IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
PROGRAMA DEL DIPLOMA DEL BI

Mathematics
Standard level

Specimen paper 1 and paper 2

For first examinations in 2006

CONTENTS

SPEC/5/MATSL/SP1/ENG/TZ0/XX

MATHEMATICS
STANDARD LEVEL
PAPER 1

SPECIMEN

1 hour 30 minutes

Candidate session number

0	0							

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. All students should therefore be advised to show their working. Working may be continued below the lines, if necessary.

1. Let $\boldsymbol{A} = \begin{pmatrix} a & b \\ c & 0 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 1 & 0 \\ d & e \end{pmatrix}$. Giving your answers in terms of *a, b, c, d* and *e*,

(a) write down $\boldsymbol{A} + \boldsymbol{B}$;

(b) find $\boldsymbol{AB}$.

....

....

....

....

....

....

....

....

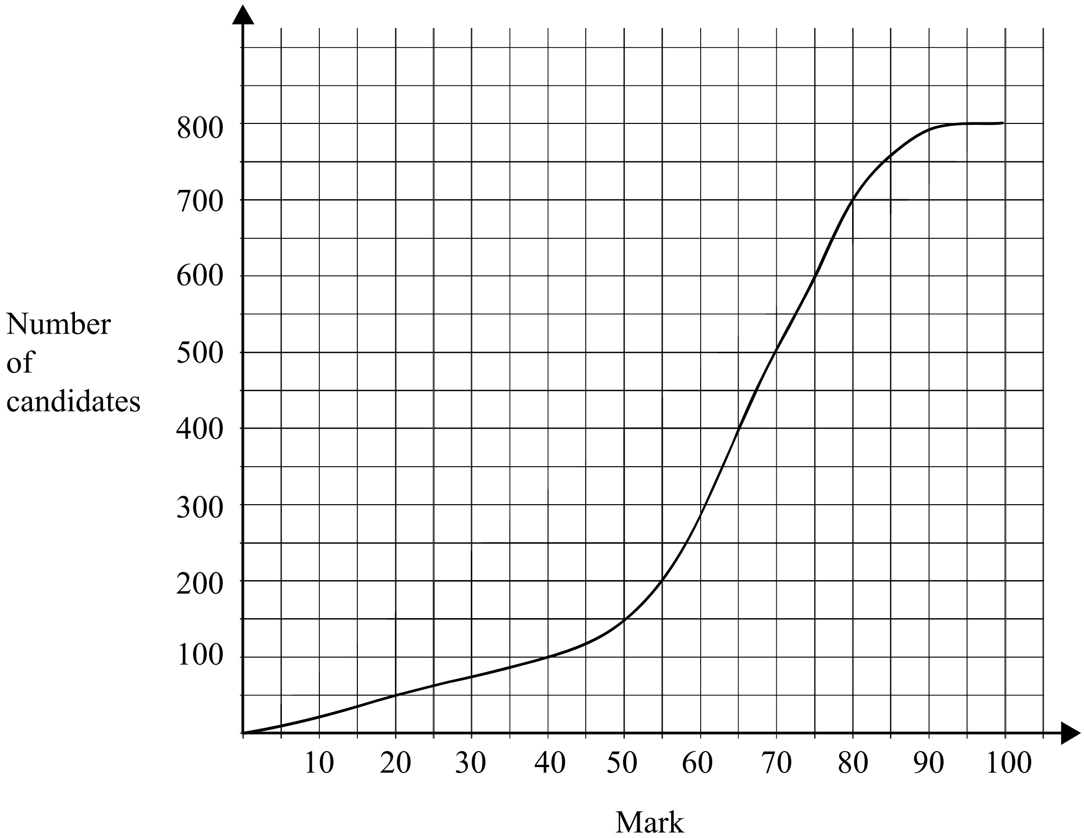
....

....

....

....

2. A test marked out of 100 is written by 800 students. The cumulative frequency graph for the marks is given below.



(a) Write down the number of students who scored 40 marks or less on the test.

