Mathematical studies
Standard level

Specimen paper 1 and paper 2
Bank of specimen questions for paper 1 and paper 2

For first examinations in 2006

CONTENTS

Mathematical studies standard level paper 1 specimen paper

Mathematical studies standard level paper 1 specimen markscheme

Bank of specimen questions for paper 1 and markscheme

Mathematical studies standard level paper 2 specimen paper

Mathematical studies standard level paper 2 specimen markscheme

Bank of specimen questions for paper 2 and markscheme

IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
PROGRAMA DEL DIPLOMA DEL BI

MATHEMATICAL STUDIES
STANDARD LEVEL
PAPER 1

SPECIMEN

1 hour 30 minutes

Candidate session number

0	0								

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.

16 pages

Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Working may be continued below the box, if necessary. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. The age in months at which a child first starts to walk is observed for a random group of children from a town in Brazil. The results are

$$14.3, \quad 11.6, \quad 12.2, \quad 14.0, \quad 20.4, \quad 13.4, \quad 12.9, \quad 11.7, \quad 13.1.$$

(a) (i) Find the mean of the ages of these children.

(ii) Find the standard deviation of the ages of these children.

(b) Find the median age.

Working:

Answers:

(a) (i) _____

(ii) _____

(b) _____

2.   A field is 91.4 m long and 68.5 m wide.

(a)   Calculate the area of the field in m^2.

(b)   Calculate the area of the field in cm^2.

(c)   Express your answer to (b) in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.
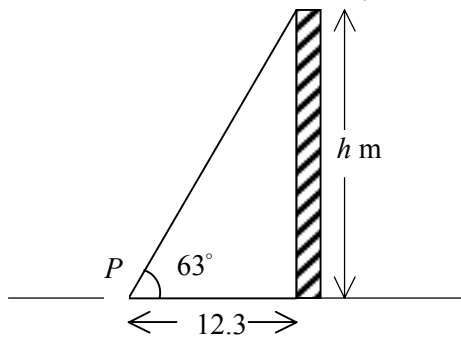
Working:

Answers:

(a)   _____

(b)   _____

(c)   _____

3. The diagram shows a point *P*, 12.3 m from the base of a building of height *h* m. The angle measured to the top of the building from point P is $63°$.

(a) Calculate the height *h* of the building.

Consider the formula $h = 4.9t^2$, where *h* is the height of the building and *t* is the time in seconds to fall to the ground from the top of the building.

(b) Calculate how long it would take for a stone to fall from the top of the building to   the ground.

Working:

Answers:

(a) _____

(b) _____

4. Two logic propositions are given.

p : Paula eats chocolates.

q : Paula watches television.

Write in words

(a) $p \wedge \neg q$,

(b) $p \veebar q$,

(c) $q \Rightarrow \neg p$.
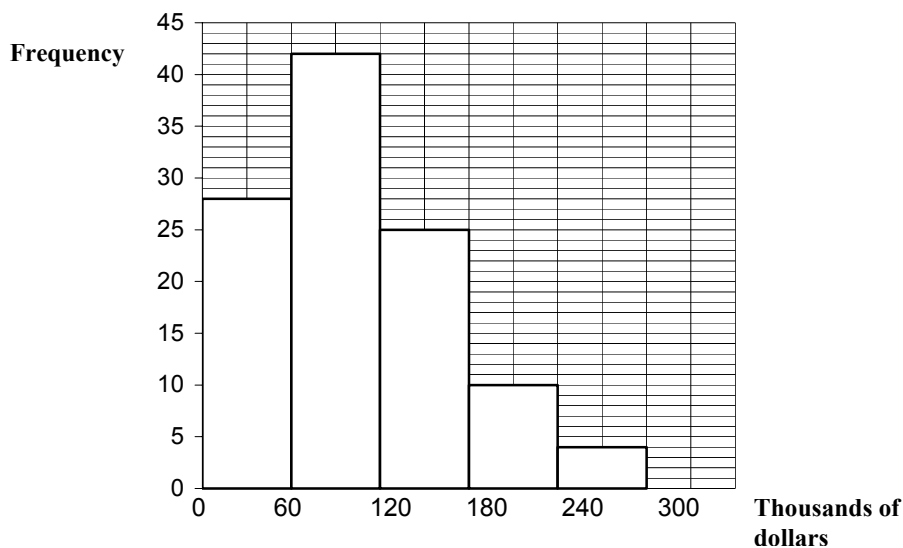
Working:

Answers:

(a) _____

_____

_____

(b) _____

_____

_____

(c) _____

_____

_____

5.   The following histogram shows the house prices in thousands of Australian dollars (AUD) of a random sample of houses in a certain town in Australia.

(a)   How many houses are there in the sample?

(b)   Write down the modal group for house prices.

(c)   Find the probability of choosing a house at random which costs less than 60 000 AUD or more than 240 000 AUD.

(d)   Given that a house costs more than 120 000 AUD, find the probability that it costs between 180 000 and 240 000 AUD.

Working:

Answers:

(a)   _____

(b)   _____

(c)   _____

(d)   _____

6.   A researcher consulted 500 men and women to see if the colour of the car they drove was independent of gender. The colours were red, green, blue, black and silver. A χ^2 test was conducted at the 5 % significance level and the value found to be 8.73.

(a)   Write down the null hypothesis.

(b)   Find the number of degrees of freedom for this test.

(c)   Write down the critical value for this test.

(d)   Is car colour independent of gender? Give a clear reason for your answer

Working:

Answers:

(a) _____

(b) _____

(c) _____

(d) _____

7. The following stem and leaf diagram gives the heights in cm of 39 schoolchildren.

Stem	Leaf	Key 13 $\mid$ 2 represents 132 cm.
13	2, 3, 3, 5, 8,	
14	1, 1, 1, 4, 5, 5, 9,	
15	3, 4, 4, 6, 6, 7, 7, 7, 8, 9, 9,	
16	1, 2, 2, 5, 6, 6, 7, 8, 8,	
17	4, 4, 4, 5, 6, 6,	
18	0,	

(a)   (i)   State the lower quartile height.

(ii)   State the median height

(iii)   State the upper quartile height.

(b)   Draw a box-and-whisker plot of the data using the axis below.

120   130   140   150   160   170   180   190

height in cm
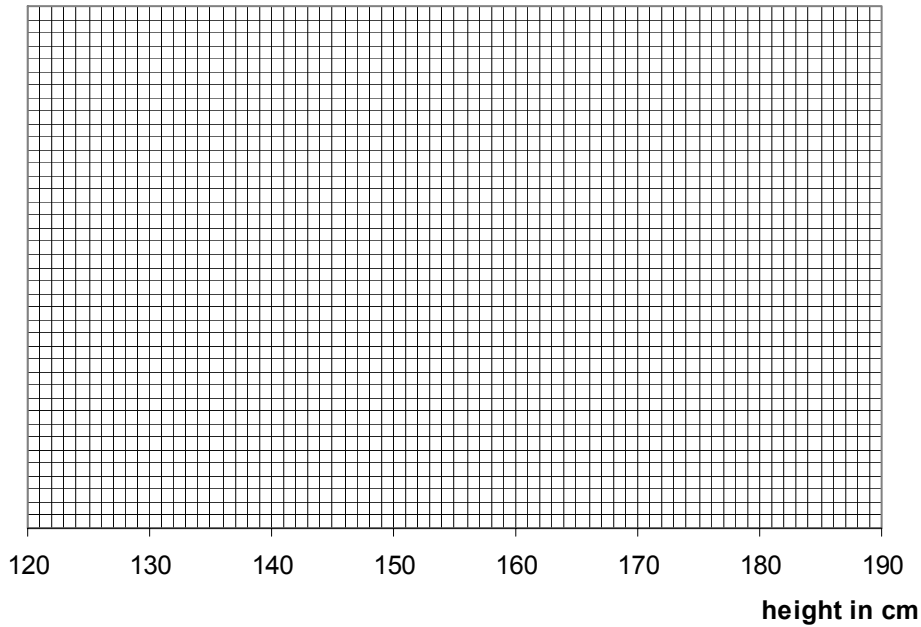
Working:

Answers:

(a) (i) _____

(ii) _____

(iii) _____

8. (a) Represent the function $y = 2x^2 - 5$, where $x \in \{-2, -1, 0, 1, 2, 3\}$ by a mapping diagram.
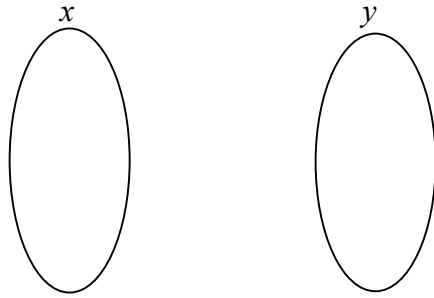
(b) List the elements of the domain of this function.

(c) List the elements of the range of this function.

Working:

Answers:

(b) _____

(c) _____

9.   A geometric sequence has all its terms positive. The first term is 7 and the third term is 28.

(a)   Find the common ratio.

(b)   Find the sum of the first 14 terms.

Working:

Answers:

(a) _____

(b) _____

10. Given $\mathbb{Z}$ the set of integers, $\mathbb{Q}$ the set of rational numbers, $\mathbb{R}$ the set of real numbers.

(a)   Write down an element that belongs to $\mathbb{R} \cap \mathbb{Z}$.

(b)   Write down an element that belongs to $\mathbb{Q} \cap \mathbb{Z}'$.

(c)   Write down an element that belongs to $\mathbb{Q}'$.

(d)   Use a Venn diagram to represent the sets $\mathbb{Z}$, $\mathbb{Q}$ and $\mathbb{R}$.

