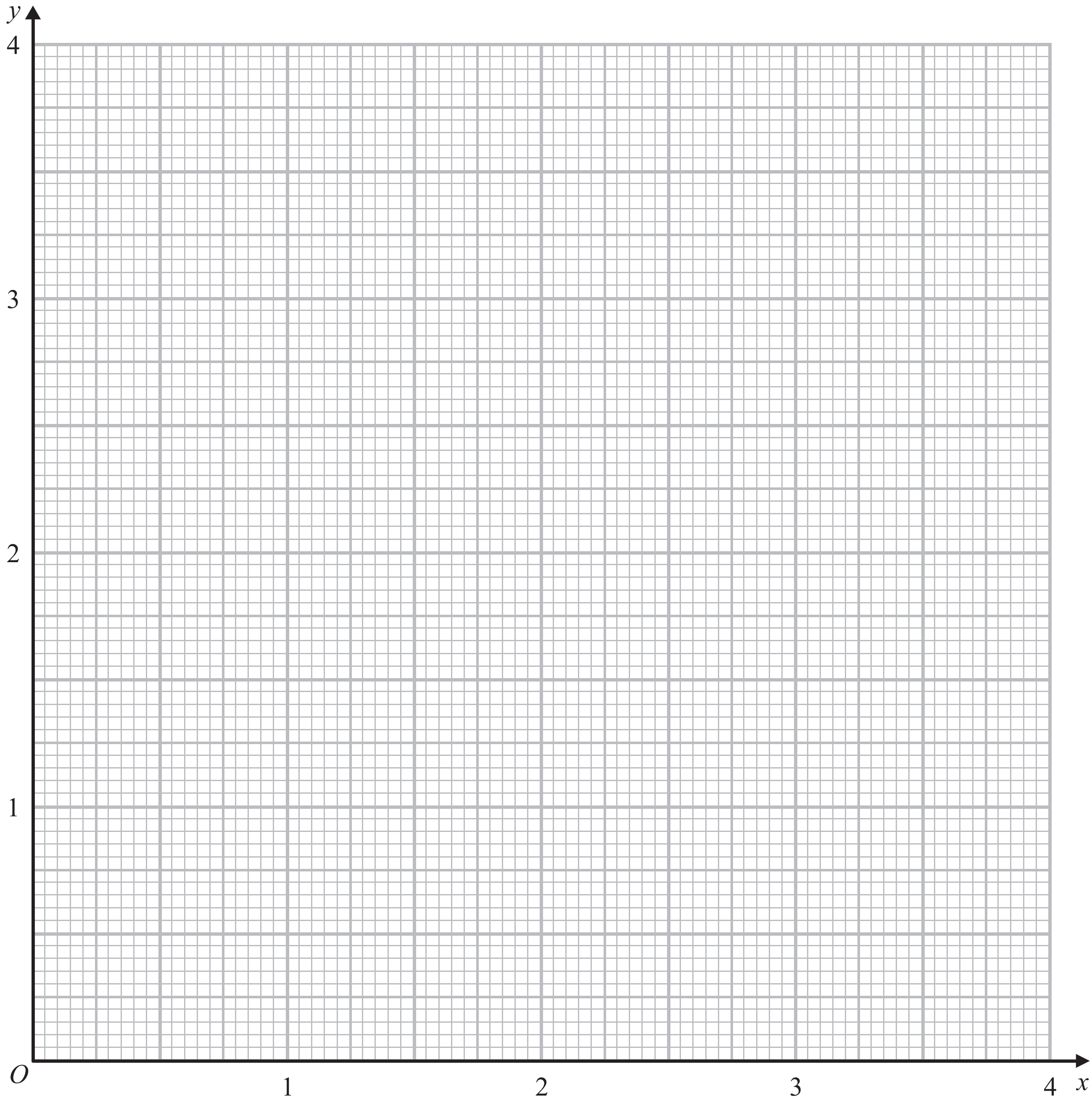
(c) By drawing an appropriate straight line on the grid, obtain an estimate, to one decimal place, of the root of the equation $\ln(2x + 1) = 3x - 4$ in the interval $0 \leqslant x \leqslant 3$

(3)

