ADDITIONAL MATHEMATICS

Paper 0606/01

Paper 1

General comments

Although there were some good scripts, the new style of syllabus and paper presented many candidates with considerable difficulty. There were obviously some topics, particularly "matrices" which some candidates had covered if no real depth, if at all. It was very noticeable that the more stereotyped topics from the previous syllabus, notably binomial expansions, solution of simultaneous linear and quadratic equations, coordinate geometry and topics within the calculus sections, were most accessible to the candidates.

Comments on specific questions

Question 1

This question was very well answered by nearly all candidates. The standard of algebra shown in the elimination of either x or y and in solving the resulting quadratic equation was excellent. Candidates seemed comfortable either with the manipulation of fractions in eliminating x or by the squaring required to eliminate

y. A lot of candidates lost the last mark through inaccurate use of decimals when a y-value of $\frac{1}{2}$ was

expressed as 0.34.

Answer: $(3\frac{1}{3}, \frac{1}{3})$ and $(2\frac{1}{2}, 2)$.

Question 2

Attempts varied considerably. Most candidates realised the need to integrate but such attempts to integrate $\frac{kx^2}{2}$

 e^{kx} as $e^{\frac{n}{2}}$ or e^{kx+1} showed a considerable lack of understanding of the exponential function. At least a third of all attempts also ignored the use of the point (0, 3) to evaluate the constant of integration.

Answer:
$$y = \frac{e^{4x}}{4} - e^{-x} + \frac{15}{4}$$
.

Question 3

Attempts also varied considerably and there were very few correct answers. A significant number of candidates failed to realise that calculators were of no use in this type of question. Most candidates seemed

to be familiar with one of the two processes needed, that is to express $\sqrt{18}$ as $3\sqrt{2}$ or $\frac{4}{\sqrt{2}}$ as $2\sqrt{2}$, but very

few seemed confident at using both. The common error of expressing $(a + b)^2$ as $a^2 + b^2$ also led to loss of marks in part (ii).

Answers: (i) $-2 + 11\sqrt{2}$; (ii) $55 - 8\sqrt{2}$.

Question 4

This produced a large number of perfectly correct solutions and although most candidates worked by finding **PQ** first, several went directly to the answer from using **OR** = 5**OQ** - 4**OP**. Use of **PQ** = **p** - **q**, and even **p** + **q**, was seen and over a quarter of all candidates did not appreciate the term "unit vector".

Answer:
$$\frac{1}{75} \begin{pmatrix} 21\\72 \end{pmatrix}$$
.

Part (i) was badly answered with very few candidates sketching both graphs correctly. In a large number of cases $y = \frac{1}{4} + \sin x$ was sketched as either sinx or as $\frac{1}{4} \sin x$ and $\frac{1}{2} \cos 2x$ was only shown in the range 0 to π . A surprising number of graphs were shown in which curves were replaced by straight lines, even at the turning points! Very few candidates realised that the value of *k* in part (ii) could be obtained by equating the two equations. Most candidates attempted to read a value of *x* from their sketches and to deduce a value for *k* from this. Such attempts received no credit unless an accurate graph had been drawn.

Answers: (i) Sketch; (ii) k = 4.

Question 6

This was also poorly answered and several candidates ignored the question altogether. In part (a), the total number of 720 was often obtained without candidates being able to cope with the number not beginning with "0". Part (b) proved to be more successfully answered with many candidates appreciating the need to consider four different cases (though often the case with "all women" was ignored). A surprising number of

candidates also realised the need to calculate such expressions as $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$ but then found the sum

rather than the product. Very rarely was the solution "Total – no women" seen.

Answers: (a) 600; (b) 121.

Question 7

This was well answered and generally a source of high marks. Candidates were able to write down the expansion unsimplified and had no real problems with the binomial coefficients. Subsequent errors with $(-x^2)^n$ were however considerable. Often the minus sign was ignored completely, at times all terms were negative apart from the first and $(-x^2)^n$ was in many instances taken as x^{2+n} or as $-x^{2+n}$. Part (ii) also suffered from the obvious error of expressing $(1 + x^2)^2$ as $1 + x^4$, but it was pleasing to note that most candidates realised the need to consider more than one term in finding the coefficient of x^6 .