Working:

Answers:

(a) _____

(b) _____

(c) _____

(d) _____

11. The following table gives the amount of fuel in a car's fuel tank, and the number of kilometres travelled after filling the tank.

Distance travelled (km)	0	220	276	500	680	850
Amount of fuel in tank (litres)	55	43	30	24	10	6

(a) On the scatter diagram below, plot the remaining points.
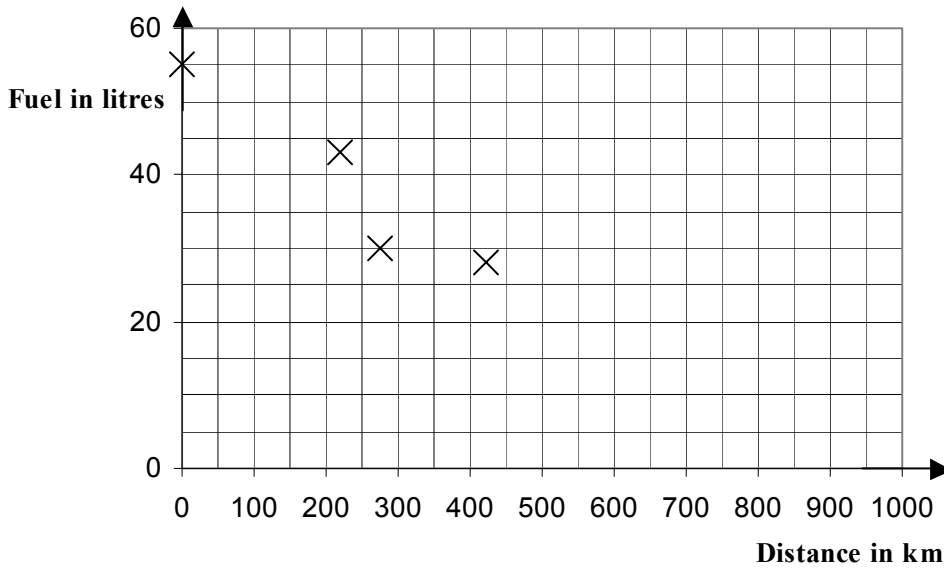
The mean distance travelled is 421 km $(\bar{x})$, and the mean amount of fuel in the tank is 28 litres $(\bar{y})$. This point is plotted on the scatter diagram.

(b) Sketch the line of best fit.

A car travelled 350 km.

(c) Use your line of best fit to estimate the amount of fuel left in the tank.

Working:

Answer:

(c) _____

12. Consider the line *l*: $2x + y + 4 = 0$.

   (a)   Write down the gradient of *l*.

   (b)   Find the gradient of a line perpendicular to *l*.

   (c)   Find the equation of a line perpendicular to *l*, passing through the point (5, 3). Give your answer in the form $ax + by + d = 0$, where a, b, d $\in \mathbb{Z}$.

Working:

Answers:

(a) _____

(b) _____

13. Events *A* and *B* have probabilities $P(A) = 0.4$, $P(B) = 0.65$, and $P(A \cup B) = 0.85$.

(a) Calculate $P(A \cap B)$.

(b) State with a reason whether events *A* and *B* are independent.

(c) State with a reason whether events *A* and *B* are mutually exclusive.

Working:

Answers:

(a) _____

(b) _____

_____

_____

(c) _____

_____

_____

14. A bank in Canada offers the following exchange rate between Canadian dollars (CAD) and Euros (EUR). The bank sells 1 CAD for 1.5485 EUR and buys 1 CAD for 1.5162 EUR. A customer wishes to exchange 800 Canadian dollars for Euros.

(a) Find how many Euros the customer will receive.

(b) The customer has to cancel his trip and changes his money back later when the rates are "sells 1 CAD = 1.5546 EUR, buys 1 CAD = 1.5284 EUR". Use the "we sell" information to find how many Canadian dollars he receives.

(c) How many Canadian dollars has he lost on the transaction?

Working:

Answers:

(a) _____

(b) _____

(c) _____

15. It is thought that a joke would spread in a school according to an exponential model $N = 4 \times (1.356)^{0.4t}$, $t \geq 0$; where N is the number of people who have heard the joke, and t is the time in minutes after the joke is first told.

(a) How many people heard the joke initially?

(b) How many people had heard the joke after 16 minutes?

There are 1200 people in the school.

(c) Estimate how long it would take for everybody in the school to hear this joke.

Working:

Answers:

(a) _____

(b) _____

(c) _____

IB DIPLOMA PROGRAMME
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MARKSCHEME

SPECIMEN

MATHEMATICAL STUDIES

Standard Level

Paper 1

7 pages

QUESTION 1

(a) (i)   Mean = 13.7   *(M1)(A1)*   *(G2)*

(ii) sd = 2.52   *(M1)(A1)*   *(G2)*

(b)   For attempting to put their numbers in order   *(M1)*
13.1   *(A1)*   *(G2)*

QUESTION 2

(a)   For multiplying 2 lengths   *(M1)*
6260.9 sq m (accept 6260 or 6261)   *(A1)*   *(C2)*

(b)   For multiplying each length by 100   *(M1)*
Award (M0)(A0) if multiplying their (a) by 100.
$9140 \times 6850 = 62609000$ (accept 62,600,000 or 62,610,000)   *(A1)*   *(C2)*

(c) 6.26×10^7 For 6.26   *(A1)*
For 10^7 *(A1)*   *(C2)*

QUESTION 3

(a)   For using tan   *(M1)*
$h = 12.3 \times \tan 63$ For using tan something   *(A1)*
$h = 24.1$ *(A1)*   *(G3)*

(b) $24.1 = 4.9t^2$ For substituting for h in the formula and attempting to solve   *(M1)*
For taking a square root (can be implied)   *(M1)*
2.22 sec   *(A1)*   *(C3)*

QUESTION 4

(a)   Paula eats chocolates and does not watch television
For "and"   *(A1)*
For the rest correct   *(A1)*   *(C2)*

(b)   Paula eats chocolates or watches television but not both
For correct…or…   *(A1)*
For "but not both"   *(A1)*   *(C2)*

(c)   If Paula watches television then she does not eat chocolates
for if….then   *(A1)*
for antecedent and consequent both correct   *(A1)*   *(C2)*

QUESTION 5

(a)   109   *(A1)*   *(C1)*

(b)   60–120 thousand dollars   *(A1)*   *(C1)*

(c) $\dfrac{32}{109}$ For correct numerator   *(A1)*

For correct denominator   *(A1)*   *(C2)*

(d) $\dfrac{10}{39}$ For correct numerator   *(A1)*

For correct denominator   *(A1)*   *(C2)*

QUESTION 6

(a)   Colour of car and gender are independent   *(A1)*   *(C1)*

(b) $(2-1)(5-1)$ *(M1)*

$= 4$ *(A1)*

OR

4   *(A2)*   *(C2)*

(c) $\chi^2 = 9.488$ *(A1)*   *(C1)*

(d)   Yes. Test statistic is smaller than the critical value.   *(A1)(R1)*   *(C2)*

QUESTION 7

(a)   (i)   145   *(A1)*

(ii)   157   *(A1)*

(iii)   167   *(A1)*   *(C3)*

(b)   For median in correct place   *(A1)*
For both quartiles in the correct places   *(A1)*
For correct 2 whiskers   *(A1)*   *(C3)*
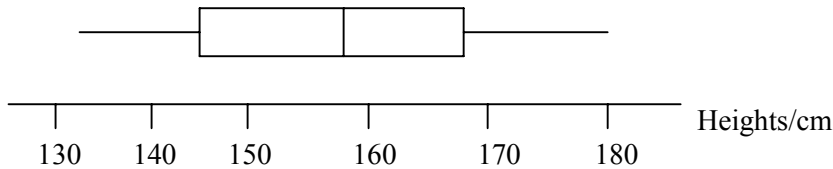
QUESTION 8

(a)
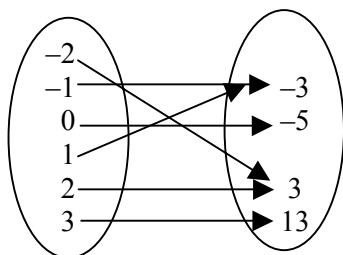
For six single lines going to correct y **(M1)**
(y-value can be repeated)
Correct diagram (y-values not repeated)   **(A1)**   **(C2)**

(b) $x \in \{-2, -1, 0, 1, 2, 3\}$ **(A2)**   **(C2)**
Award (A1) if one value omitted.

(c) $y \in \{-3, -5, 3, 13\}$ **(A2)**   **(C2)**

QUESTION 9

(a)   For obtaining an equation in r^2, can be implied   **(M1)**
$28 = 7r^2$ **(A1)**
$r = 2$ **(A1)**   **(C3)**

(b)   For using their value of r in the GP sum formula   **(M1)**
For obtaining 114681 (accept fewer sig fig up to 115000)   **(M1)(A1)**   **(C3)**

QUESTION 10

(a)   For example, 2, –3 *etc*   **(A1)**   **(C1)**

(b)   For example, $\dfrac{3}{5}\left(\text{not } \dfrac{6}{1}\right)$ **(A1)**   **(C1)**

(c)   For example, $\sqrt{2}, \pi$ **(A1)**   **(C1)**

(d) U
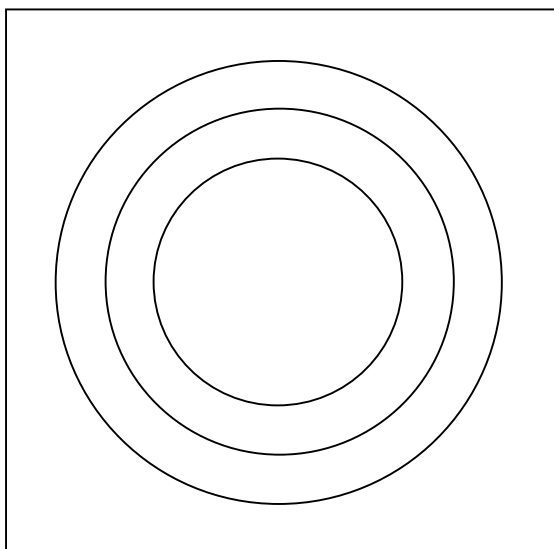
For $\mathbb{Z} \subset \mathbb{Q}$ **(A1)**
For $\mathbb{Z} \subset \mathbb{R}$ **(A1)**
For $\mathbb{Q} \subset \mathbb{R}$ **(A1)**   **(C3)**
Accept $\mathbb{R}$ as U.