(d) By drawing an appropriate straight line on the grid, obtain an estimate, to one decimal place, of the root of the equation $e^{(6-x)} - (2x + 1)^2 = 0$ in the interval $0 \leqslant x \leqslant 3$

(4)

Question 8 continued

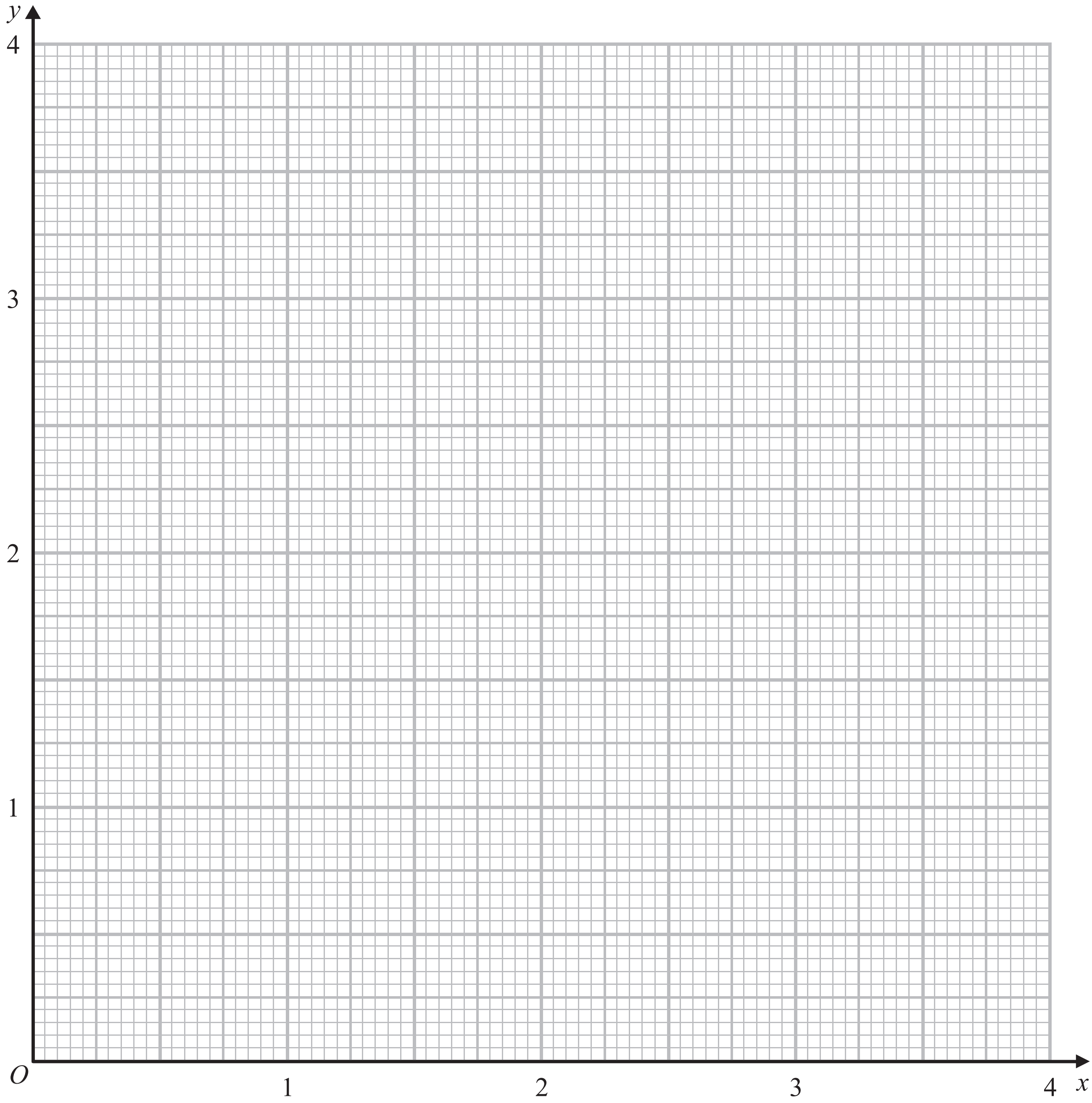


Turn over for a spare grid if you need to redraw your graph.