Answers: (i) $32 - 80x^2 + 80x^4 - 40x^6 + 10x^8 - x^{10}$; (ii) 40.

Question 8

There were very few completely correct solutions, though candidates did better on part (ii) than on part (i). The majority of attempts at the area of triangle *SXY* attempted to find the sides by Pythagoras's Theorem – no progress was made! Only about a half realised the need to evaluate A by subtracting the sum of the areas of three right-angled triangles from the area of the square. In part (ii), most candidates realised the

need to differentiate and to set the differential to zero. Errors in misusing the " $\frac{1}{2}$ " or in differentiation,

particularly with $\frac{d}{dx}(1) = 1$ and $\frac{d}{dx}(kx^2) = 3qx$ were surprisingly common. Of those obtaining $x = \frac{1}{2q}$, most realised that QX = XR but many forget to substitute this value of x into the expression for A

realised that QY = YR but many forgot to substitute this value of x into the expression for A.

Answers: (i) Proof; (ii) Proof, $A = \frac{1}{2} - \frac{1}{8q}$.

This was well answered and a source of high marks. In part (i) most candidates realised the need to use the product rule, though at least a quarter of all attempts ignored the "2" from the differential of $\sqrt{(2x+5)}$. Part (ii) was well answered, though a considerable number took δx to be 10 - p or failed to substitute x = 10 to obtain a numerical value for the gradient. Part (iii) also presented few problems and it was pleasing to see the

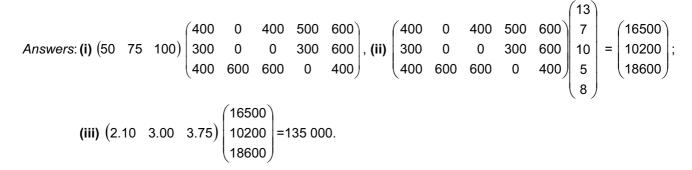
number of attempts correctly using the chain rule and realising either the need to either divide by $\frac{dy}{dx}$ or to

invert $\frac{dy}{dx}$ in order to obtain $\frac{dx}{dt}$.

Answers: (i) Proof, k = 3; (ii) $\pm 6p$; (iii) 0.5 units per second.

Question 10

There were very few correct solutions. Most candidates had obviously had little, if any, experience in manipulating matrices and were unable to set up the basic matrices needed for each part. The basic rule of compatibility of matrices for multiplication – namely that for multiplication to be possible, the number of columns of the first matrix must equal the number of rows of the second, was only sketchily known by many candidates. Many candidates failed to realise that the blanks in the given arrays had to be replaced by "0" in the matrix prior to multiplication.



Question 11

Attempts at this question were very variable and rarely produced high marks. Most candidates were confident in completing the square of the quadratic, though having removed the "2" many left the "-8*x*" and found *b* to be -4. Very few candidates realised that the answers to parts (i) to (iv) proceeded directly from the completion of the square. At least a third of all attempts gave the range in part (i) as $5 \le f(x) \le 15$, using the endpoints of the domain. Even worse were the attempts that just gave a table of values for f(x) for $0 \le x \le 5$. Only a few candidates realised that a function needed to be one-one over the whole domain to have an inverse. It was obvious from the answers that many candidates did not realise that a quadratic function could have an inverse providing that the domain did not include the value of *x* at which the graph of the function had a stationary value. Only a handful of solutions were as x = -b. Only a few realised in part (iv) that the inverse of a quadratic could be obtained directly once the quadratic was written in the 'completed square' form.

Answers:
$$a = 2, b = -2, c = -3$$
; (i) $-3 \le f(x)$ 15; (ii) Not one-one; (iii) $k = 2$; (iv) $g^{-1} : x \mapsto \sqrt{\frac{x+3}{2}} + 2$.