QUESTION 11

(a)

For all 3 points correct   *(A2)*   *(C2)*
If only 2 points correct award (A1).

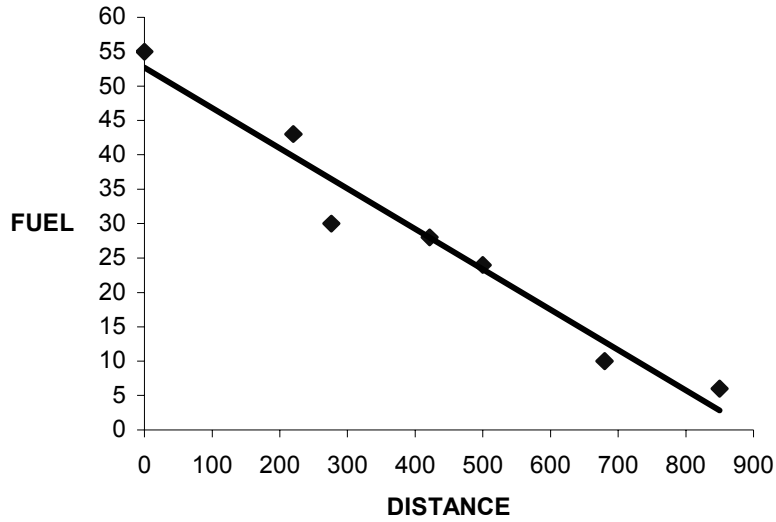(b)   For straight line **with –ve gradient** for passing through the mean   *(A1)(A1)*
      For straight line intercept on *y*-axis between 50 and 55   *(A1)*   *(C3)*

(c)   32 (read answer from candidate's line)   *(A1)*   *(C1)*

QUESTION 12

(a)   –2   *(A1)*   *(C1)*

(b)   gradient $= \dfrac{1}{2}$ *(A1)*   *(C1)*

(c)   Using $y = mx + c$ with their m to find c *(M1)*

$c = \dfrac{1}{2}$ *(A1)*

$y = \dfrac{1}{2}x + \dfrac{1}{2}$ *(A1)*

$x - 2y + 1 = 0$ *(A1)*   *(C4)*

QUESTION 13

(a)   For solving for $P(A \cap B)$ from the formula in their tables   *(M1)*
      $P(A \cap B) = 0.2$ *(A1)*   *(C2)*

(b)   Because $0.4 \times 0.65 \neq 0.2$ need to see the numbers, not just a statement   *(R1)*
      Therefore no, not independent   *(A1)*   *(C2)*
      Cannot award (A1) if (R1) not awarded

(c)   Because $P(A \cap B) \neq 0$ *(R1)*
      Not mutually exclusive   *(A1)*   *(C2)*
      Cannot award (A1) if (R1) not awarded.

QUESTION 14

    (a) 800×1.5162 for multiplying by 1.5162   ***(M1)***
              $= 1212.96$ EUR (accept 1213)   ***(A1)***   ***(C2)***

    (b)   1212.96 / 1.5546   ***(M1)***
              $= 780.24$ (accept 780)   ***(A2)***   ***(C3)***

    (c)   19.76 CAD   ***(A1)***   ***(C1)***

QUESTION 15

    (a)   4   ***(A1)***   ***(C1)***

    (b)   For raising to a power of 6.4   ***(M1)***
              28   ***(A1)***   ***(C2)***

    (c) $1200 = 4 \times (1.356)^{0.4t}$ (for substituting in the formula)   ***(M1)***
              $300 = (1.356)^{0.4t}$ ***(A1)***
              $t = 46.8$ (by trial and error)   ***(A1)***

             OR

              $t = 46.8$ ***(G3)***   ***(C3)***

Bank of specimen questions for mathematical studies – Paper 1

1. (a) The exam results for 100 boys are displayed in the following diagram:
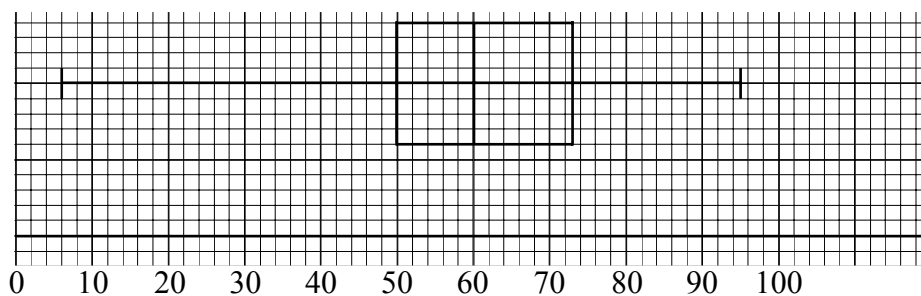
      (i) Find the range of the results.

      (ii) Find the inter quartile range.

      (iii) Write down the median.

(b) The exam results for 100 girls are displayed in the diagram below:
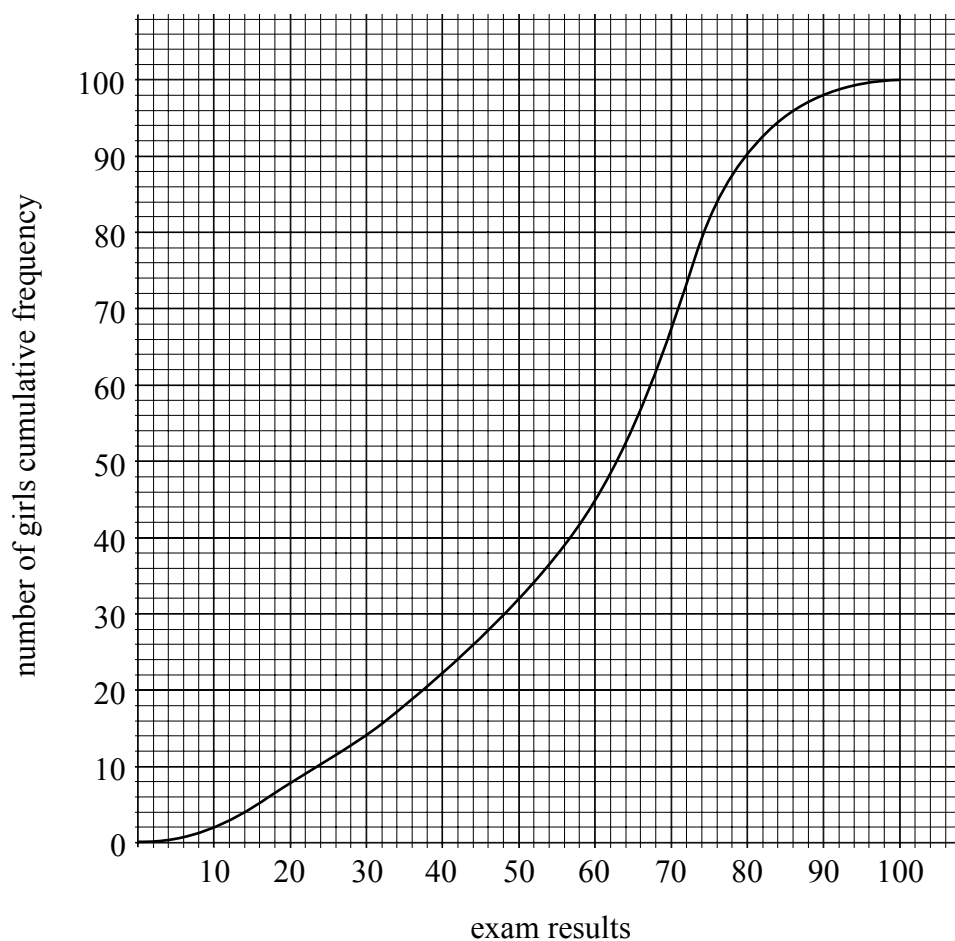
      (i) Write down the median.

      (ii) Find the inter quartile range.

(c) Write down the set of marks that are the most spread out and give a reason for your answer.

2. Write down the values for **a**, **b**, **c**, **d**, **e** and **f** from the table below:

p	q	¬p	p ∧ q	p ∨ q	p ⊻ q	p ⇒ q	p ⇔ q
T	T	a			d		
T	F		b				f
F	T			c			
F	F					e	

3. (a) Differentiate the following function with respect to x:

$$f(x) = 2x - 9 - 25x^{-1}$$

(b) Calculate the *x*-coordinates of the points on the curve where the gradient of the tangent to the curve is equal to 6.

4. Bob invests 600 EUR in a bank that offers a rate of 2.75 % compounded annually. The interest is added on at the end of each year.

(a) Calculate how much money Bob has in the bank after 4 years.

(b) Calculate the number of years it will take for the investment to double.

