Cambridge IGCSE

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Cambridge International General Certificate of Secondary Education

ADDITIONAL MATHEMATICS **0606/13**

Paper 1 **October/November 2018**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2018 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1(a)	${}^5C_3 \times 2^2 \times (px)^3$	**B1**	
	$40p^3 = -\frac{8}{25}$ $p^3 = -\frac{8}{1000}$	**M1**	equating *their* coefficient of x^3 to $-\frac{8}{25}$ and finding p^3
	$p = -\frac{1}{5}$ or $p = -0.2$	**A1**	
1(b)	${}^8C_4 \times (2x^2)^4 \times \left(\frac{1}{4x^2}\right)^4$	**B1**	
	$70 \times 16 \times \frac{1}{256}$	**M1**	at least two of $70, 16, \frac{1}{256}$ correct in an evaluation of a three-term product
	$\frac{35}{8}$, 4.375, $4\frac{3}{8}$	**A1**	cao
2(i)	$\theta = \frac{20-2r}{r}$	**B1**	
	Area = $\frac{1}{2}r^2\left(\frac{20-2r}{r}\right)$	**M1**	use of *their* θ in terms of r in formula for sector area
	$A = 10r - r^2$	**A1**	simplification to get given answer
	Alternative		
	$s = 20 - 2r$	**B1**	
	$= \frac{1}{2}r(20-2r)$	**M1**	use of formula for sector area using *their* expression for s in terms of r
	$A = 10r - r^2$	**A1**	simplification to get given answer
2(ii)	$\frac{dA}{dr} = 10 - 2r$ When $\frac{dA}{dr} = 0$, $r = 5$	**M1**	for $\frac{dA}{dr} = 10 - kr$, equating to zero and solving for r
	$\theta = \frac{(20-2\times5)}{5}$	**M1**	**Dep** substitution of *their* value of r to get θ
	$\theta = 2$	**A1**	

Question	Answer	Marks	Guidance
3(i)	$AC^2 = \left(5\sqrt{3}+5\right)^2 + \left(5\sqrt{3}-5\right)^2$	**M1**	correct use of Pythagoras or correct use of cosine rule with cos90
	$= 75+25+50\sqrt{3}+75+25-50\sqrt{3}$ $= 200$	**M1**	correct expansion to 6 or 8 terms
	$AC = 10\sqrt{2}$	**A1**	from $AC^2 = 200$
3(ii)	$\tan BCA = \dfrac{5\sqrt{3}+5}{5\sqrt{3}-5}$ oe	**B1**	
	$= \dfrac{\left(5\sqrt{3}+5\right)\left(5\sqrt{3}+5\right)}{\left(5\sqrt{3}-5\right)\left(5\sqrt{3}+5\right)}$ oe $= \dfrac{100+50\sqrt{3}}{50}$ oe	**M1**	for rationalisation
	$= 2+\sqrt{3}$	**A1**	
4(i)		**M1**	for $10\left(1+\cos 3x\right)^9$ f(x)
		M1	for $k\sin 3x\left(1+\cos 3x\right)^9$
	$\dfrac{dy}{dx} = -30\sin 3x\left(1+\cos 3x\right)^9$	**A1**	
	When $x = \dfrac{\pi}{2}$, $\dfrac{dy}{dx} = 30$	**A1**	
4(ii)	Use of $\dfrac{dy}{dx} \times \dfrac{dx}{dt} = \dfrac{dy}{dt}$ with $\dfrac{dy}{dt} = 6$	**M1**	*their* $\dfrac{dy}{dx} \times \dfrac{dx}{dt} = 6$
	$\dfrac{dx}{dt} = \dfrac{1}{5}$ or 0.2	**A1**	**FT** from *their* answer from part **(i)**

Question	Answer	Marks	Guidance
5(i)	$\log_9 4 = \dfrac{\log_3 4}{\log_3 9}$	**B1**	change of base
	$= \dfrac{1}{2}\log_3 4$ $= \dfrac{1}{2}\log_3 2^2$ or $\log_3 \sqrt{4}$ $= \log_3 2$	**B1**	**Dep** must have B1 for change of base and full working
	Alternative A		
	$\log_9 4 = 2\log_9 2$	**B1**	use of power rule
	$= \dfrac{2\log_3 2}{\log_3 9}$ $= \dfrac{2\log_3 2}{2\log_3 3}$ $= \log_3 2$	**B1**	**Dep** change of base and full working
	Alternative B		
	$x = \log_9 4 \Rightarrow 9^x = 4$ $9^x = 4 \Rightarrow 3^{2x} = 4$	**B1**	correct use of indices to reach $3^{2x} = 4$
	$\Rightarrow 3^x = 2 \Rightarrow x = \log_3 2$ $\therefore \log_9 4 = \log_3 2$	**B1**	**Dep** full working
	Alternative C		
	$\log_9 4 = \dfrac{\log_{10} 4}{\log_{10} 9}$ $= \dfrac{2\log_{10} 2}{2\log_{10} 3}$	**B1**	change of base and use of power rule
	$= \log_3 2$	**B1**	**Dep** change of base and full working