(b) The middle 50 % of test results lie between marks a and b, where $a < b$. Find a and b.

....

....

....

....

....

....

....

....

....

....

....

....

3. A theatre has 20 rows of seats. There are 15 seats in the first row, 17 seats in the second row, and each successive row of seats has two more seats in it than the previous row.

(a) Calculate the number of seats in the 20^{th} row.

(b) Calculate the **total** number of seats.

....

....

....

....

....

....

....

....

....

....

....

....

4. The function f is given by $f(x) = 2\sin(5x - 3)$.

(a) Find $f''(x)$.

(b) Write down $\int f(x)\mathrm{d}x$.

....

....

....

....

....

....

....

....

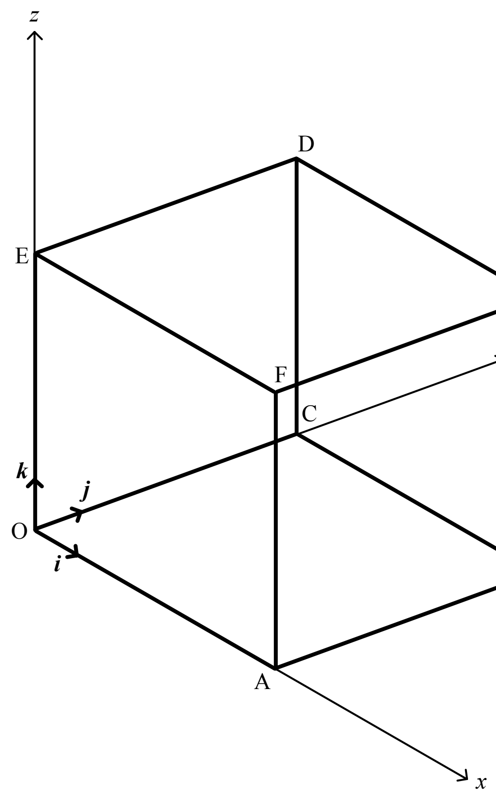
....

....

....

....

5. The diagram shows a cube, OABCDEFG where the length of each edge is 5cm. Express the following vectors in terms of ***i***, ***j*** and ***k*** .



(a) $\overrightarrow{OG}$;

(b) $\overrightarrow{BD}$;

(c) $\overrightarrow{EB}$.

....

....

....

....

....

....

....

....

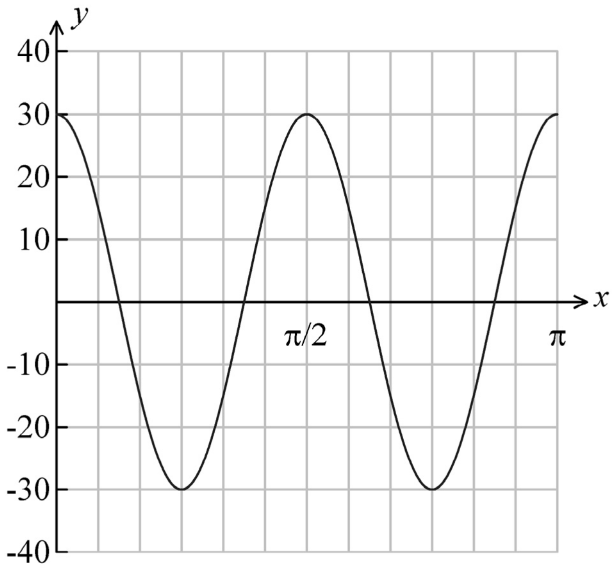
....

....

....

....

6. The graph of a function of the form $y = p\cos qx$ is given in the diagram below.



(a) Write down the value of p.

(b) Calculate the value of q.

....

....

....

....

....

....

....

....

....

....

....

....

7. The velocity $v\ \mathrm{m\,s^{-1}}$ of a moving body at time t seconds is given by $v = 50 - 10t$.

(a) Find its acceleration in $\mathrm{m\,s^{-2}}$.