Ann invests 600 EUR in another bank that offers interest compounded annually. Her investment doubles in 20 years.

(c) Find the rate that the bank is offering.

5. (a) Differentiate the function $y = x^2 + 3x - 2$.

(b) At a certain point (*x, y*) on this curve the gradient is 5. Find the co-ordinates of this point.

6. Three points A (1, 3), B (4, 10) and C (7, –1) are joined to form a triangle. The mid-point of AB is D and the mid-point of AC is E.

(a) Plot the points A, B, C, on the grid.
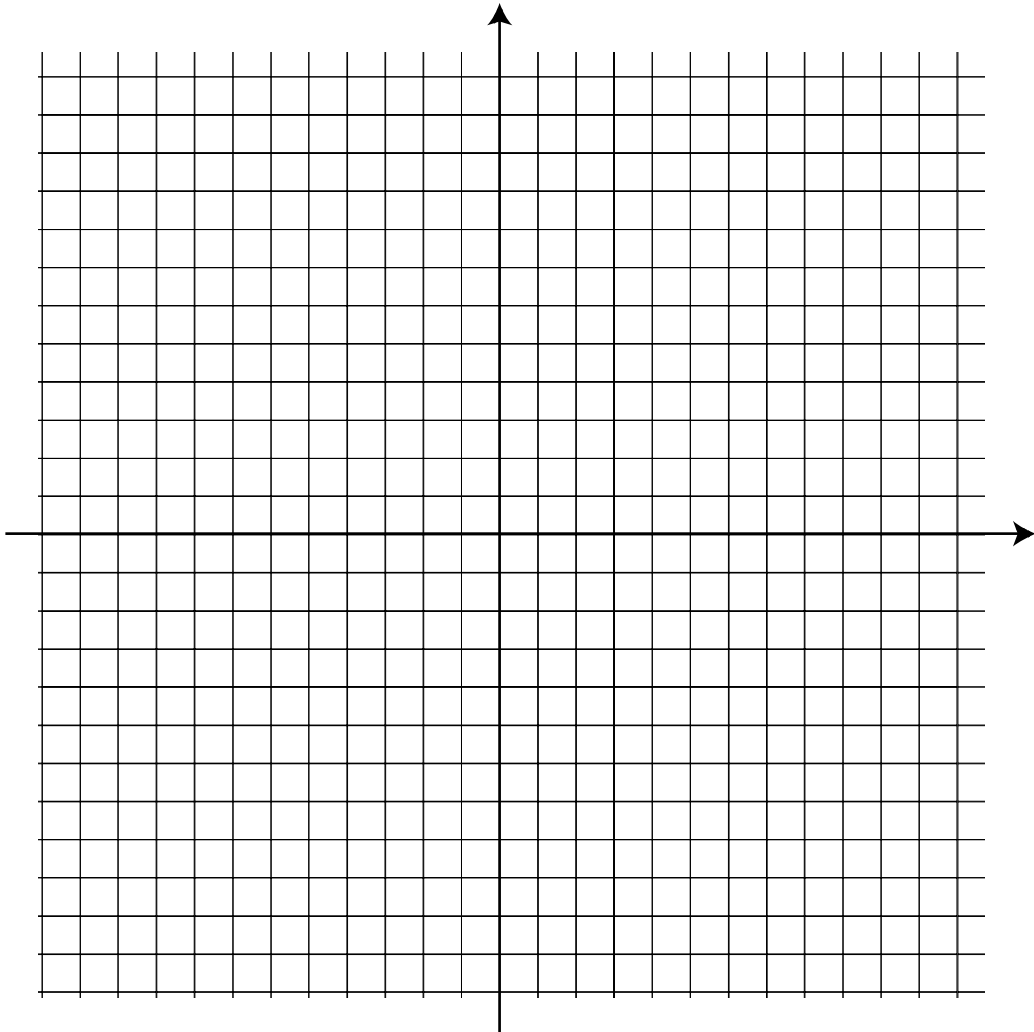
(b) Find the distance DE.

7. (a) Write $\dfrac{3}{x^2}$ in the form $3x^a$ where $a \in \mathbb{Z}$.

(b) Hence differentiate $y = \dfrac{3}{x^2}$ giving your answer in the form $\dfrac{b}{x^c}$ where $c \in \mathbb{Z}^+$.

8. (a) Sketch the graph of the function $y = 2x^2 - 6x + 5$.

(b) Write down the coordinates of the local maximum or minimum of the function.

(c) Find the equation of the axis of symmetry of the function.

9.   William invests $1200 for 5 years at a rate of 3.75 % compounded annually.

(a)   Calculate the amount of money he has in total at the end of the 5 years.

(b)   The interest rate then drops to 3.25 %. If he continues to leave his money in the bank find how much it will be worth after a further 3 years.

10.   Two students Ann and Ben play a game. Each time Ann passes GO she receives $15. Each time Ben passes GO he receives 8 % of the amount he already has. Both students start with $ 100.
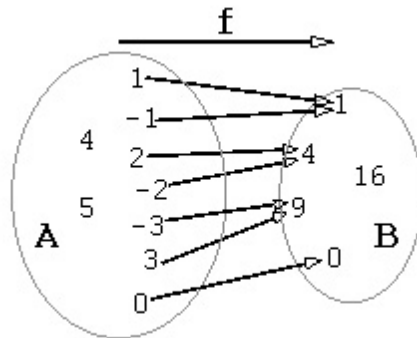
(a)   How much money will Ann have after she has passed GO 10 times?

(b)   How much money will Ben have after he passes GO 10 times?

(c)   How many times will the students have to pass GO for Ben to have more money than Ann?

11.   (a)   Sketch a graph of $y = \dfrac{x}{2+x}$ for $-10 \le x \le 10$.

(b)   Hence write down the equations of the horizontal and vertical asymptotes.

12.

The diagram shows a function f, mapping members of set A to members of set B.

(a)   (i)   Using set notation write down all members of the domain of f.

(ii)   Using set notation, write down all members of the range of f.

(iii)   Write down the equation of the function f.

The equation of a function g is $g(x) = x^2 + 1$. The domain of g is $\mathbb{R}$.

(b)   Write down the range of g.

13. Below is a graph of the function $y = a + b\sin(cx)$ where a, b and c are positive integers and x is measured in degrees.
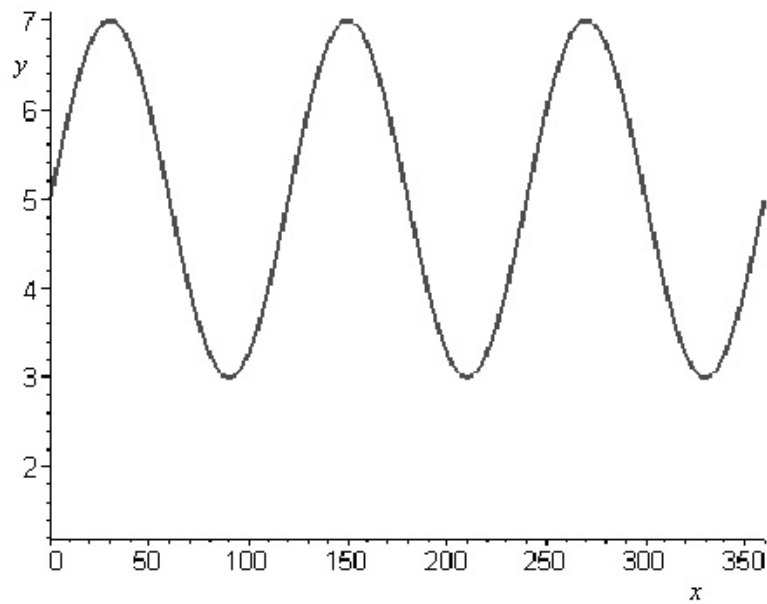
Find the values of a, b and c.

14. The following results give the heights of sunflowers in centimetres.

180   184   195   177   175   173   169   167   197   173   166   183   161   195   177
192   161   165

Represent the data by a stem and leaf diagram.

15. Tom performs a chi-squared test to see if there is any association between the time to prepare for a penalty kick (short time, medium time and long time) and the outcome (scores a goal, doesn't score a goal). Tom performs this test at the 10% level.

(a)   Write down the null hypothesis.

(b)   Find the number of degrees of freedom for this test.

(c)   The p-value for this test is 0.073. What conclusion can Tom make? Justify your answer.

Markscheme for bank of specimen questions for mathematical studies – Paper 1

QUESTION 1

   (a)   (i) $95 - 6 = 89$ **(A1)**

          (ii) $73 - 50 = 23$ **(A1)**

          (iii) 60   **(A1)**   **(C3)**

   (b)   (i)   62   **(A1)**

          (ii) $73 - 43 = 30$ **(A1)**   **(C2)**

   (c)   The girls as the IQR is larger   **(R1)**   **(C1)**

QUESTION 2

a = F   **(A1)**
b = F   **(A1)**
c = T   **(A1)**
d = F   **(A1)**
e = T   **(A1)**
f = F   **(A1)**

QUESTION 3

   (a) $f'(x) = 2 + 25x^{-2}$ **(A2)**   **(C2)**

   (b) $2 + 25x^{-2} = 6$ **(M1)**

$$25 = 4x^2 \qquad \textbf{(M1)}$$

$$x^2 = \frac{25}{4}$$

               $x = \pm 2.5$ **(A1)(A1)**   **(C4)**

QUESTION 4

(a) $600\left(1+\dfrac{2.75}{100}\right)^{4}=668.77$ (accept 669) *(M1)(A1)*

OR

669 *(G2)* *(C2)*

(b) $600\left(1+\dfrac{2.75}{100}\right)^{n}=1200$ *(M1)*

$n=25.6$
$n=26$ *(A1)*

OR

26 *(G2)* *(C2)*

(c) $600\left(1+\dfrac{r}{100}\right)^{20}=1200$ *(M1)*

$1+\dfrac{r}{100}=1.03526$
$r=3.53\,\%$ *(A1)*

OR

3.53 % *(G2)* *(C2)*

QUESTION 5

(a) $2x+3$ (–1 for each extra term) *(A2)* *(C2)*
If correct and an extra term included, award (A1) only.

