Write your name here


| Further Pure Mathematics |
| :--- | :--- |
| Paper 1 |$\quad$| Paper Reference |
| :--- |
| 4PM0/01 |
| Tuesday 14 June 2016 - Morning <br> Time: 2 hours |

Calculators may be used.
Total Marks

## Instructions

- Use black ink or ball-point pen.
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
- there may be more space than you need.


## Information

- The total mark for this paper is 100.
- The marks for each question are shown in brackets - use this as a guide as to how much time to spend on each question.


## Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.


## Answer all TEN questions.

## Write your answers in the spaces provided.

## You must write down all the stages in your working.

1

$$
\mathrm{f}(x)=x^{3}-7 x+6
$$

(a) Show that $(x-2)$ is a factor of $\mathrm{f}(x)$
(b) Hence, or otherwise, factorise $\mathrm{f}(x)$ completely.

Question 1 continued

2 (a) Expand $\left(1+3 x^{2}\right)^{-\frac{1}{3}}, 3 x^{2}<1$, in ascending powers of $x$, up to and including the term in $x^{6}$, simplifying each term as far as possible.

$$
\mathrm{f}(x)=\frac{1-k x^{2}}{\left(1+3 x^{2}\right)^{\frac{1}{3}}} \text { where } k \text { is a constant }
$$

(b) Obtain a series expansion for $\mathrm{f}(x)$ in ascending powers of $x$ up to and including the term in $x^{4}$.

Given that the coefficient of $x^{2}$ in the expansion of $\mathrm{f}(x)$ is -5
(c) find the value of $k$.

Question 2 continued

3 A right pyramid $A B C D E$ has a square base $A B C D$ of side 10 cm . The height of the pyramid is 8 cm .
(a) Find, to 3 significant figures, the length of $A E$.
(b) Find, in degrees to the nearest degree, the size of the angle between the plane $A B E$ and the base $A B C D$.

Question 3 continued

4 The $n$th term of an arithmetic series is $t_{n}$ and the sum of the first $n$ terms of the series is $S_{n}$
Given that $S_{2}=\frac{2}{3} t_{5}$ and that $S_{4}=t_{10}+3$
(a) find
(i) the common difference of the series,
(ii) the first term of the series.

Given also that $S_{p+2}-S_{p}=110$
(b) find the value of $p$.

Question 4 continued

5 Using the identities

$$
\begin{gathered}
\sin (A+B)=\sin A \cos B+\cos A \sin B \\
\tan A=\frac{\sin A}{\cos A}
\end{gathered}
$$

(a) show that the equation

$$
3 \sin (x+\alpha)=5 \sin (x-\alpha)
$$

can be written in the form $\tan x=4 \tan \alpha$
(b) Hence solve, to the nearest integer, the equation

$$
\begin{equation*}
3 \sin (2 y+30)^{\circ}=5 \sin (2 y-30)^{\circ} \quad \text { for } 90 \leqslant y<180 \tag{4}
\end{equation*}
$$

Question 5 continued

Question 5 continued

Question 5 continued

6 Solve
(a) $\log _{x} 1024=5$
(b) $\log _{3}(7 y-3)=4$
(c) $\log _{a} 25+2 \log _{a} 625=10$
(d) $\log _{b} 7-2 \log _{7} b+1=0$

Question 6 continued

Question 6 continued

Question 6 continued

7 (a) Complete the table of values for $y=2^{x}-4$, giving your answers to 2 decimal places.

| $x$ | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 2.75 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | -3 |  | -2 |  | 0 |  | 2.73 | 4 |

(b) On the grid opposite, draw the graph of $y=2^{x}-4$ for $0 \leqslant x \leqslant 3$
(c) Use your graph to obtain an estimate, to one decimal place, of the value of $\log _{2} 7$ Show clearly how you used the graph.
(d) By drawing a straight line on your graph, obtain an estimate to one decimal place of the root of the equation $2^{x}+3 x=7$ in the interval $0 \leqslant x \leqslant 3$

## Question 7 continued



Turn over for a spare grid if you need to redraw your graph.

Question 7 continued

Question 7 continued
Only use this grid if you need to redraw your graph

(Total for Question 7 is 11 marks)


Figure 1
In Figure $1, \overrightarrow{O A}=\mathbf{a}, \overrightarrow{O B}=\mathbf{b}$ and $\overrightarrow{O D}=\frac{2}{3} \mathbf{b}$
The point $E$ divides $A D$ in the ratio $2: 3$
(a) Find as simplified expressions in terms of $\mathbf{a}$ and $\mathbf{b}$
(i) $\overrightarrow{A D}$
(ii) $\overrightarrow{O E}$
(iii) $\overrightarrow{B E}$

The point $F$ lies on $O A$ such that $\overrightarrow{O F}=\lambda \overrightarrow{O A}$ and $F, E$ and $B$ are collinear.
(b) Find the value of $\lambda$.

The area of triangle $O F B$ is 5 square units.
(c) Find the area of triangle $O A D$.

Give your answer in the form $\frac{p}{q}$, where $p$ and $q$ are integers.

Question 8 continued

Question 8 continued

Question 8 continued

$$
f(x)=3 x^{2}-5 x-4
$$

The roots of the equation $\mathrm{f}(x)=0$ are $\alpha$ and $\beta$
(a) Without solving the equation $\mathrm{f}(x)=0$, form an equation, with integer coefficients, which has
(i) roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$
(ii) roots $2 \alpha+\beta$ and $\alpha+2 \beta$
(b) Express $\mathrm{f}(x)$ in the form $A(x+B)^{2}+C$, stating the values of the constants $A, B$ and $C$.
(c) Hence, or otherwise, show that the equation $\mathrm{f}(x)=-8$ has no real roots.

Question 9 continued

Question 9 continued

Question 9 continued

10 The points $A$ and $B$ have coordinates $(2,4)$ and $(5,-2)$ respectively. The point $C$ divides $A B$ in the ratio 1:2
(a) Find the coordinates of $C$.

The point $D$ has coordinates $(1,1)$
(b) Show that $D C$ is perpendicular to $A B$.
(c) Find the equation of $D C$ in the form $p y=x+q$

The point $E$ is such that $D C E$ is a straight line and $D C=C E$.
(d) Find the coordinates of $E$.
(e) Calculate the area of quadrilateral $A D B E$.

Question 10 continued

Question 10 continued

