

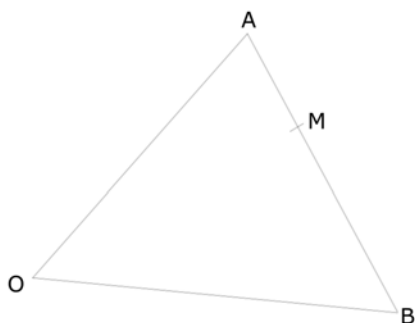
As a guideline this paper should be completed in 1 hour.

No Calculator to be used in this examination.

Section A [42 marks]

1. [Maximum mark 4]

In the diagram below the position vectors of A and B are represented by the vectors \mathbf{a} and \mathbf{b} respectively.



Given that $2AM = BM$, find the position vector of M .

2. [Maximum mark 6]

$$p = \begin{pmatrix} -1 \\ -5 \end{pmatrix} + s \begin{pmatrix} -1 \\ 4 \end{pmatrix} \text{ and } q = \begin{pmatrix} -5 \\ 0 \end{pmatrix} + t \begin{pmatrix} -2 \\ -3 \end{pmatrix}.$$

Find the position vector at the point where the lines p and q intersect.

3. [Maximum mark 7]

A die is biased such that the probability of getting a six is $\frac{1}{4}$. The die is rolled 2000 times. Let X be the number of sixes obtained. Find,

- the mean of X ,
- the standard deviation of X , leaving your answer as a surd.

4. [Maximum mark 6]

The function $f(x)$ is defined as $f(x) = \frac{\cos x}{e^{2x}}$

Find $f'(x)$.

5. [Maximum mark 10]

a) Write the function $f(x) = 3x^2 - 24x + 47$ in the form $f(x) = a(x - p)^2 + q$.

b) Hence find the vertex of $f(x)$.

c) Find the inverse of $f(x)$.

6. [Maximum mark 6]

The matrix A is

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 4 & 5 & -2 \\ 1 & -1 & x \end{bmatrix}$$

Find the value of x such that the matrix is singular, e.g. it has no inverse.

7. [Maximum mark 3]

A particle is moving from a fixed point such that its displacement from the point is given by the equation $s = 4t - t^2 - e^t$, where s is displacement in metres after t seconds.

a) Find the equation of the velocity of the particle at time t .

b) Find the equation of the acceleration of the particle at time t .

Section B [18 marks]

8. [Maximum mark 18]

The sets A , B , and C are subsets of U . they are defined as:

$$U = \{\text{the numbers from 1 to 20 inclusive}\}$$
$$A = \{\text{square numbers}\}$$
$$B = \{\text{multiples of 2}\}$$
$$C = \{\text{prime numbers}\}$$

i) List the elements (if any) of,

a) A ,b) B ,c) C ,d) $(A \cup B \cup C)'$.

[4 marks]

ii) a) Draw a Venn diagram showing the relationship between the sets U , A , B and C .b) Write the elements of sets U , A , B and C in the appropriate places on the Venn diagram.

[6 marks]

iii) On your diagram shade the area represented by $(A \cup B) \cap C$.

[2 marks]

iv) Find the probability that a number chosen from the universal set, U , will be:

a) a prime number;

b) a square and a prime number;

c) a multiple of 2, given that the number is prime;

d) prime, given that the number is a multiple of 2.

[6 marks]