(b) Equating the gradient to 5 $(2x+3=5)$ *(M1)*
For solving attempt *(M1)*
For $x=1$ *(A1)*
Co-ordinates (1, 2) *(A1)* *(C4)*

QUESTION 6

(a)
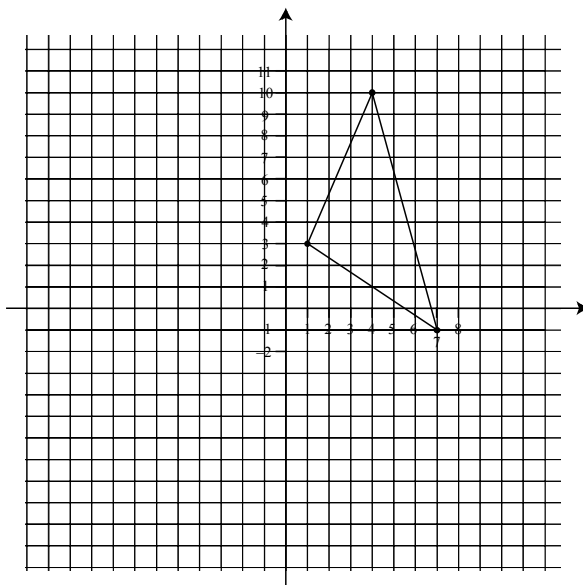
(A2)   *(C2)*

For 3 correctly plotted points.
For 2 correctly plotted points award (A1) only.

(b)   Co-ordinates D are (2.5, 6.5), ft on their AB   *(A1)*
       Co-ordinates E are (4, 1), ft on their AC   *(A1)*

       Distance $DE = \sqrt{(1.5^2 + 5.5^2)}$ for using Pythagoras   *(M1)*

           $= 5.70$ *(A1)*   *(C4)*

QUESTION 7

(a) $3x^{-2}$ *(A1)*   *(C1)*
       Award mark for −2.

(b) $-2 \times 3x^{-3}$ *(A1)(A1)*
       Award (A1) for -2×3, (A1) for −3.

       $= -6x^{-3}$ *(A1)*

       $= -\dfrac{6}{x^3}$ *(A1)(A1)*   *(C5)*

       Award (A1) for positive power on denominator, (A1) for 3.

QUESTION 8

(a)

(0, 5)

(A3)   *(C3)*

Award (A1) for point (0,5) *indicated.*
Award (A2) for correct shape.

(b)   (1.5, 0.5)   *(A1)(A1)*   *(C2)*

(c) $x = 1.5$ *(A1)*   *(C1)*

QUESTION 9

(a)   For attempting to find 5 years by compound interest formula or any
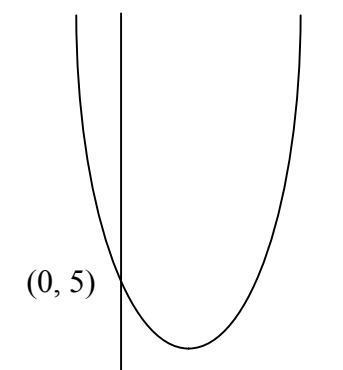   alternative method.   *(M1)*
      For using (1.0375)   *(M1)*
      $ 1442.52 accept 3 s.f.   *(A1)*   *(G3)*
      Accept $ 1440 *or* $ 1443.

(b)   For using answer in part (a) in an expression.   *(M1)*
      For multiplying by $(1.0325)^3$ *(M1)*
      $ 1587.79 accept $ 1588 or $ 1590   *(A1)*   *(G3)*

QUESTION 10

(a) $100 + 15 \times 10$ *(M1)*
      $= 250$ *(A1)*

   OR

      250 (using table function of the GDC)   *(G2)*   *(C2)*

(b) $100(1.08)^{10}$ *(M1)*
      $= 215.89$ *(A1)*

   OR

      215.89 (using table function of the GDC)   *(G2)*   *(C2)*

(c) $100 + 15x = 100(1.08)^x$ *(M1)*
      After 16 years   *(A1)*
      Candidate can use trial and error so not necessary to see the first line
      to award (A2).

   OR

      16 years (using table function of the GDC).   *(G2)*   *(C2)*

QUESTION 11

(a)
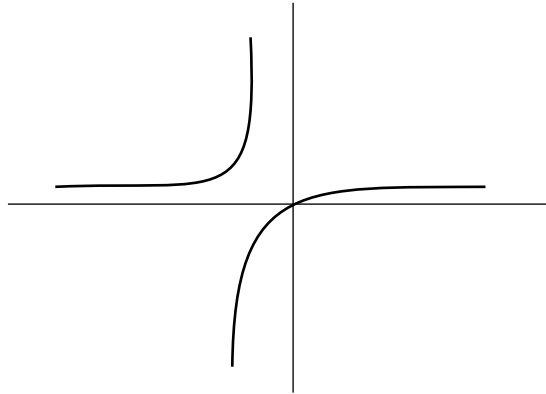
(A4) *(C4)*

Award (A1) for correct scales.
Award (A1)(A1) for two correct parts to the graph.
Award (A1) if asymptotes are shown.

(b)   Horizontal asymptote $y = 1$.   *(A1)* *(C1)*

(c)   Vertical asymptote $x = -2$.   *(A1)* *(C1)*

QUESTION 12

(a)   (i) $\{-3, -2, -1, 0, 1, 2, 3\}$ *(A1)(A1)*
Award (A1) for set brackets.
Award (A1) for all and only correct numbers.

(ii) $\{0, 1, 4, 9\}$ *(A1)*
Award (A1) for all and only correct numbers.
If domain and range reversed, can follow through in (ii).

(iii) $f(x) = x^2$ *(A2)* *(C5)*
Allow any other rule that works.

(b) $[1, \infty)$ or $\{x \in \mathbb{R} \mid x \geq 1\}$ *(A1)* *(C1)*

QUESTION 13

a = 5   *(A2)*
b = 2   *(A2)*
c = 3   *(A2)*

QUESTION 14

Unsorted	
stem	leaf
16	976115
17	75337
18	043
19	5752

Sorted	
stem	leaf
16	115679
17	33577
18	034
19	2557

Key: 16│1 represents 161 cm

For sorted diagram attempt   *(M2)*
For an unsorted diagram attempt award (M1) only.
All entries correct   *(A2)*
For one error in entries award (A1) only.
For key   *(A1)*
For correct key with units   *(A1)*

QUESTION 15

(a)   Time to prepare is independent of outcome, or, there is no association
      between time to prepare and the outcome   *(A1)*   *(C1)*

(b)   2   *(A1)*   *(C1)*

(c) $0.073 < 0.10$ For comparing 0.073 with 0.10 or 10 %   *(M1)*
      For < or saying 'less than'   *(M1)*
      Reject H_0 *(A1)*
      Time and outcome are not independent of each other or equivalent in words
      relating to the question   *(A1)*   *(C4)*

MATHEMATICAL STUDIES
STANDARD LEVEL
PAPER 2

SPECIMEN

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.

8 pages

Please start each question on a new page. You are advised to show all working, where possible. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. *[Maximum mark: 18]*

    (i) Celia has $ 20 000 to invest. There are two different options that she can choose.

    Option 1: The investment grows at a rate of 3.5 % compound interest each year.
    Option 2: The total value of the investment increases by $ 800 each year.

    The money is to be invested for 15 years.

    (a) Copy and complete the table below giving the values of the investments to the nearest dollar for the first 4 years.   *[3 marks]*

Year	0	1	2	3	4
Option 1	20 000	20 700			
Option 2	20 000	20 800			

    (b) Calculate the values of each investment at the end of 15 years.   *[4 marks]*

    (c) If Option 1 is chosen find the total number of complete years before the values of the investment is first greater than $ 25 000.   *[2 marks]*

    (d) If Option 2 is chosen calculate the percentage increase in the investment for the final year.   *[2 marks]*

    (This question continues on the following page)

(Question 1 continued)

(ii)   Two more Options are available to Celia. After 7 years she can change the investment conditions.

Option 3:   If Celia has chosen Option 1 she can change and then receives $800 each year until the end of the 15 years.

Option 4:   If Celia has chosen Option 2 she can change and then receive 3.5 % interest compounded annually.

If Celia wishes to receive the maximum amount of money at the end of the 15 years which option should she choose?   *[7 marks]*

2.   *[Maximum mark: 16]*

Consider the function $f(x) = \dfrac{3}{x^2} + x - 4$.