(b) The initial displacement s is 40 metres. Find an expression for s in terms of t.

....

....

....

....

....

....

....

....

....

....

....

....

8. The functions f and g are defined by $f: x \mapsto 3x,\ g: x \mapsto x + 2$.

(a) Find an expression for $(f \circ g)(x)$.

(b) Show that $f^{-1}(18) + g^{-1}(18) = 22$.

....

....

....

....

....

....

....

....

....

....

....

....

9. A sum of \$5 000 is invested at a compound interest rate of 6.3% per annum.

(a) Write down an expression for the value of the investment after *n* full years.

(b) What will be the value of the investment at the end of five years?

(c) The value of the investment will exceed \$10 000 after *n* full years.

(i) Write down an inequality to represent this information

(ii) Calculate the minimum value of *n*.

....

....

....

....

....

....

....

....

....

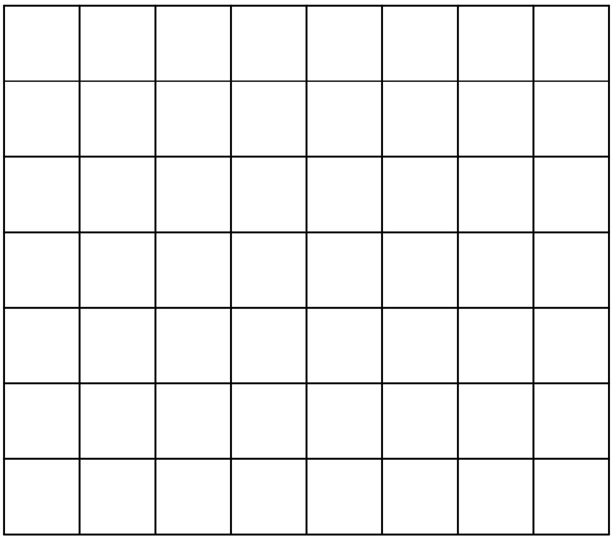
....

....

....

10. The function f is defined by $f(x) = \dfrac{3}{\sqrt{9 - x^2}}$, for $-3 < x < 3$.

(a) On the grid below, sketch the graph of f.



(b) Write down the equation of each vertical asymptote.

(c) Write down the range of the function f.

11. A triangle has its vertices at A(–1, 3) , B(3, 6) and C(–4,4).

(a) Show that $\overrightarrow{AB} \bullet \overrightarrow{AC} = -9$.

(b) Show that, to three significant figures, $\cos B\hat{A}C = -0.569$.

....

....

....

....

....

....

....

....

....

....

....

....

12. A factory makes calculators. Over a long period, 2 % of them are found to be faulty. A random sample of 100 calculators is tested.

(a) Write down the expected number of faulty calculators in the sample.

(b) Find the probability that three calculators are faulty.

(c) Find the probability that more than one calculator is faulty.

....

....

....

....

....

....

....

....

....

....

....

....

13. The quadratic function f is defined by $f(x) = 3x^2 - 12x + 11$.

(a) Write f in the form $f(x) = 3(x-h)^2 - k$.

(b) The graph of f is translated 3 units in the positive x-direction and 5 units in the positive y-direction. Find the function g for the translated graph, giving your answer in the form $g(x) = 3(x-p)^2 + q$.

....

....

....

....

....

....

....

....

....

....

....

....

14. (a) Write down the inverse of the matrix $\boldsymbol{A} = \begin{pmatrix} 1 & -3 & 1 \\ 2 & 2 & -1 \\ 1 & -5 & 3 \end{pmatrix}$

(b) **Hence** solve the simultaneous equations

$$\begin{aligned} x - 3y + z &= 1 \\ 2x + 2y - z &= 2 \\ x - 5y + 3z &= 3 \end{aligned}$$

....

....

....

....

....

....

....

....

....

....

....

....