Question	Answer	Marks	Guidance
5(ii)	$\log_3 2 + \log_3 x = 3$ $\log_3 2x = 3$	B1	for $\log_3 2x = 3$
	$3^3 = 2x$	B1	
	$x = 13.5,\ \ x = \frac{27}{2}$	B1	
	Alternative		
	$\log_3 x = \log_3 27 - \log_3 2$	B1	
	$= \log_3 \frac{27}{2}$	B1	
	$x = 13.5,\ \ x = \frac{27}{2}$	B1	
6(i)	$\frac{\mathrm{d}s}{\mathrm{d}t} = -6\mathrm{e}^{-0.5t} + 4$	M1	for $k\mathrm{e}^{-0.5t} + 4$
	When $\frac{\mathrm{d}s}{\mathrm{d}t} = 0,\ \mathrm{e}^{-0.5t} = \frac{2}{3}$ $-0.5t = \ln\frac{2}{3}$ $t = -2\ln\frac{2}{3}$	M1	**Dep** equating to zero and correct order of operations to solve for t
	$t = 0.811$	A1	
6(ii)		M1	for $k\mathrm{e}^{-0.5t}$
	$\frac{\mathrm{d}^2 s}{\mathrm{d}t^2} = 3\mathrm{e}^{-0.5t}$	A1	
6(iii)	$3\mathrm{e}^{-0.5t} = 0.3$ $\mathrm{e}^{-0.5t} = 0.1$ $t = \frac{\ln 0.1}{-0.5}$	M1	correct order of operations and correct use of ln to solve $k\mathrm{e}^{-0.5t} = 0.3$ for t
	$s = 12\mathrm{e}^{-0.5\times 4.605} + 4\times 4.605 - 12$	M1	**Dep** use of t to obtain s
	$s = 7.62$	A1	
6(iv)	$\mathrm{e}^{-0.5t}$ is always positive or $\mathrm{e}^{-0.5t}$ can never be zero or negative	B1	correct comment about $\mathrm{e}^{-0.5t}$

Question	Answer	Marks	Guidance
7(i)	$\overrightarrow{AD} = m(\mathbf{c}-\mathbf{a})$	**B1**	
7(ii)	$\overrightarrow{AD} = \overrightarrow{OD} - \mathbf{a}$	**B1**	for $\overrightarrow{OD} = \frac{2}{3}\mathbf{b}$
	$= \frac{2}{3}\mathbf{b} - \mathbf{a}$	**B1**	**FT** *their* $\overrightarrow{OD}$ if $\overrightarrow{OD} = k\mathbf{b}$
7(iii)	$m(\mathbf{c}-\mathbf{a}) = \frac{2}{3}\mathbf{b} - \mathbf{a}$	**M1**	equating parts **(i)** and **(ii)**
	$24\mathbf{a}(1-m) + 24m\mathbf{c} = 16\mathbf{b}$ Comparing with $15\mathbf{a} + 9\mathbf{c} = 16\mathbf{b}$	**M1**	attempt to eliminate or compare like vectors using given condition
	$m = \frac{3}{8}$	**A1**	
8(i)	$5 \leqslant f(x) \leqslant 6$ or $[5,6]$ oe	**B2**	**B1** for $5 \leqslant f(x) \leqslant p$ ($p > 5$) or for $q \leqslant f(x) \leqslant 6$ ($q < 6$)
8(ii)	$x = \sin\frac{y}{4} + 5$	**M1**	complete valid attempt to obtain the inverse with operations in correct order.
	$y = 4\sin^{-1}(x-5)$	**A1**	
	Range $0 \leqslant y \leqslant 2\pi$	**B1**	
8(iii)	$2\left(\sin\frac{\left(x-\frac{\pi}{3}\right)}{4} + 5\right)$ $(=11)$	**B1**	for $\sin\frac{\left(x-\frac{\pi}{3}\right)}{4} + 5$
	$\sin\frac{\left(x-\frac{\pi}{3}\right)}{4} = \frac{1}{2}$	**M1**	for $\sin\frac{\left(x-\frac{\pi}{3}\right)}{4} = k$
	$x = 4\sin^{-1}\left(\frac{1}{2}\right) + \frac{\pi}{3}$ oe	**M1**	**Dep** for use of $\sin^{-1}$ and correct order of operations to obtain x. Allow one +/– or $\times/\div$ sign error
	$x = \pi$ or 3.14	**A1**	$x = \pi$ and no other solutions in range