(a)   Calculate the value of $f(x)$ when $x = 1$.   *[2 marks]*

(b)   Differentiate $f(x)$.   *[4 marks]*

(c)   Find $f'(1)$.   *[2 marks]*

(d)   Explain what $f'(1)$ represents.   *[2 marks]*

(e)   Find the equation of the tangent to the curve $f(x)$ at the point where $x = 1$.   *[3 marks]*

(f)   Determine the x-coordinate of the point where the gradient of the curve
       is zero.   *[3 marks]*

3.   *[Maximum mark: 20]*

(i)   The diagram below shows a field ABCD with a fence BD crossing it.
AB $= 15\,$m, AD $= 20$ m and angle BAD $= 110°$.   BC $= 22$ m   and   angle
BDC $= 30°$.

diagram not to scale
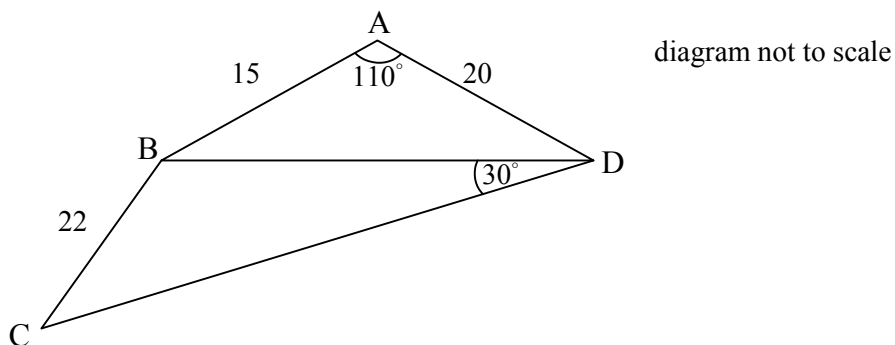
(a)   Calculate the length of BD.   *[3 marks]*

(b)   Calculate the size of angle $\hat{BCD}$ *[3 marks]*

One student gave the answer to (a) "correct to 1 significant figure" and
used this answer to calculate the size of angle $\hat{BCD}$.

(c)   Write down the length of BD correct to 1 significant figure.   *[1 mark]*

(d)   Find the size of angle $\hat{BCD}$ that the student calculated, **giving
your answer correct to 1 decimal place.**   *[2 marks]*

(e)   Hence find the percentage error in his answer for angle $\hat{BCD}$.   *[3 marks]*

(This question continues on the following page)

(Question 3 continued)

(ii) It is decided to take a random sample of 10 students to see if there is any linear relationship between height and shoe size. The results are given in the table below.

Height (cm) (x)	Shoe size (y)
175	8
160	9
180	8
155	7
178	10
159	8
166	9
185	11
189	10
173	9

(a) Write down the equation of the regression line of shoe size (y) on height (x), giving your answer in the form $y = mx + c$.   *[3 marks]*

(b) Use your equation in part (a) to predict the shoe size of a student who is 162 cm in height.   *[2 marks]*

(c) Write down the correlation coefficient.   *[1 mark]*

(d) Describe the correlation between height and shoe size.   *[2 marks]*

4.   *[Maximum mark: 17]*

(a)   Sketch the graph of the function $f : x \mapsto 1 + 2\sin x$, where $x \in \mathbb{R}$,
       $-360° \leq x \leq 360°$.   *[4 marks]*

(b)   Write down the range of this function for the given domain.   *[2 marks]*

(c)   Write down the amplitude of this function.   *[1 mark]*

(d)   On the same diagram sketch the graph of the function $g : x \mapsto \sin 2x$,
       where $x \in \mathbb{R}$, $-360° \leq x \leq 360°$.   *[4 marks]*

(e)   Write down the period of this function.   *[1 mark]*

(f)   Use the sketch graphs drawn to find the number of solutions to the
       equation $f(x) = g(x)$ in the given domain.   *[1 mark]*

(g)   Hence solve the equation $1 + 2\sin x = \sin 2x$ for $\mathbf{0^o \leq x \leq 360^o}$.   *[4 marks]*

5.   *[Maximum mark: 19]*

A child's toy is made by combining a hemisphere of radius 3 cm and a right circular cone of slant height *l* as shown on the diagram below.

diagram not to scale
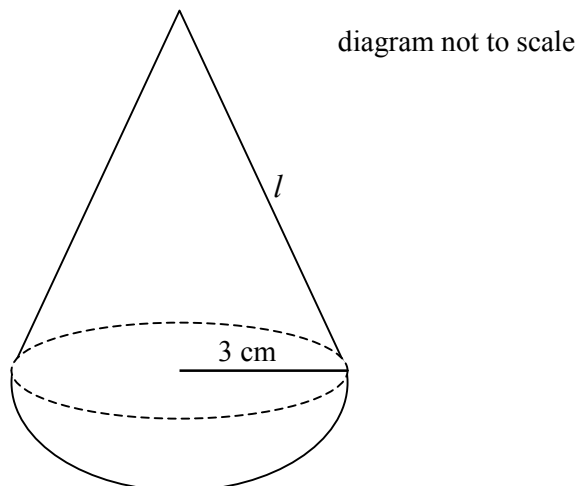
(a)   Show that the volume of the hemisphere is $18\pi \text{ cm}^3$.   *[2 marks]*

The volume of the cone is two-thirds that of the hemisphere.

(b)   Show that the vertical height of the cone is 4 cm.   *[4 marks]*

(c)   Calculate the slant height of the cone.   *[2 marks]*

(d)   Calculate the angle between the slanting side of the cone and the flat surface of the hemisphere.   *[3 marks]*

(e)   The toy is made of wood of density 0.6 g per cm^3 . Calculate the weight of the toy.   *[3 marks]*

(f)   Calculate the total surface area of the toy.   *[5 marks]*

MARKSCHEME

SPECIMEN

MATHEMATICAL STUDIES

Standard Level

Paper 2

7 pages

QUESTION 1

(i) (a)

	0	1	2	3	4
Option 1	20000	20700	**21425**	**22174**	**22950**
Option 2	20000	20800	**21600**	**22400**	**23200**

(A3)

*Award **(A2)** for Option 1 all correct.*
*Award **(A1)** for Option 2 all correct.*

[3 marks]

(b) $A = 20000(1.035)^{15}$ *(A1)*
Option 1 $= 33507$ *(A1)*
$A = 20000 + 15 \times 800$ *(A1)*
Option 2 $= 32000$ *(A1)*

OR

Option 1 $= 33507$ *(G2)*
Option 2 $= 32000$ *(G2)*

[4 marks]

(c) 7 years (from reading the table from GDC) *(G2)*

[2 marks]

(d) $\dfrac{800}{31200} \times 100$ *(M1)*
*Award **(M1)** for candidate's difference divided by their original times 100.*
$= 2.56\,\%$ *(A1)*

[2 marks]

(ii) Option 3: $20000 \times (1.035)^7 + 800 \times 8$ *(M1)(A1)*
$= \$\,31845.59$ *(A1)*
Option 4: $(20000 + 7 \times 800) \times (1.035)^8$ *(M1)(A1)*
$= \$33710.31$ *(A1)*

OR

$\$\,31845.59$ *(G3)*
$\$\,33710.31$ *(G3)*

She should choose Option 4 *(R1)*

[7 marks]

Total [18 marks]

QUESTION 2

(a) $f(1) = \dfrac{3}{1^2} + 1 - 4$ **(M1)**

$\quad\quad = 0$ **(A1)**

OR

$f(1) = 0$ **(G2)**

[2 marks]

(b) $f'(x) = -\dfrac{6}{x^3} + 1$ **(A4)**

*Award **(A2)** for $\dfrac{3}{x^2}$ correctly differentiated and **(A1)** for each other term correctly differentiated.*

[4 marks]

(c) $f'(1) = -\dfrac{6}{1} + 1$ for substituting $f'(x)$ **(M1)**

$\quad\quad = -5$ **(A1)**

OR

$f'(1) = -5$ **(G2)**

[2 marks]

(d) The gradient of the curve where $x = 1$. **(A2)**
*Award **(A1)** for gradient and **(A1)** for $x = 1$ or at point $(1, 0)$.*

[2 marks]

(e) $y = 0$, $x = 1$, $m = -5$ for using $y = mx + c$ with their correct values of m , x and y. **(M1)**
$0 = -5 \times 1 + c$
$c = 5$ **(A1)**
$y = -5x + 5$ **(A1)**

OR

$y = -5x + 5$ **(G3)**

[3 marks]

(f) $f'(x) = 0$

$1 - \dfrac{6}{x^3} = 0$ **(M1)(A1)**

$x^3 = 6$
$x = \sqrt[3]{6}$ (1.82) **(A1)**

OR

1.82 **(G3)**

[3 marks]

Total [16 marks]

QUESTION 3

(i) (a) $BD^2 = 15^2 + 20^2 - 2 \times 15 \times 20 \times \cos 110°$ **(M1)(A1)**
Award (M1) for using the cosine rule, award (A1) for correct substitution.
$BD^2 = 830.212$
$BD = 28.8$ **(A1)**

OR

$BD = 28.8$ **(G3)**
[3 marks]

(b) $\dfrac{28.81}{\sin C} = \dfrac{22}{\sin 30°}$ **(M1)(A1)**
$C = 40.9°$ **(G1)**

OR

$C = 40.9°$ **(G3)**
[3 marks]

(c) $BD = 30$ **(A1)**
[1 mark]

(d) $\dfrac{30}{\sin C} = \dfrac{22}{\sin 30°}$ **(M1)**
$C = 43.0°$ **(A1)**

OR

$C = 43.0°$ **(G2)**
[2 marks]

(e) Percentage error $= \dfrac{43.0 - 40.9}{40.9} \times 100$ **(M1)(A1)**
$= 5.13\,\%$ **(A1)**
[3 marks]

(ii) (a) $y = 0.070x - 3.22$ **(G3)**
Award (G1) for correct m value, (G1) for 3.22, (G1) for negative sign.
Accept $0.07x$.
[3 marks]

(b) $y = 0.070 \times 162 - 3.22$ **(M1)**
$= 8.12$
Therefore shoe size 8 or 9 (8.12). **(A1)**

OR

$y = 8$ or 9 **(G2)**
[2 marks]

(c) $r = 0.681$ **(A1)**
[1 mark]

(d) Moderately strong, positive correlation. **(A1)(A1)**
[2 marks]

Total [20 marks]

QUESTION 4

(a)
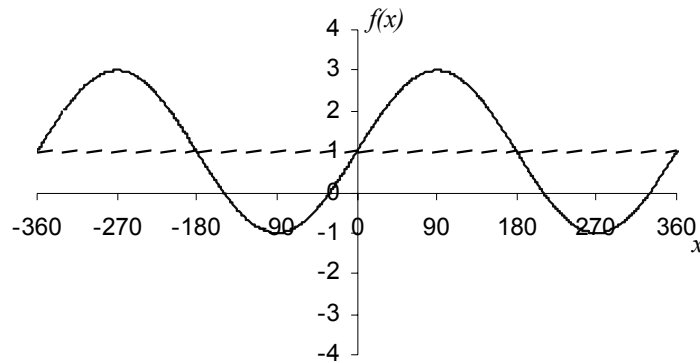
x-axis from $-360°$ to $360°$	*(A1)*
2 maxima at $y = 3$	*(A1)*
2 minima at $y = -1$	*(A1)*
Correct shape of graph with reasonable axes intercepts.	*(A1)*
	[4 marks]

(b)   Range $-1 \le y \le 3$ or $[-1, 3]$ *(A2)*

Award (A1) for –1 to 3.