15. The speeds of cars at a certain point on a straight road are normally distributed with mean μ and standard deviation σ. 15 % of the cars travelled at speeds greater than 90 km h^{-1} and 12 % of them at speeds less than 40 km h^{-1}. Find μ and σ.

....

....

....

....

....

....

....

....

....

....

....

....

SPEC/5/MATSL/SP1/ENG/TZ0/XX/M

MARKSCHEME

SPECIMEN

MATHEMATICS

Standard Level

Paper 1

11 pages

Markscheme Instructions

A. Abbreviations

M Marks awarded for attempting to use a correct **Method:** the working must be seen.

(M) Marks awarded for **Method**: this may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**, usually dependent on preceding ***M*** marks: the working must be seen.

(A) Marks awarded for an **Answer** or for **Accuracy**: this may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**

N Marks awarded for **correct** answers, if **no** working (or no relevant working) shown: in general, these will not be all the marks for the question. Examiners should only award these ***N*** marks for correct answers where there is no working (or if there is working which earns no other marks).

B. Using the markscheme

Follow through (ft) marks: Only award **ft** marks when a candidate uses an incorrect answer in a subsequent **part**. Any exceptions to this will be noted on the markscheme. Follow through marks are now the exception rather than the rule within a question or part question. Follow through marks may only be awarded to work that is seen. Do **not** award ***N*** **(ft)** marks. If the question becomes much simpler then use discretion to award fewer marks.
If a candidate mis-reads data from the **question** apply follow-through.

Discretionary (d) marks: There will be rare occasions where the markscheme does not cover the work seen. In such cases, **(d)** should used to indicate where an examiner has used discretion. It must be accompanied by a brief note to explain the decision made.

It is important to understand the difference between "**implied**" marks, as indicated by the brackets, and marks which can only be awarded for work seen - no brackets. The implied marks can only be awarded if **correct** work is seen or implied in subsequent working. Normally this would be in the next line.

Where ***M1 A1*** are awarded on the same line, this usually means ***M1*** for an attempt to use an appropriate formula, ***A1*** for correct substitution.

As ***A*** marks are normally **dependent** on the preceding ***M*** mark being awarded, it is not possible to award ***M0 A1.***

As ***N*** marks are only awarded when there is no working, it is not possible to award a mixture of ***N*** and other marks.

Accept all correct alternative methods, even if not specified in the markscheme Where alternative methods for complete questions are included, they are indicated by **METHOD 1**, **METHOD 2**, *etc*. Other alternative (part) solutions, are indicated by **EITHER**…**OR**. Where possible, alignment will also be used to assist examiners to identify where these alternatives start and finish.

Unless the question specifies otherwise, accept **equivalent forms**. On the markscheme, these equivalent numerical or algebraic forms will generally be written in brackets after the required answer The markscheme indicate the required answer, by allocating full marks at that point. Once the correct answer is seen, ignore further working, unless it contradicts the answer.

Brackets will also be used for what could be described as the well-expressed answer, but which candidates may not write in examinations. Examiners need to be aware that the marks for answers should be awarded for the form preceding the brackets eg in differentiating $f(x) = 2\sin(5x-3)$, the markscheme says

$$f'(x) = \left(2\cos(5x-3)\right)5 \quad \left(=10\cos(5x-3)\right)$$ **A1**

This means that the ***A1*** is awarded for seeing $\left(2\cos(5x-3)\right)5$, although we would normally write the answer as $10\cos(5x-3)$.

As this is an international examination, all **alternative forms of notation** should be accepted.

Where the markscheme specifies ***M2*, *A3***, etc, for an answer do NOT split the marks unless otherwise instructed.

Do **not** award full marks for a correct answer, all working must be checked.

Candidates should be penalized **once IN THE PAPER** for an accuracy error (**AP).** There are two types of accuracy error:

- **Rounding errors:** only applies to final answers not to intermediate steps.
- **Level of accuracy**: when this is not specified in the question the general rule is *unless otherwise stated in the question all numerical answers must be given exactly or to three significant figure*s.