EITHER

Rather surprisingly, especially considering the stereotyped part (b), this was not selected by many candidates and marks were low. In part (a) most candidates converted $y = ax^n$ to $\lg y = \lg a + n \lg x$, realised 64 - 27

that the gradient was *n*, but then took this to be $\frac{64-27}{4-2.25}$ instead of using logarithms of these numbers. The

few correct solutions came from solving a pair of simultaneous equations but expressing the logarithms to an insufficient accuracy meant that the final answers were often inaccurate. The graphs drawn in part (b) were of a high standard and most realised that the *y*-intercept was $\ln m_0$. Unfortunately errors over sign, either through taking the gradient of the line as positive or by thinking that the gradient was +*k*, meant that full marks were rarely achieved.

Answers: (a) n = 1.5, a = 8, p = 125; (b) Graph, k = 0.04, $m_0 = 60$.

Question 12

OR

This was the more popular option and proved to be a source of high marks, even from weaker candidates. Point C was usually obtained from solving simultaneously the line equations for BC and CD, and the standard of algebra was very good. Many weaker candidates obtained the equation of BC in the form

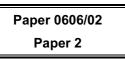
 $\frac{y-11}{x-4} = \frac{1}{2}$ and then assumed that "y - 11 = 1 and x - 4 = 2". Only a handful of solutions were seen in

which a ratio method was used to find *E*, most preferring to solve the simultaneous equations for *AB* and *CD*. A common error was to assume that *C* was the mid-point of *ED*. Attempts at part (ii) were pleasing, though again it was rare, but not unseen, to see solutions coming from considerations of the ratio of (length)². The

more common solution was to use Pythagoras's Theorem along with the formulae " $\frac{1}{2}bh$ " and " $\frac{1}{2}(a + b)h$ "

but many others preferred to use the matrix method for area.

Answers: (i) C (14, 16); (ii) E (9, 26); (iii) 25:39.



General comments

The overall performance of candidates was somewhat lower than in previous years. This was to be expected, perhaps, from the change in style of the examination to one in which the candidates' choice was much more restricted. Another contributory factor was the inclusion in the syllabus of a number of new, and less familiar, topics.

Comments on specific questions

Question 1

Most candidates could find the adjoint matrix correctly and the idea of multiplying by the reciprocal of the determinant was generally well known. Combination of the various minus signs caused difficulty for weaker candidates. Most candidates showed they understood how to combine matrices as required by $\mathbf{A} - 3\mathbf{A}^{-1}$. Many candidates did not understand the identity matrix, some took it to be $\begin{pmatrix} 0 & 1 \end{pmatrix}$ while others took it to be

 $\begin{pmatrix} 5 & 7 \\ 4 & 5 \end{pmatrix}$. Some treated I as though it was 1 leading to $k = \begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix}$; the same result was obtained by some candidates arriving at $\begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix} = k \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

Answer: 10.

Relatively few candidates scored full marks. Some drew graphs of y = x + 1 and y = 2x - 3, whilst others drew graphs for positive values of x only. Attempts at the graph of y = |2x - 3| were generally better than attempts at y = |x| + 1. Many candidates failed to understand the shape of the graph between (1, 1) and (2, 1), joining these points by means of a straight line or a curve. Attempts at y = |x| + 1 frequently resulted in graphs depicting y = |x + 1| or |y - 1| = x. Some candidates produced diagrams showing the four lines y = x + 1, y = -x + 1, y = 2x - 3 and y = -2x + 3.

Answer: (ii) 2.

Question 3

Candidates generally showed a lack of clarity and understanding in their use of set notation. This was particularly true in part (i) where $(H \cap P)'$ was frequently given as the answer. Part (ii) resulted in considerably more correct answers with most candidates offering $P \subset M$, which was accepted, rather than $P \subseteq M$. Incorrect answers were usually either $P \in M$ or $M \subset P$. A few candidates offered perfectly correct alternative answers e.g., $P \cap M = P$ or $P \cap M' = \emptyset$. Parts (iii) and (iv) presented some language difficulties as demonstrated e.g., by the answer "Only students studying mathematics" to part (iii). In general part (iii) was answered correctly but in part (iv) $H \cup M$ was almost always taken to indicate either "students taking History or Mathematics" or "students taking History and Mathematics".

Answers: (i) $H \cap P = \emptyset$; (ii) $P \subseteq M$; (iii) Students studying Mathematics only; (iv) Students studying History or Mathematics or both, but not Physics.