[2 marks]

(c)   Amplitude $= 2$ *(A1)*

[1 mark]

(d)
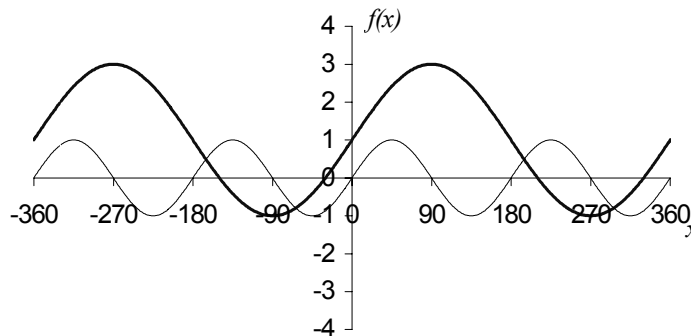
Correct maximum	*(A1)*
Correct minimum	*(A1)*
Correct period	*(A1)*
Correct shape with reasonable axes intercepts.	*(A1)*
	[4 marks]

(e)   Period $180°$ *(A1)*

[1 mark]

(f)   4 solutions   *(A1)*

[1 mark]

(g) $x = 195°$ *(G2)*

$x = 296°$ *(G2)*

If more than two solutions given award (A2).

[4 marks]

Total [17 marks]

QUESTION 5

(a) $V = \dfrac{1}{2} \times \dfrac{4}{3}\pi r^3$ For using $\dfrac{4}{3}\pi r^3$ (with or without $\dfrac{1}{2}$) **(M1)**

$= \dfrac{2}{3} \times \pi \times 3^3$ For using $\dfrac{1}{2}$ (their sphere formula) **(M1)**

$= 18\pi \text{ cm}^3$ **(AG)**

[2 marks]

(b) $V = \dfrac{2}{3} \times 18\pi$ For using $\dfrac{2}{3} \times$ their answer to (a) **(M1)**

$= 12\pi$ **(A1)**

$12\pi = \dfrac{1}{3}\pi \times 3^2 \times h$ For equating the volumes **(M1)**

$\dfrac{36\pi}{9\pi} = h \quad \left(\dfrac{113.097}{28.27}\right)$ **(A1)**

$h = 4 \text{ cm}$ **(AG)**

[4 marks]

(c) $l^2 = 4^2 + 3^2$ For using Pythagoras theorem **(M1)**

$l = 5$ **(A1)**

[2 marks]

(d) For identifying the correct angle **(M1)**

$\tan\theta = \dfrac{4}{3}$ or $\sin\theta = \dfrac{4}{5}$ or $\cos\theta = \dfrac{3}{5}$ **(M1)**

$\theta = 53.1°$ (0.927 radians) **(A1)**

[3 marks]

(e) For summing volume of cone and hemisphere. **(M1)**
Volume $= 12\pi + 18\pi$
$\quad = 30\pi \text{ cm}^3$ (94.2 cm^3)

For multiplying the volume by 0.6 **(M1)**
Weight $= 0.6 \times 30\pi$
$\quad = 56.5\text{g}$ **(A1)**

[3 marks]

(f) Surface area of cone $= \pi r l$
$\quad = \pi \times 3 \times 5 = 15\pi$ **(M1)(A1)**

Surface area of a hemisphere $= \dfrac{1}{2} \times 4\pi r^2$

$\quad = \dfrac{1}{2} \times 4 \times \pi \times 3^2$

$\quad = 18\pi$ **(M1)(A1)**

Total surface area $= 15\pi + 18\pi$
$\quad = 103.67$
$\quad = 104 \text{ cm}^2$ **(A1)**

[5 marks]

Total [19 marks]

Total for paper [90 marks]

Bank of specimen questions for mathematical studies – paper 2

1.   *[Maximum mark: 15]*

(a)   On the same graph sketch the curves $y = x^2$ and $y = 3 - \dfrac{1}{x}$ for values of x from 0 to 4 and values of y from 0 to 4. Show your scales on your axes.   *[4 marks]*

(b)   Find the points of intersection of these two curves.   *[4 marks]*

(c)   (i)   Find the gradient of the curve $y = 3 - \dfrac{1}{x}$ in terms of x.

    (ii)   Find the value of this gradient at the point (1,2).   *[4 marks]*

(d)   Find the equation of the tangent to the curve $y = 3 - \dfrac{1}{x}$ at the point (1, 2).   *[3 marks]*

2.   *[Maximum mark: 16]*

The functions f and g are defined by

$$f : x \mapsto \frac{x+4}{x}, \ x \in \mathbb{R}, \ x \neq 0$$
$$g : x \mapsto x, \ x \in \mathbb{R}$$

(a)   Sketch the graph of f for $-10 \leq x \leq 10$.   *[4 marks]*

(b)   Write down the equations of the horizontal and vertical asymptotes of the function f.   *[4 marks]*

(c)   Sketch the graph of g on the same axes.   *[2 marks]*

(d)   Hence, or otherwise, find the solutions of $\dfrac{x+4}{x} = x$.   *[4 marks]*

(e)   Write down the range of function f.   *[2 marks]*

3. *[Maximum mark: 8]*

The figure below shows the graphs of the functions $y = x^2$ and $y = 2^x$ for values of x between -2 and 5.
The points of intersection of the two curves are marked as B, C and D.
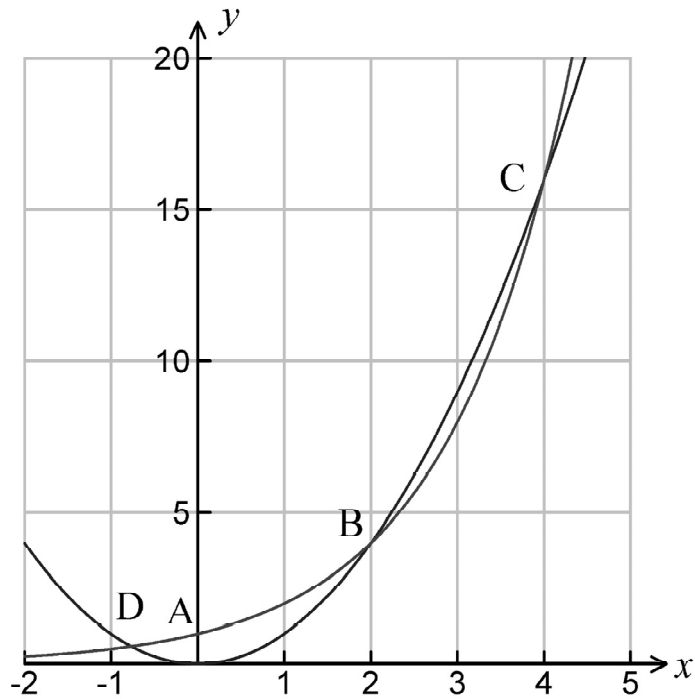
(a) Write down the coordinates of the point A. *[2 marks]*

(b) Write down the coordinates of the points B and C. *[2 marks]*

(c) Find the x-coordinate of the point D. *[1 mark]*

(d) Write down, using interval notation, all values of x for which $2^x \leq x^2$. *[3 marks]*

4. *[Maximum mark: 16]*

At the circus a clown is swinging from an elastic rope. A student decides to investigate the motion of the clown. The results can be shown on the graph of the function $f(x) = (0.8^x)(5\sin 100x)$, where x is the horizontal distance in metres.

(a) Sketch the graph of $f(x)$ for $0 \le x \le 10$ and $-3 \le f(x) \le 5$. *[5 marks]*

(b) Find the coordinates of the first local maximum point. *[2 marks]*

(c) Find the coordinates of one point where the curve cuts the y-axis. *[1 mark]*

Another clown is fired from a cannon. The clown passes through the points given in the table below:

Horizontal distance (x)	Vertical distance (y)
0.00341	0.0102
0.0238	0.0714
0.563	1.69
1.92	5.76
3.40	10.2

(d) Find the correlation coefficient, r, and comment on the value for r. *[3 marks]*

(e) Write down the equation of the regression line of y on x. *[2 marks]*

(f) Sketch this line on the graph of $f(x)$ in part (a). *[1 mark]*

(g) Find the coordinates of one of the points where this line cuts the curve. *[2 marks]*

5. *[Maximum mark: 15]*

A closed box has a square base of side x and height h.

