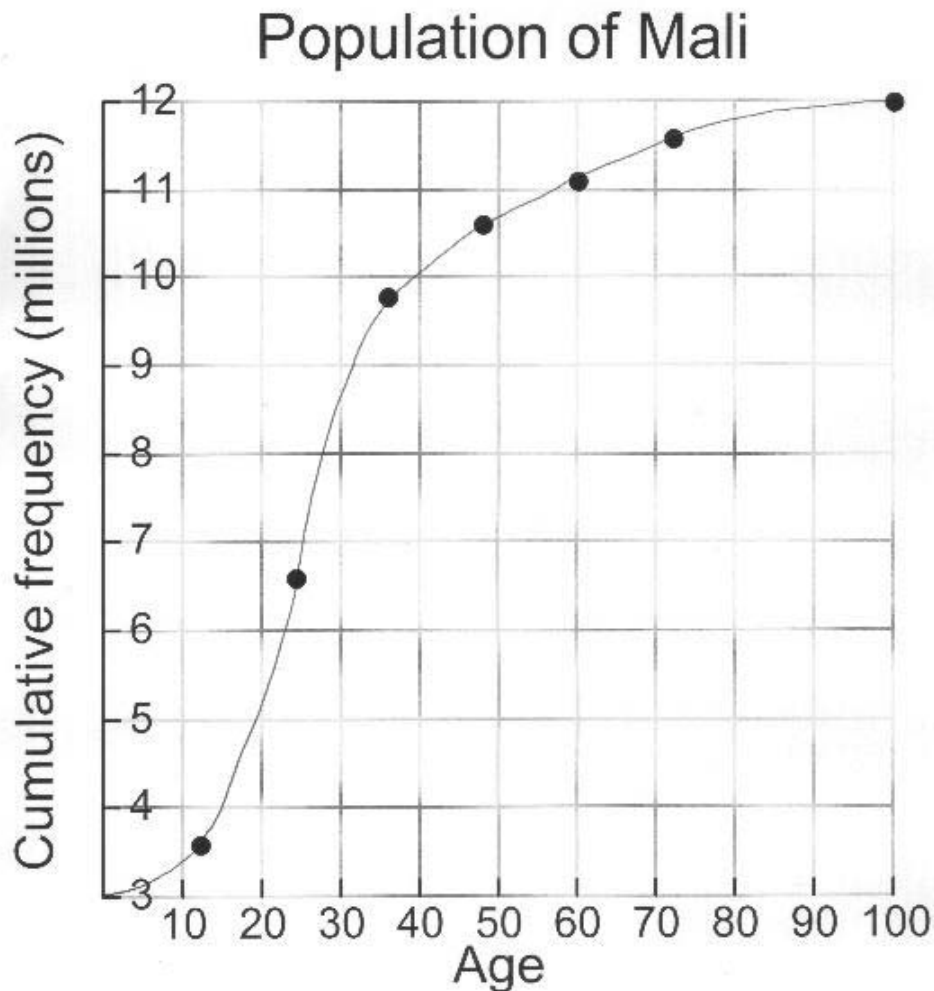# Paper D

**As a guideline, this paper should be completed in 1 hour.**

**You will need a Graphics Display Calculator (GDC) for this examination.**

**Section A [21 marks]**

1.    *[Maximum 6 marks]*

The cumulative frequency curve shows the ages of a population of Mali.

a)   Use the cumulative frequency curve to complete the table below.

| Age | ≤ 12 | ≤ 24 | ≤ 36 | ≤ 48 | ≤ 60 | ≤ 72 | ≤ 100 |
|---|---|---|---|---|---|---|---|
| Cumulative frequency (millions) | ……… | ……… | ……… | ……… | ……… | ……… | 12 |

b)   Find an estimate of the median average of the age of Mali's population.

c)   Find an estimate of the mean average age of the population of Mali.

2.   *[Maximum 5 marks]*

The table below shows three different types of number sequences.

Type I       $\dfrac{1}{2}, \dfrac{3}{8}, \dfrac{9}{32}, \dfrac{27}{128}, \ldots\ldots$

Type II      0, 3, 15, 24, 35, ……

Type III     $\dfrac{1}{2}, \dfrac{3}{4}, 1, \dfrac{5}{4}, \dfrac{3}{2}, \ldots\ldots$

a)   Identify which one is an arithmetic sequence and give a formula for the $n^{th}$ term of this sequence.

b)   Identify which one is a geometric sequence and calculate the sum to infinity of the sequence.

3.   *[Maximum 5 marks]*

A function $f$ is defined by,

$$f(x) = \sqrt{4 + 3x} - 2, \qquad \text{where } x \geq -\dfrac{4}{3}.$$

Evaluate $f^{-1}(5)$.

4.    *[Maximum 5 marks]*

Solve the equation $4 \sin x = 2 + 3 \cos^2 x$, for $0 \leq x \leq 180°$.