Question 4

All but the weakest candidates scored reasonably well on this question. The factor x+ 2 was usually spotted and the quadratic factor $x^2 - 6x + 1$ almost always followed. Many candidates took x + 2 to be a solution of the given equation with the result that x = -2 never appeared. Most candidates proceeded from $6 \pm \sqrt{32}$

 $x^2 - 6x + 1 = 0$ to $x = \frac{6 \pm \sqrt{32}}{2}$ but many could not then give the answer in the required form – some gave

decimal answers and other offerings were $\frac{6 \pm 4\sqrt{2}}{2}$, $3 \pm 4\sqrt{2}$, $3 \pm \sqrt{8}$, $6 \pm 2\sqrt{2}$.

Answers: -2, $3 \pm 2\sqrt{2}$.

Question 5

This proved to be the most difficult question on the paper, mainly because candidates appeared unable to handle vectors in this situation. Many candidates omitted the question completely or made feeble attempts, sometimes introducing a spurious right-angled triangle of velocities. The relatively few candidates who quickly obtained 50i – 100j almost invariably quoted this as the speed of the plane; some then found a relevant angle but the correct bearing was very rarely obtained. The majority of those making mainly successful attempts followed the tortuous route of calculating two speeds and the difference of two angles, constructing a triangle of velocity and then applying the cosine rule to find the speed, followed by the sine rule to obtain an angle leading to the bearing. But even those who managed to perform all these calculations correctly usually gave the bearing as 153.4° rather than 333.4°. Some candidates did not appreciate the significance of the 4 hours and inevitably became confused, trying to combine distance with velocity. Others ignored the unit vectors taking, for instance, the velocity (250i + 160j) kmh⁻¹ to indicate a speed of 410 km.

Answers: 112 kmh⁻¹, 333.4°.

Question 6

Although the better candidates produced a large number of correct evaluations of k, usually via the quotient rule, weaker candidates often failed to do so, the usual errors being misquoting the quotient rule, applying incorrect signs to the derivative of cos x and/or sin x, and spurious cancellations. Many candidates ignored the "Hence" and attempted the integration of part (ii) directly, resulting in answers involving $\ln(1 - \sin x)$ or $(x - \cos x)^{-1}$. Strangely, many who understood that part (ii) involved the reversal of the result from part (i)

i.e.,
$$\int \frac{1}{1-\sin x} dx = \frac{\cos x}{1-\sin x} \text{ took } \int \frac{\sqrt{2}}{1-\sin x} dx \text{ to be } \frac{1}{\sqrt{2}} \left(\frac{\cos x}{1-\sin x} \right).$$

Answers: (i) 1; (ii) 2.

Candidates generally scored well on this question. Part (i) caused little difficulty to the large majority of candidates although many found it necessary to find angle *AOB* in degrees and then convert to radians. Some candidates used laborious methods, finding *OX* and then applying the sine rule or even the cosine rule. The ideas of arc length and area of sector were almost always correct, as was part (ii). Part (ii) caused more difficulty with a fairly large percentage of candidates attempting to obtain the answer by subtracting the perimeter of the sector from the perimeter of the triangle.

Answers: (ii) 21.8m; (iii) 11.5m².

Question 8

Better candidates were able to obtain full marks with relative ease. Some of the weaker candidates used an incorrect trigonometrical ratio in one of the triangles, but of those who used $\sin\theta$ and $\tan\theta$ correctly a considerable number were unable, or ignored, the request to "express *AB* in terms of θ ". Quite a few

candidates applied the sine rule to triangle *ADB* arriving at $AB = \frac{5\sin(90^\circ - \theta)}{\sin\theta}$ which was acceptable; unfortunately $\sin(90^\circ - \theta)$ rarely, if ever, resulted in $\cos\theta$, almost invariably becoming $\sin90^\circ - \sin\theta$. Expressing $\tan\theta$ in terms of $\sin\theta$ and $\cos\theta$ led some candidates to $6\sin\theta = \frac{5\sin\theta}{\cos\theta}$ and hence $\cos\theta = \frac{5}{6}$, but those candidates arriving at $6\sin^2\theta = 5\cos\theta$ were usually able to complete the question successfully, although there were some errors in sign, and hence factorisation, and also a few candidates who took

 $\cos\theta(6\cos\theta + 5) = 6$ to imply $\cos\theta = 6$ or $6\cos\theta + 5 = 6$.