Question	Answer	Marks	Guidance
9	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln\left(3x^2+1\right)\right)=\frac{6x}{3x^2+1}$	**B1**	for $\frac{6x}{3x^2+1}$
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{x^2\frac{6x}{3x^2+1}-2x\ln\left(3x^2+1\right)}{x^4}$ or $\frac{\mathrm{d}y}{\mathrm{d}x}=\left(\frac{-2}{x^3}\right)\ln\left(3x^2+1\right)+\left(\frac{1}{x^2}\right)\frac{6x}{\left(3x^2+1\right)}$	**M1**	differentiation of a quotient or product
	$\frac{x^2\,\mathrm{f}(x)-2x\ln\left(3x^2+1\right)}{x^4}$ or for $\left(-\frac{2}{x^3}\right)\ln\left(3x^2+1\right)+\left(\frac{1}{x^2}\right)\mathrm{f}(x)$	**A1**	
	When $x=2$, $\frac{\mathrm{d}y}{\mathrm{d}x}=-0.410$	**A1**	
	Gradient of perp = 2.436…	**M1**	use of $-\frac{1}{m}$ with a gradient obtained by differentiation
	When $x=2,\ y=0.641$ or $\frac{1}{4}\ln 13$	**B1**	
	Normal: $y-0.641=2.436\left(x-2\right)$	**M1**	**Dep**
	$y=2.44x-4.23$	**A1**	
10(i)	$x+8=12+x-x^2$ $x^2=4$, $x=\pm 2$ or $y^2-16y+60=0$ $y=6$ or $y=10$	**M1**	correct method of solution to obtain x or y
	$x=2,\ y=10$ $x=-2,\ y=6$	**A2**	**A1** for $x=-2$ and $x=2$ or for $y=6$ and $y=10$ or for either point from a correctly solved equation.
10(ii)		**M1**	for $12x+px^2+qx^3\quad(+c)$
	$12x+\frac{x^2}{2}-\frac{x^3}{3}\quad(+c)$	**A1**	

Question	Answer	Marks	Guidance
10(iii)	$\left[12x+\frac{x^2}{2}-\frac{x^3}{3}\right]_{-2}^{2}-\left(\frac{1}{2}(6+10)\times 4\right)$	B1	**FT** area of the trapezium unsimplified $\left(\frac{1}{2}(6+10)\times 4\right)$ or $\left[\frac{2^2}{2}+8\times 2\right]-\left[\frac{(-2)^2}{2}+8\times(-2)\right]$ (= 32)
	$\left[12\times 2+\frac{2^2}{2}-\frac{2^3}{3}\right]-\left[12\times -2+\frac{(-2)^2}{2}-\frac{(-2)^3}{3}\right]$	M1	correct use of correct limits for area under the curve using *their* integral of the form $12x+px^2+qx^3$
	$=\frac{128}{3}$ oe	A1	
	$=\frac{32}{3}$ oe	A1	
	Alternative		
	$\int_{-2}^{2}12+x-x^2-x-8\ \mathrm{d}x$ $=\int_{-2}^{2}4-x^2\ \mathrm{d}x$	M1	subtraction of the two equations with intent to integrate the result
	$=\left[4x-\frac{x^3}{3}\right]_{-2}^{2}$	A1	
	$\left[4\times 2-\frac{8}{3}\right]-\left[4\times -2+\frac{8}{3}\right]$	M1	**Dep** for correct application of limits
	$=\frac{32}{3}$ oe	A1	
11(i)	$\mathrm{p}\left(\frac{1}{2}\right)=a\left(\frac{1}{2}\right)^3+17\left(\frac{1}{2}\right)^2+b\left(\frac{1}{2}\right)-8$	M1	expression for $\mathrm{p}\left(\frac{1}{2}\right)$
	$\mathrm{p}(-3)=a(-3)^3+17(-3)^2+b(-3)-8$	M1	expression for $\mathrm{p}(-3)$
	$\frac{a}{8}+\frac{17}{4}+\frac{b}{2}-8=0$ $-27a+153-3b-8=-35$	A1	both equations correct (allow equivalents and terms not collected but powers should be evaluated)
	Leading to $a=b=6$	A1	from correct equations with evidence that both have been found correctly in order to verify that $a=b$

Question	Answer	Marks	Guidance
11(ii)	$(2x-1)(3x^2+10x+8)$	**B2**	**B1** for $3x^2$ and +8 from factorisation or for $3x^2+10x...$ from long division
11(iii)	$(2x-1)(x+2)(3x+4)$	**B1**	cao
11(iv)	$\sin\theta=\frac{1}{2}$	**B1**	
	$\theta=30^\circ,\ 150^\circ$	**B2**	**B1** for a first correct solution **B1** for a second correct solution with no extras in range $0\leqslant\theta\leqslant180$ and no solution arising from other factors.