1. A right-angled triangle has a base of 2.9×10^4, and a height of 4.2×10^3. Find each of the following giving your answer in the form $a \times 10^k$, where $1 \leq a \leq 10$ and $k \in \Re$. Write your answer correct to 3 significant figures.

    a)   The area of the triangle.

    b)   The length of the hypotenuse of the triangle.

    Answers:

    a)   ....

    b)   ....

2. In the following set of data the numbers are ordered by size. The median of the numbers is 9 and their mean is 10.

    4, 5, b, 7, 7, a, 12, 13, 16, 20

    Find each of the following,

    a)   the value of a,

    b)   the value of b.

    Answers:

    a)   ....

    b)   ....

3.   A game warden stands at a point *A*. A zebra, *Z*, is located 150 metres north of the game warden and Tammy, *T*, is stood 130 metres due west of the game warden.

    a)   Draw a diagram to show the information above. The diagram does not have to be to scale.

    b)   Find the distance between the zebra and Tammy.

    c)   Tammy is looking at the game warden and then turns through $\theta°$ to look at the zebra. Calculate the value of θ.

Answers:

a) ....

b) ....
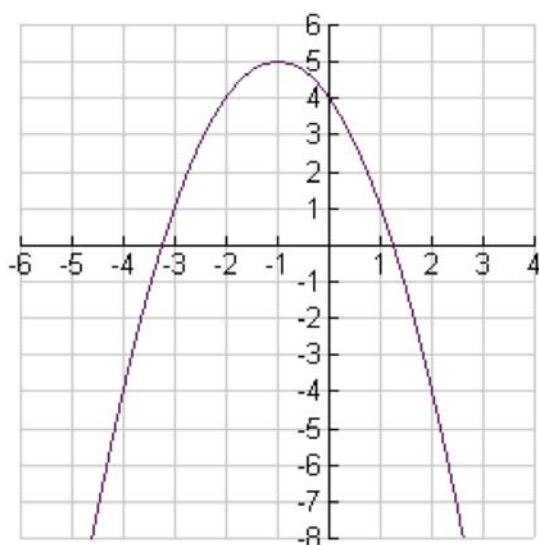
c) ....

4.   Consider the following graphs:

(i) $y = 3x^2 - 2x$
(ii) $y = 4 - 2x - x^2$
(iii) $y = (x - 2)(x + 3)$
(iv) $y = 2x^2 - 3x + 7$

Which of these graphs
a)   has a y intercept below the x-axis,
b)   does not cross the x-axis,
c)   passes through the origin,
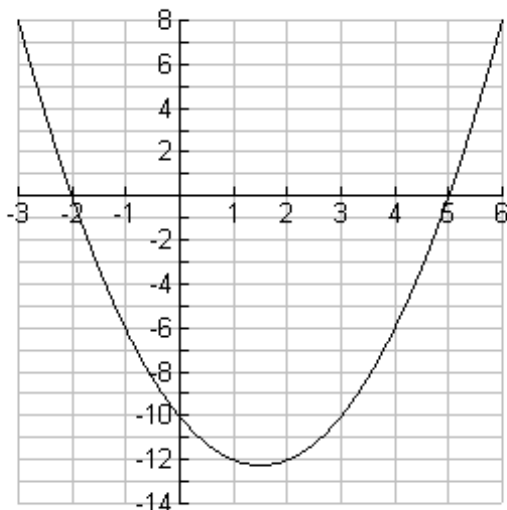d)   can be represented by the graph below.

Answers:

a) ....

b) ....

c) ....

d) ....

5.   The graph of $y = x^2 - 3x - 10$ is drawn below. The points A and B are where the curve intercepts the x-axis. The point C is the minimum point of the graph.

a)   Factorise $x^2 - 3x - 10$
b)   Write down the coordinates of A.
c)   Write down the coordinates of B.
d)   Write down the coordinates of C.

Answers:

a) ....

b) ....

c) ....

d) ....

6. The amount of drugs present in the body, N, after time t, where t is measured hours, is modelled by the formula

$$N = 120 \times 0.5^{t}$$

Use this formula to calculate

a) the amount of drug present initially,

b) the amount of drug present after 4 hours,

c) the time taken for the amount of drugs to 1% of the original taken.

Answers:

a) ....

b) ....

c) ....

7.   Zadie has a savings account which her parents add to on each
      birthday. On her first birthday they deposit $1200, her second
      $1800, her third $2400, on so on.

   a)   Calculate how much her parents deposit on her 18th birthday.

   b)   Find the sum of money in the account on Zadie's 19th birthday,
         if her parents make the final payment on this birthday.