Answers: (i) $6\sin\theta$, $\frac{5}{\tan\theta}$; (ii) 48.2° .

Question 9

- (a) Few of the candidates who chose to consider the discriminant took the simpler route of eliminating x to obtain a quadratic in y, the vast majority preferring to eliminate y, obtaining $(x + k)^2 = 4x + 8$. A variety of errors then occurred, with $(x + k)^2$ becoming $x^2 + k^2$ or $x^2 + 2k + k^2$ or, most frequently, the equation above becoming $(x + k)^2 4x + 8 = 0$. Some of the weaker candidates were unable to identify correctly the elements a, b and c of the discriminant $b^2 4ac$. Some candidates successfully applied the calculus; implicit differentiation was occasionally seen, but it was most usual for candidates to attempt, not always correctly, to differentiate $(4x + 8)^{\frac{1}{2}}$. Differentiation of $(4x + 8)^{\frac{1}{2}}$, whether correct or not, was as far as some candidates could go in that they did not understand the need to equate their result to 1, the gradient of the tangent y = x + k.
- (b) There was a widespread failure to identify this question with the routine solution of a quadratic inequality. The small minority of candidates who recognised that $\{x : x > 2\} \cup \{x : x < -4\}$ implied (x 2)(x + 4) > 0 almost always proceeded quickly to the correct solution. Some candidates obtained the correct answers by solving 4 + 2a = b and 16 4a = b but many eschewed the equality signs attempting to solve 4 + 2a > b and 16 4a > b, arriving at a > 2, b < 8. Many others took note of x > 2 and x < -4 and substituted x = 3 and x = -5 (or -3) in the equation $x^2 + ax = b$.

Answers: (a) 3; (b) 2, 8.

Question 10

Part (i) produced many correct solutions but also a fair number of inept attempts e.g., 2x - (x - 3) = 1 or 10, $\frac{2x}{x-3} = 1$ and $\frac{\lg 2x}{\lg(x-3)} = 1$ or 10 followed by the "cancellation" of Ig. In part (ii) most candidates appreciated that change of base was necessary but many could not profitably proceed any further. The most successful candidates were those who replaced 4 $\log_y 3$ by $\frac{4}{\log_3 y}$ and then used a further symbol (often *y*) to represent $\log_3 y$. The alternative, replacing $\log_3 y$ by $\frac{1}{\log_y 3}$, was seen infrequently, but many candidates changed both terms

on the left-hand side of the equation to logarithms to the base 10. Candidates frequently made complications for themselves by rendering the 4 as $\log_3 81$ or $\log_y y^4$ or $\log_1 0000$ depending on the base chosen. The most commonly occurring error was to write what should have been $(\log_3 y)^2$ as $\log_3 y^2$; this then became $2\log_3 y$ or was combined with $4\log_3 y$, i.e., $\log_3 y^4$ to give $\log_3 y^4 - \log_3 y^2 = \log_3 y^2$.

Answers: (i) 3.75; (ii) 9.

Question 11

This question was a good source of marks for many candidates. Nearly all candidates were capable of finding f^{-1} correctly, the only error occurring when x = 3y - 7 became x - 7 = 3y. Similarly, apart from the occasional arithmetic error, g^{-1} was usually correct, although candidates were quite often unable to give 0 as the value of x for which g^{-1} is not defined; alternative offers were 2, 6 or – 6 while some candidates failed to offer any value. Relatively few candidates had any difficulty with part (ii). Poor algebra spoiled some

attempts with $3\left(\frac{12}{x-2}\right)$ becoming $\frac{36}{3x-6}$ or $\frac{36}{x-2}$ - 7 = x becoming 36 - 7 = x(x-2). A few candidates

omitted the *x*, thus solving fg(x) = 0, whilst some confused the order of operation and, in effect, solved gf(x) = x. One or two of the weakest candidates attempted to solve $f(x) \times g(x) = 0$ and the solution of $f^{-1}g^{-1}(x) = x$ was also seen. Graphs usually contained correct segments of both lines but the choice of axes was such that all the points of intersection with the axes could not be shown, the coordinates often being calculated separately. Some of the weakest candidates clearly did not appreciate that y = 3x - 7 and