(a) Write down an expression for the volume, V, of the box. *[1 mark]*

(b) Write down an expression for the total surface area, A, of the box. *[1 mark]*

The volume of the box is 1000 cm^3

(c) Express h in terms of x. *[2 marks]*

(d) Hence show that $A = 4000x^{-1} + 2x^2$. *[2 marks]*

(e) Find $\dfrac{dA}{dx}$. *[2 marks]*

(f) Calculate the value of x that gives a minimum surface area. *[4 marks]*

(g) Find the surface area for this value of x. *[3 marks]*

6. *[Maximum mark: 7]*

The eye colour and gender of 500 students are noted and the results are indicated in the table below.

	Blue	Brown	Green
Male	18	152	50
Female	40	180	60

It is believed that eye colour is related to gender in a school in Banff. It is decided to test this hypothesis by using a χ^2 test at the 5% level of significance.

(a) Write down the null hypothesis for this experiment. *[1 mark]*

(b) Show that the number of degrees of freedom is 2. *[1 mark]*

(c) Write down the χ^2 critical value for the degrees of freedom. *[1 mark]*

(d) Calculate the χ^2 test statistic for this data. *[2 marks]*

(e) Does the evidence suggest that eye colour is related to gender in this school? Give a clear reason for your answer. *[2 marks]*

Markscheme for bank of specimen questions – paper 2

1. (a)
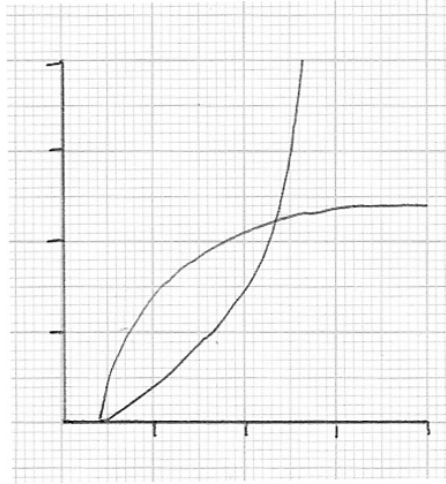
For correct axes from 0 to 4.	*(A1)*
For correct curve $y = x^2$.	*(A1)*
For correct curve $y = 3 - \dfrac{1}{x}$.	*(A1)*
For two intersections.	*(A1)*
	[4 marks]

(b) $(0.347, 0.121)$ or $x = 0.347$, $y = 0.121$ (by GDC)   *(G1)(G1)*

   $(1.53, 2.35)$ or $x = 1.53$, $y = 2.35$.   *(G1)(G1)*

   [4 marks]

(c) (i) $\dfrac{dy}{dx} = \dfrac{1}{x^2}$ for losing the constant.   *(A1)*

   For attempting to write $\dfrac{1}{x}$ as a power (can be implied).   *(M1)*

   For correct answer $\dfrac{1}{x^2}$ or x^{-2}.   *(A1)*

   (ii)   1   *(A1)*

   [4 marks]

(d) For using $y = mx + c$ or equivalent with their *m,* to find *c.*   *(M1)*

   $c = 1$ *(A1)*

   $y = x + 1$ *(A1)*

   [3 marks]

   Total [15 marks]

2.　(a)
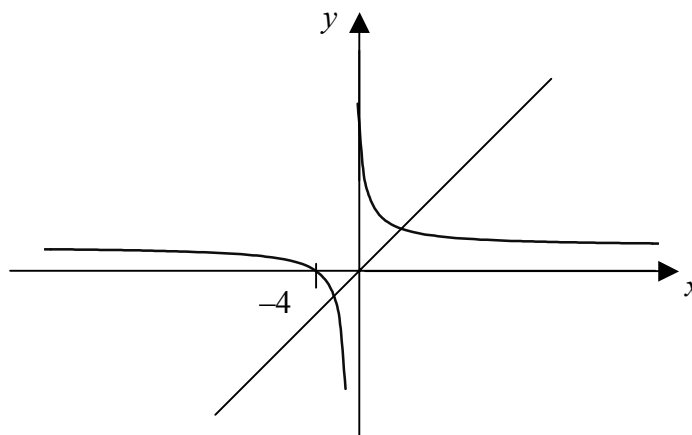
　　　For *x*-axis from –10 to 10.　　　　　　　　　　　　　　　　　　*(A1)*
　　　For –4 marked.　　　　　　　　　　　　　　　　　　　　　　*(A1)*
　　　For correct shape of graph.　　　　　　　　　　　　　　　　*(A1)(A1)*
　　　　　　　　　　　　　　　　　　　　　　　　　　　　　[4 marks]

　(b)　Horizontal asymptote　　　　　　　　　　　　　　　　　　*(A1)*
　　　$y = 1$　　　　　　　　　　　　　　　　　　　　　　　　*(A1)*
　　　Vertical asymptote　　　　　　　　　　　　　　　　　　　*(A1)*
　　　$x = 0$　　　　　　　　　　　　　　　　　　　　　　　　*(A1)*
　　　　　　　　　　　　　　　　　　　　　　　　　　　　　[4 marks]

　(c)　Line drawn on sketch　　　　　　　　　　　　　　　　　　*(A2)*
　　　　　　　　　　　　　　　　　　　　　　　　　　　　　[2 marks]

　(d)　(2.56, 2.56) (–1.56, –1.56)　　　　　　　　　　*(A1)(A1)(A1)(A1)*
　　　　　　　　　　　　　　　　　　　　　　　　　　　　　[4 marks]

　(e)　Range $y \in \mathbb{R}$, $y \neq 1$　　　　　　　　　　　　　　*(A1)(A1)*
　　　　　　　　　　　　　　　　　　　　　　　　　　　　　[2 marks]

　　　　　　　　　　　　　　　　　　　　　　　　Total [16 marks]

3.   (a)   A = (0,1)

             For parentheses   *(A1)*

             For numbers   *(A1)*

                                                                            [2 marks]

    (b)   B = (2,4), C = (4,16)

             For 2,4   *(A1)*

             For 4,16   *(A1)*

                                                                            [2 marks]

    (c)   At D, $x = -0.767$ *(A1)*

                                                                            [1 mark]

    (d) $x \leq -0.767$ *(A1)*

             $2 \leq x \leq 4$

             For inequalities   *(A1)*

             For numbers   *(A1)*

             OR

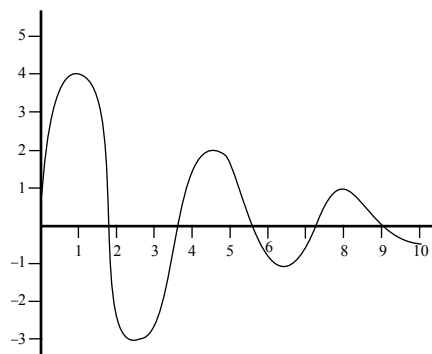 $(-\infty, -0.767] \cup [2, 4]$ *(A3)*

                                                                            [3 marks]

                                                                    Total [8 marks]

4. (a)

For labels and scales.	*(A1)*
3 maxima drawn.	*(A1)*
2 minima drawn.	*(A1)*
General shape	*(A2)*

[5 marks]

(b)   (0.827, 4.12)   *(G2)*

[2 marks]

(c)   0, 1.8, 3.6, 5.4, 7.2, 9 (for any one of these answers).   *(G1)*

[1 mark]

(d) $r = 1$ *(G2)*
      Perfect positive correlation.   *(R1)*

[3 marks]

(e) $y = 3x$ (accept $y = 3x + 0.000274$)   *(G2)*

[2 marks]

(f)   line on graph   *(A1)*

[1 mark]

(g)   (0, 0) or (1.16, 3.48)   *(G1)(G1)*

[2 marks]

Total [16 marks]

5.   (a) $V = x^2 h$ *(A1)*

[1 mark]

(b) $A = 2x^2 + 4xh$ *(A1)*

[1 mark]

(c) $1000 = x^2 h$ *(M1)*

$$h = \frac{1000}{x^2}$$ *(A1)*

[2 marks]

(d) $A = 2x^2 + 4x\left(\dfrac{1000}{x^2}\right)$ *(M1)*

$$A = 2x^2 + \frac{4000}{x}$$ *(A1)*

$$= 2x^2 + 4000x^{-1}$$ *(AG)*

[2 marks]

(e) $\dfrac{\mathrm{d}A}{\mathrm{d}x} = 4x - 4000x^{-2}$ *(A2)*

[2 marks]

(f) $4x - 4000x^{-2} = 0$ *(M1)*

$4x^3 - 4000 = 0$ *(M1)*

$4x^3 = 4000$

$x^3 = 1000$ *(A1)*

$x = 10$ *(A1)*

OR

$x = 10$ *(G4)*

[4 marks]

(g) $h = \dfrac{1000}{100} = 10$ *(A1)*

$A = 2(100) + 4(10)(10)$ *(M1)*

$= 200 + 400 = 600$ *(A1)*

OR

$A = 600$ *(G3)*

[3 marks]

Total [15 marks]

6. (a) Eye colour and gender are independent.

OR

There is no relationship (association) between eye colour and gender. *(A1)*

[1 mark]

(b) $(2-1)(3-1)$ *(M1)*
$= 2$ *(AG)*

[1 mark]

(c) 5.991 (5.99) *(A1)*

[1 mark]

(d) 4.48 *(G2)*

[2 marks]

(e) For comparing χ^2 test statistic with χ^2 critical value *(A1)*
No, eye colour is not related to gender
χ^2 test statistic $< \chi^2$ critical value *(R1)*

OR

For comparing their *p*-value with 0.05
No, eye colour is not related to gender *(A1)*
p-value of $0.106 > 0.05$ *(R1)*

[2 marks]

Total [7 marks]