                                   Answers:

                                   a) ....

                                   b) ....

8.   The radius in the centre of a football field is measured to the nearest
      metre and is known to be 3 metres.

   a)   Write the range of possible values for the radius of the circle.

   b)   Calculate the smallest possible circumference that the circle
         has.

                                   Answers:

                                   a) ....

                                   b) ....

9.　　$f(x) = \dfrac{x+3}{2}$ and $g(x) = x^2 - 5$

a)　Calculate f(4).

b)　Find $f(x)^{-1}$.

c)　Calculate (g o f) (9).

Answers:

a)　...

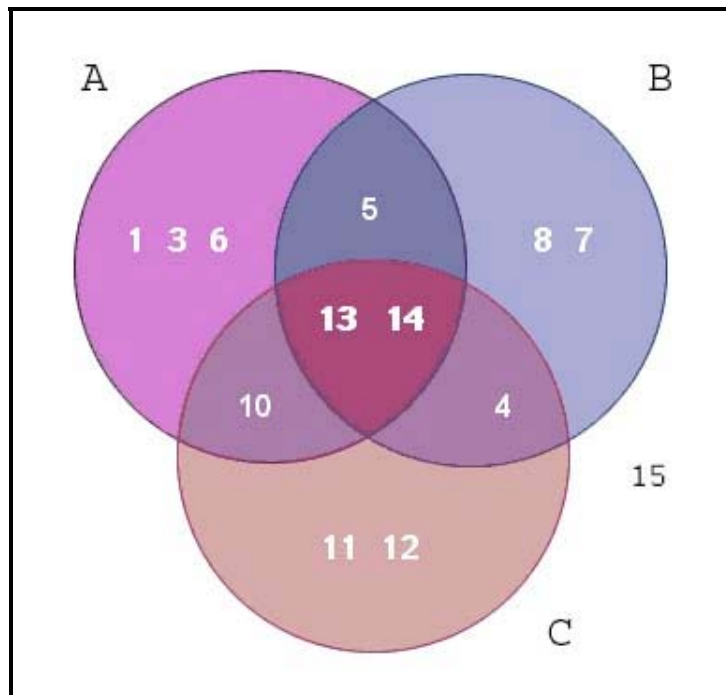b)　...

c)　...

10.   Below is a Venn diagram with the universal set $\{\ 1 \le x \le 15\}$.

a)   Describe in words where the number 5 is on the diagram.

List the elements of,
b)   A $\cap$ B $\cap$ C

c)   (A $\cup$ B $\cup$ C)'.

Answers:

a) ....

b) ....

c) ....

11. The table shows 200 students in a school with the number who have dark eyes or light eyes and the gender of the students.

	Dark eyes	Light eyes	Total
Female	50	30	80
Male	85	35	120
	135	65	200

If a student was selected at random from the group, find the probability that

a)   the student is male;

b)   the students has dark eyes;

c)   the students has dark eyes given that the student is male.

Answers:

a) ....

b) ....

c) ....

12. The diagram below shows the distance between 3 towns.
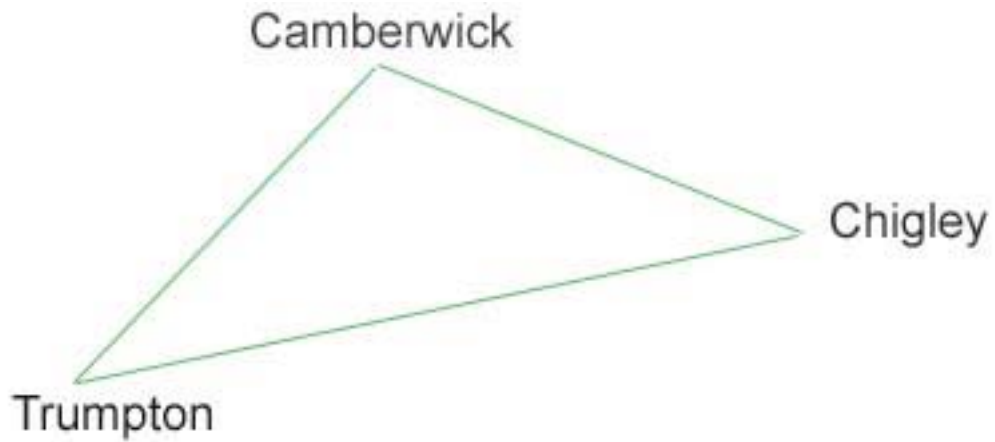
The distance between Trumpton and Camberwick is 90km, Trumpton and Chigley is 140km, and Camberwick to Chigley 105km.

a)   Find the angle made inside the triangle at Camberwick.

b)   Find the area (in km^2) enclosed by the 3 towns.