**Section B [39 marks]**

5.    *[Maximum mark 19]*

Bag A has 4 green and 2 blue balls. Two balls are picked at random. Let *X* be the number of green balls. A table is produced to show the probability of *X*.

| *X* | 0 | 1 | 2 |
|-----|---|---|---|
| Prob (*X=x*) | $\dfrac{1}{15}$ | $\dfrac{8}{15}$ | $\dfrac{6}{15}$ |

i)    Calculate $E(X)$.                                    *[3 marks]*

Bag B contains 5 green balls and 2 blue balls. Two balls are picked at random without replacement, and the number of green balls is denoted by the letter *Y*.

ii)   a)   Draw a tree diagram to show the possibilities of *Y*.

      b)   Use your tree diagram to produce a probability density table for *Y*.                           *[8 marks]*

iii)  An unbiased die is rolled. If the die lands on a 1 or a 2 then *X* is selected. If not *Y* is selected.

      a)   Under these conditions calculate the probability of obtaining 2 green balls,

      b)   Given that 2 green balls were obtained, find the probability of the die landing on a 1 or a 2.                    *[8 marks]*

6.    *[Maximum mark 20]*

   i)   a)   Find the 3$^{rd}$ term in the expansion of the function
             $f(x) = (2x - 5)^6$.                              *[3 marks]*

        b)   Differentiate the function $f(x) = (2x - 5)^6$.    *[2 marks]*

   ii)  a)   Calculate the value correct to 2 decimal places of the area
             of the region bounded by the curve $y = x^2 - \dfrac{3}{x^{\frac{1}{2}}}$ and the
             lines $x = 2$ and $x = 3$.

        b)   Find the equation of the tangent to the curve $y = x^2 - \dfrac{3}{x^{\frac{1}{2}}}$ at
             the point where $x = 1$. Give your answer in the form
             $ax + by + c = 0$.                                *[10 marks]*

   iii)  A ball on an elastic band travels along a straight line. A model
         of the equation has been set up to show the displacement from
         a point, $s$ cm, at time, $t$ seconds.

$$s = 32 - 2t^2, \quad 0 \le t \le 8.$$

        a)   Write down an equation for the velocity at time $t$.

        b)   Find the total distance travelled by the ball.   *[5 marks]*

Answers

1.  a)

| Age | ≤ 12 | ≤ 24 | ≤ 36 | ≤ 48 | ≤ 60 | ≤ 72 | ≤ 100 |
|---|---|---|---|---|---|---|---|
| Cumulative frequency (millions) | 3.5 | 6.6 | 9.8 | 10.6 | 11.1 | 11.6 | 12 |

   b)   $\approx 22$                    c)   $\approx 25$

2.  a)   $\dfrac{1}{4}n + \dfrac{1}{4}$                    b)   2

3.  15

4.  $x = 52°$, $x = 128°$

5.  i)   $\dfrac{4}{3}$

    ii)   a)
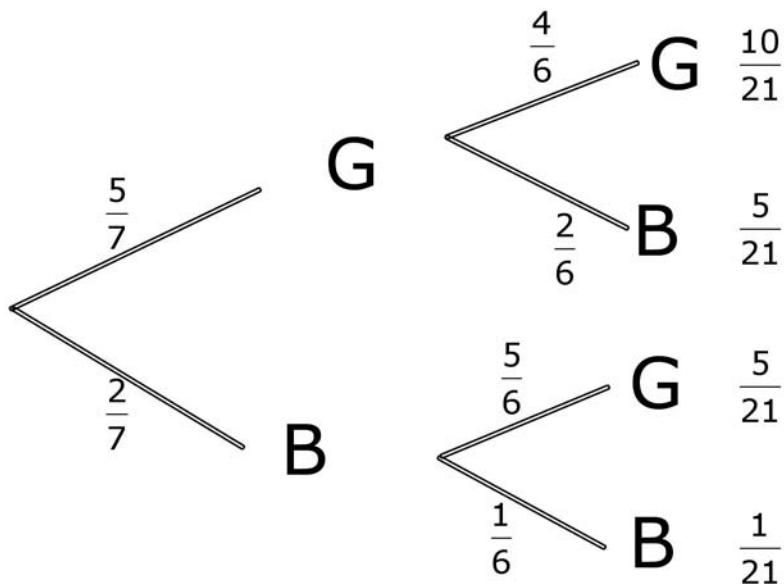
b)

| $Y$ | 0 | 1 | 2 |
|---|---|---|---|
| Prob $(Y=y)$ | $\dfrac{1}{21}$ | $\dfrac{10}{21}$ | $\dfrac{10}{21}$ |

iii)  a)  $\dfrac{142}{315} = 0.451$        b)  $\dfrac{21}{71} = 0.296$

6.  i)  a)  $6000x^4$        b)  $f'(x) = 12(2x - 5)^5$

    ii)  a)  4.43 units$^2$        b)  $7x - 2y - 11 = 0$

    iii)  a)  $v = -4t$        b)  $341\dfrac{1}{3}$ units$^2$