 $y = \frac{1}{3}(x + 7)$ were linear equations with their graphical representations being straight lines. Although it was

not essential, candidates might have used the reflective property of f and f⁻¹ in the line y = x as a confirmation of the correctness of their graph but knowledge of this property was rarely in evidence although some candidates attempted to make use of it despite the differing scales on their axes. Many candidates read the final phrase of part (iii) as a request for the coordinate of the point of intersection of the graphs of f and f⁻¹.

Answers: (i)
$$\frac{x+7}{3}$$
; 2 + $\frac{12}{x}$, x = 0; (ii) - 10, 5; (iii) f: $(2\frac{1}{3}, 0)$, $(0, -7)$; f⁻¹: $(-7, 0)$, $(0, 2\frac{1}{3})$.

Question 12

EITHER

This proved to be the easier of the two options with very many of the better candidates obtaining full marks. Finding the coordinates of *P* was successfully accomplished by a variety of methods, using $x_P = -\frac{b}{2a}$,

completing the square $(x - 3)^2 + 1 = 0$ leading to $y_P = 1$ when $x_P = 3$, and finding x_P via $\frac{d}{dx}(x^2 - 6x + 10) = 0$.

Some weaker candidates quickly went wrong, finding *P* to be (2, 2) through assuming $\frac{d}{dx}(x^2 - 6x + 10) = -2$ at *P*. Continuing with this line of reasoning candidates then found the equation of *PQ* to be y = 6 - 2x which

at *P*. Continuing with this line of reasoning candidates then found the equation of *PQ* to be y = 6 - 2x which, on solving with the equation of the curve, led to $(x - 2)^2 = 0$ and puzzlement. Other candidates found *P* correctly but then assumed that $\frac{d}{dx}(x^2 - 6x + 10) = -2$ at *Q*. The integration, usually of $x^2 - 6x + 10$ although quite frequently of $(7 - 2x) - (x^2 - 6x + 10)$, was very good and was almost always correct. Evaluation of the integral was often correct but \int_0^3 was also seen with some frequency. A correct plan for finding the shaded area was sometimes lacking, with the area between the chord *PO* and the curve being

finding the shaded area was sometimes lacking, with the area between the chord PQ and the curve being treated as though it was the area beneath the curve or with the entire area bounded by the axes and the lines x = 3, y = 5 and the chord PQ regarded as a trapezium.

Answer: 9 $\frac{2}{3}$ units².

OR

This was clearly the less popular alternative and with good cause in that candidates rarely answered it well. There were two main reasons for this; firstly, an inability to find the value T and, secondly, a lack of understanding of the velocity-time graph. Virtually all candidates were able to evaluate v_B as 15. Many candidates made no attempt to find T; some took it to be the value of t obtained from $15 = \frac{1}{225}(20 - t)^3$ and of those who understood that the required value of t was to be obtained from $\frac{1}{225}(20 - t)^3 = 0$ most were unable to solve this equation, frequently expanding $(20 - t)^3$. In part (ii) virtually all candidates understood that $\frac{d v}{d t}$ gives the acceleration and the only commonly occurring error was the omission of the minus sign arising from $\frac{d}{dt}(20 - t)$. The sketch required in part (iii) was very rarely correct in that most offerings consisted of either a straight line joining (0, 0) to (5, 15) with a straight line joining (5, 15) to some point on the time axis, or a curve representing $\frac{1}{225}(20 - t)^3$ for $0 \le t \le 20$. In the first of these cases the distance AC was calculated as the area of a triangle and so no integration was in evidence. In the second case the integral of $\frac{1}{225}(20 - t)^3$ was usually taken to be $\frac{1}{900}(20 - t)^4$, even by those candidates who had included $\frac{d}{dt}(20 - t)$ when dealing with the differentiation of part (ii), and then evaluated from 0 to 20.

Answers: (i) $15ms^{-1}$, 20; (ii) – 0.48ms⁻²; (iv) $93\frac{3}{4}m$.