Answers:

a) ....

b) ....

13. Find the equation of the straight line that passes through the coordinates (2, 6) and (-3, 8).

Give your answer in the form $ax + by + c = 0$.

Answer:

....

14. $f(x) \rightarrow 3x^3 - \dfrac{3}{2}x^2 - 12x - 5$

Use differentiation to find,

a)   the x-coordinate at the maximum turning point on the curve,

b)   the x-coordinate at the minimum turning point on the curve.

Answers:

a) ....

b) ....

15. Simone invests $100. At the end of 3 years she has $135.

   Calculate the interest rate that Simone receives on her money.

Answer:

....

1. a)   Area of a triangle is $\frac{1}{2}$xbasexheight.

   $\frac{1}{2} \times 2.9 \times 10^4 \times 4.2 \times 10^3$

   Work out the calculation, with the exception of the powers of 10 which can be simplified.

   6.09×10^7

   b)   Use Pythagoras theorem: $a^2 = b^2 + c^2$.

   $hyp^2 = (2.9 \times 10^4)^2 + (4.2 \times 10^3)^2$
   $hyp^2 = (8.41 + \left(16.82 \times 10^4\right) + 10^8) + (17.64 + \left(8.4 \times 10^3\right) + 10^6)$

   Simplify this with your calculator:
   $hyp^2 = 101176626.1$
   $hyp = \sqrt{101176626.1}$
   $hyp = 10058.6$
   $hyp = 1.01 \times 10^4$

2. a)   As there are 10 numbers then the median will be the middle two numbers divided by 2.

   $median = \frac{7 + a}{2} = 9$
   $7 + a = 18$
   $a = 11$

   b)   The mean is all the numbers added together divided by 10.

   $mean = \frac{95 + a}{10} = 10$
   $95 + a = 100$
   $a = 5$

3. a)
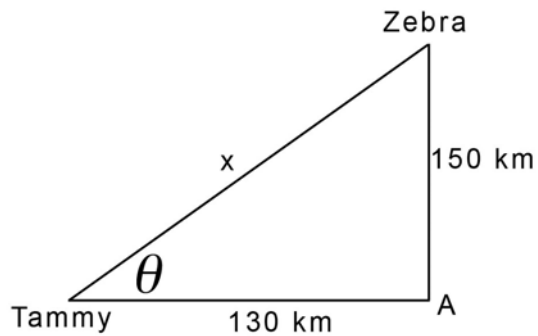
b)   The distance is the hypotenuse of the right-angled triangle, marked with *x* in the diagram.

$$x = \sqrt{150^2 + 130^2}$$
$$x = 198.5m$$

c)   The angle is marked with θ in the diagram.

Use SOHCAHTOA as the triangle is right-angled.
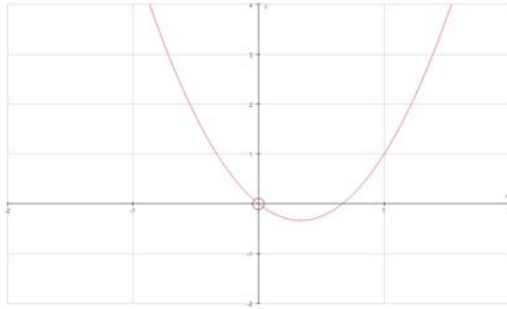
$$\tan\theta = \frac{opposite}{adjacent}$$
$$\tan\theta = \frac{150}{130}$$
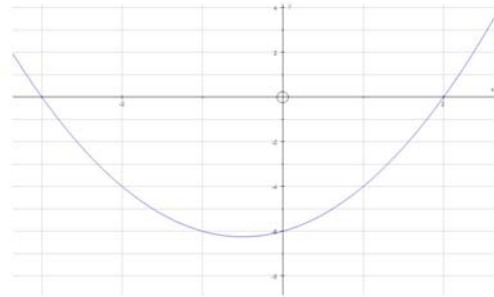$$\theta = \tan^{-1}\left(\frac{150}{130}\right)$$
$$\theta = 49.1°$$