Question 8 continued

Question 8 continued

Only use this grid if you need to redraw your graph.



(Total for Question 8 is 11 marks)

9

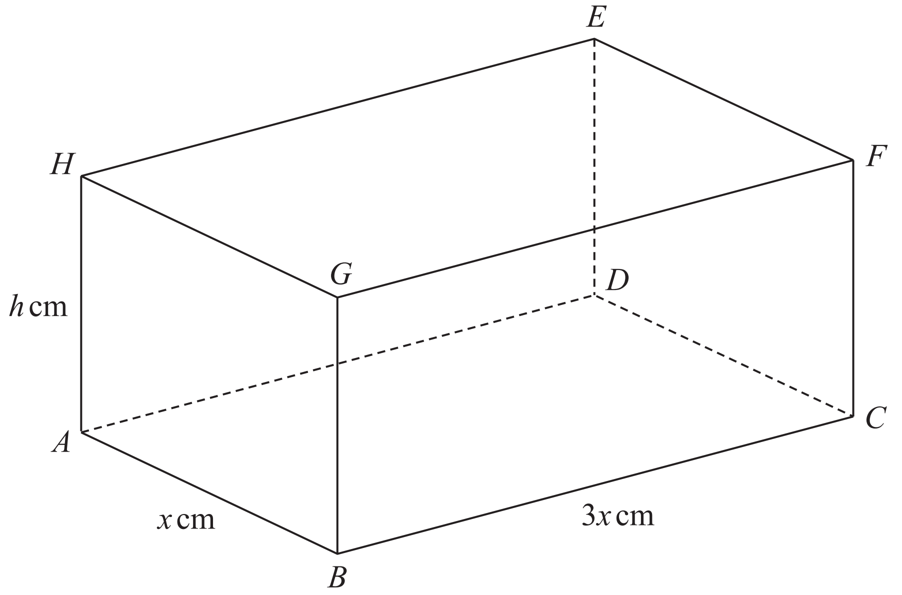


Figure 3

Figure 3 shows a solid cuboid *ABCDEFGH*

$AB = x\,\text{cm} \qquad BC = 3x\,\text{cm} \qquad AH = h\,\text{cm}$

The volume of the cuboid is $540\,\text{cm}^3$

The total surface area of the cuboid is $S\,\text{cm}^2$

(a) Show that $S = 6x^2 + \dfrac{1440}{x}$

(4)

Given that x can vary,

(b) use calculus to find, to 3 significant figures, the value of x for which S is a minimum.
Justify that this value of x gives a minimum value of S.

(5)

(c) Find, to 3 significant figures, the minimum value of S.

(1)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 10 marks)

10 $\mathrm{f}(x) = 6x - x^2 \qquad x \in \mathbb{R}$

Given that $\mathrm{f}(x)$ can be written in the form $D(x + E)^2 + F$ where D, E and F are integers,

(a) find the value of D, the value of E and the value of F.

(3)

(b) Find

(i) the maximum value of $\mathrm{f}(x)$,

(ii) the value of x for which the maximum occurs.

(2)

The curve C has equation $y = \mathrm{f}(x)$

The curve S has equation $y = x^2 - 4x + 8$

The curve S intersects the curve C at two points.

(c) Find the coordinates of each of these two points.

(4)

The finite region R is bounded by the curve C and the curve S.

(d) Use algebraic integration to find the area of R.

(4)

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 13 marks)

11 The points A and B have coordinates $(-1, 3)$ and $(5, 6)$ respectively.

(a) Find an equation for the line AB.

(2)

The point P divides AB in the ratio $2:1$

(b) Show that the coordinates of P are $(3, 5)$

(2)

The point C with coordinates (m, n), where $m > 0$, is such that CP is perpendicular to the line AB.

Given that the radius of the circle which passes through A, P and C is 5

(c) find the value of m and the value of n.

(6)

The point D with coordinates (p, q) is such that the line AD is perpendicular to the line AB and the line DC is parallel to the line AB.

(d) Find the value of p and the value of q.

(3)

(e) Find the area of trapezium $ABCD$.

(4)

Question 11 continued

Question 11 continued

Question 11 continued

Question 11 continued

(Total for Question 11 is 17 marks)

TOTAL FOR PAPER IS 100 MARKS