4.   Draw each of the graphs on your GDC, and this will answer all the questions for you.
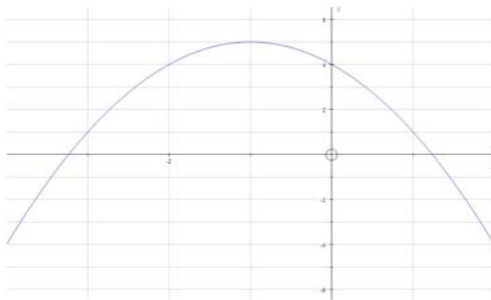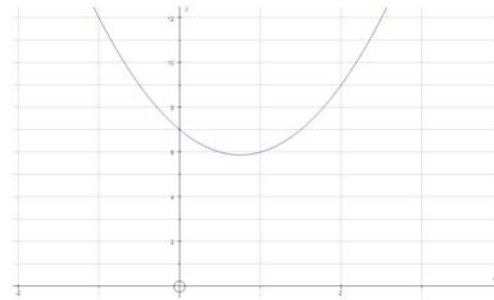
(i)

Passes through the origin so c)

(iii)

Has a y-intercept below 0, so a)

(ii)

The only graph as the same shape as the diagram in the question, so d)

(iv)

Does not cross the x-axis, so b)

5. a) $(x-5)(x+2)$

b) and As the points A and B are the x-intercepts we have:
c)
         $(-2,0)$ and $(5,0)$.

d)   The minimum point of the graph. Adding up the x-coordinate at A
         and B and dividing by 2 obtain the x-coordinate.

$$\frac{-2+5}{2} = \frac{3}{2}.$$

Now sub in the value of $\frac{3}{2}$ into the quadratic to get the y-coordinate.

$$\left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) - 10 = -12\frac{1}{4}$$

So $C=\left(\frac{3}{2}, -12\frac{1}{4}\right)$.

6. a)   Initial time $t=0$.

$$N = 120 \times 0.5^0$$
$$N = 120$$

b)   Now substitute $t=4$.

$$N = 120 \times 0.5^4$$
$$N = 7.5$$

c) $1.2 = 120(0.5)^t$

$0.01 = (0.5)^t$

By trial and error $t=9$.

7. a)   This sequence is arithmetic.
        Use the formula from the booklet for the nth term:
        $T_n = u_1 + (n-1)d$

        The first term is $u_1 = 1200$
        The common difference is $d = 600$

        $T_{18} = 1200 + 17 \times 600$
        $= 11400$

   b)   Use the formula for the sum of an arithmetic sequence:

        $S_n = \dfrac{n}{2}\left(2u_1 + (n-1)d\right)$

        $S_{19} = \dfrac{19}{2}\left(2400 + 18 \times 600\right)$
        $= 125400$

8. a)   Nearest metre is 1m÷2-=0.
        Add and subtract from 3 to get the bounds of the limit.

        $2.5 \leq radius < 3.5$

        Notes the signs used.

   b)   Circumference of a circle $= 2\pi r$

        $C = 2 \times \pi \times 2.5$
        $C = 15.71$

9. a)   Substitute 4 into the function.

$$f(4) = \frac{4 + 3}{2} = 3.5$$

   b)   Make the function = y.
           Re-arrange to make x the subject.
           Replace the y with x.

$$y = \frac{x + 3}{2}$$
$$2y = x + 3$$
$$2y - 3 = x$$
$$x = 2y - 3$$
$$f(x)^{-1} = 2x - 3$$

   c) $(g \circ f)(9)$ means substitute 9 into $f(x)$, and substitute the result
           into $g(x)$.

$$f(9) = \frac{9 + 3}{2} = 6$$
$$g(6) = 6^2 - 5 = 31$$

10. a)   The number 5 exists at the intersection of sets A and B, but not
           in set C.

   b) $A \cap B \cap C$ means that the elements that are in the intersection of
           all 3 sets.

           = 13 and 14.

   c) $(A \cup B \cup C)'$ means the elements that are not in A, B, or C.

           = 15.

11. a)   Probability is best written as fraction with the total outcomes on the bottom (denominator) and the favourable choices as the top (numerator).

   120 males out of 200 students:

   $$\frac{120}{200} = \frac{3}{5}$$

   b)   135 students out of 200 have dark eyes:

   $$\frac{135}{200} = \frac{27}{40}$$

   c)   This is a conditional probability. We are told the student is male, so the denominator is 120. Out of the 120 boys 85 have dark eyes.

   $$\frac{85}{120} = \frac{17}{24}$$

12. a)   Use the cosine rule to find the angle.

   $$\cos\theta = \frac{b^2 + c^2 - a^2}{2bc}$$
   $$\cos\theta = \frac{105^2 + 90^2 - 140^2}{2 \times 105 \times 90}$$
   $$\cos\theta = \frac{-475}{18900}$$
   $$\theta = \cos^{-1}\left(\frac{-475}{18900}\right)$$
   $$\theta = 91.4°$$

   b)   Use the formula for the area of a triangle: $\frac{1}{2}ab\sin C$.

   $$Area = \frac{1}{2} \times 90 \times 105 \times \sin(91.4)$$
   $$Area = 4723.5 km^2$$
   Area

13.   Equation of a straight line is in the form $y = mx + c$, where m is the gradient and c the intercept on the y-axis.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 6}{-3 - 2} = -\frac{2}{5}$$

$$y = -\frac{2}{5}x + c$$

Now substitute in one of the set of coordinates to find the value of c.

$$6 = \left(-\frac{2}{5} \times 2\right) + c$$
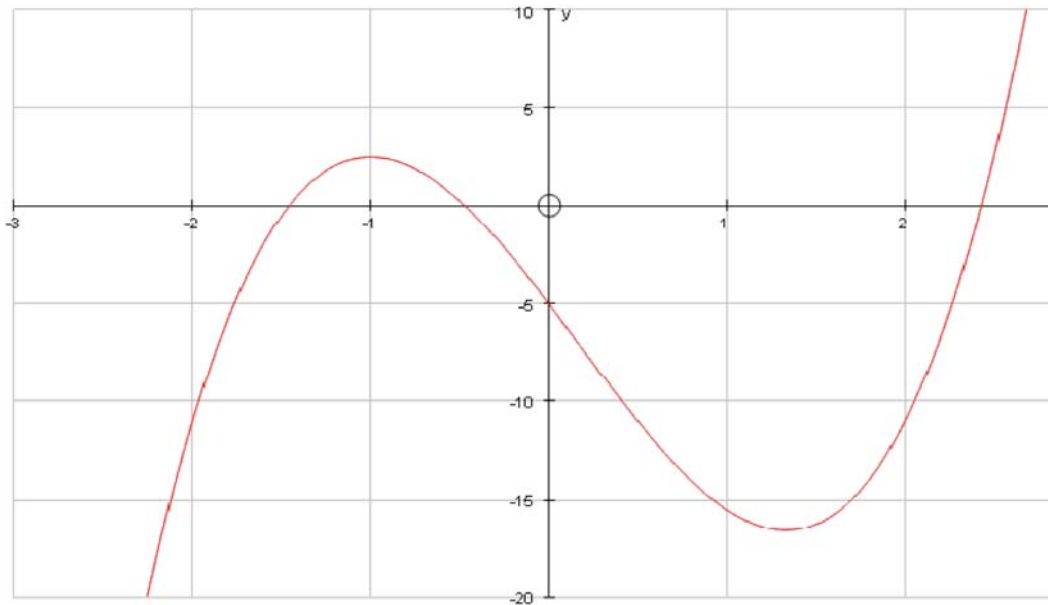
$$c = 6 + \frac{4}{5}$$

$$c = \frac{34}{5}$$

Now make a, b, and c integers.

$$y = -\frac{2}{5}x + \frac{34}{5}$$
$$5y = -2x + 34$$
$$2x + 5y - 34 = 0$$

14 a)   First step is to make a quick sketch on your GDC.
and b)

Differentiate the function and make it equal to 0.

$$y = 3x^3 - \frac{3}{2}x^2 - 12x - 5$$

$$\frac{dy}{dx} = 9x^2 - 3x - 12$$

$$9x^2 - 3x - 12 = 0$$

$$3x^2 - x - 4 = 0$$

Solve to get $x = -1, \dfrac{4}{3}$

Now calculate the *y*-coordinate by substituting in the *x* values
into the original equation.

$\left(-1, \dfrac{5}{2}\right)$ and $\left(\dfrac{4}{3}, -\dfrac{149}{9}\right)$ - verify these points from your GDC.

To prove which is the maximum point and which is the minimum,
differentiate again and put in your *x*-value.

$$\frac{d^2y}{dx^2} = 18x - 3$$

$$x = -1, \frac{d^2y}{dx^2} = -ve \Rightarrow \max$$

$$x = \frac{4}{3}, \frac{d^2y}{dx^2} = +ve \Rightarrow \min$$

$\left(-1, \frac{5}{2}\right)$ is the maximum point, $\left(\frac{4}{3}, -\frac{149}{9}\right)$ is the minimum point.

15.   The calculation for the amount of compounded interest is:

$100 \times (x)^3 = 135,$

where x is the interest rate +1. E.g. 1.2 represents a 20% increase, 1.5 represents a 50% increase.
x is to the power of 3, for three years interest.

Now solve the equation:

$100 \times (x)^3 = 135$
$x^3 = 1.35$
$x = \sqrt[3]{1.35}$
$x = 1.105$

The interest rate will be 10.5%.