QUESTION 1

(a) $\boldsymbol{A}+\boldsymbol{B}=\begin{pmatrix} a & b \\ c & 0 \end{pmatrix}+\begin{pmatrix} 1 & 0 \\ d & e \end{pmatrix}$

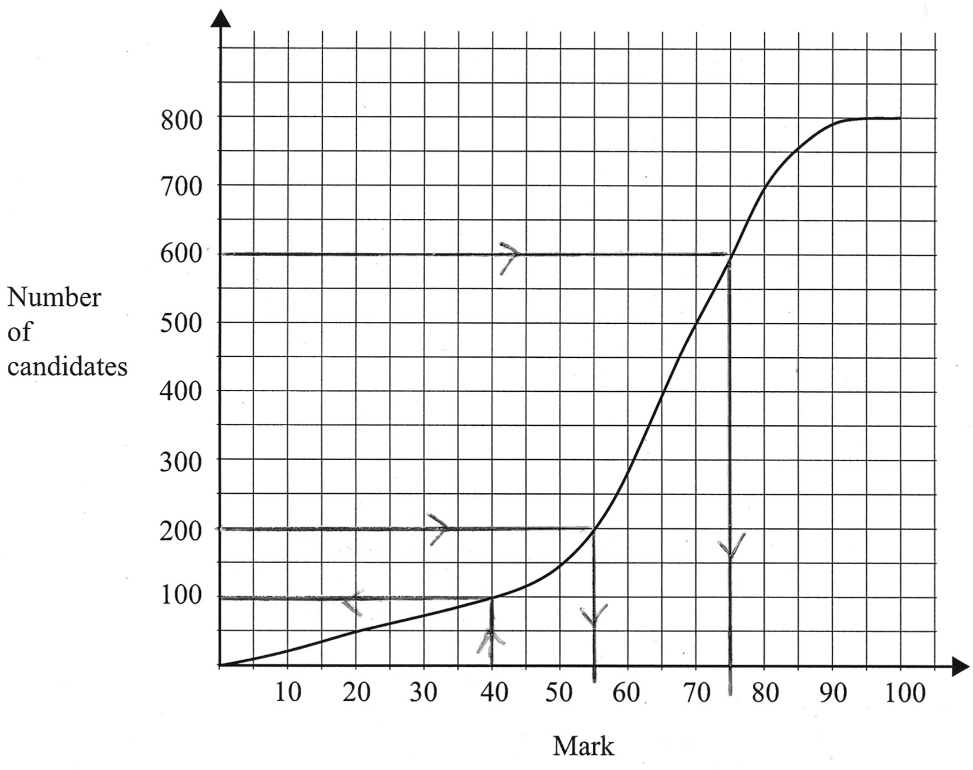
$=\begin{pmatrix} a+1 & b \\ c+d & e \end{pmatrix}$ ***A2*** ***N2***

(b) $\boldsymbol{AB}=\begin{pmatrix} a & b \\ c & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ d & e \end{pmatrix}$

$=\begin{pmatrix} a+bd & be \\ c & 0 \end{pmatrix}$ ***A1A1A1A1*** ***N4***

> **Note:** Award ***N2*** for finding $\boldsymbol{BA}=\begin{pmatrix} a & b \\ ad+ce & bd \end{pmatrix}$.

QUESTION 2



(a) Lines on graph ***(M1)***
100 students score 40 marks or fewer. ***A1*** ***N2***

(b) Identifying 200 **and** 600 ***A1***
Lines on graph. ***(M1)***
$a=55$, $b=75$. ***A1A1*** ***N1N1***

QUESTION 3

(a) Recognizing an AP *(M1)*

$u_1 = 15 \quad d = 2 \quad n = 20$ *(A1)*

substituting into $u_{20} = 15 + (20-1)\times 2$ ***M1***

$= 53$ (that is, 53 seats in the 20th row) ***A1*** ***N2***

(b) Substituting into $S_{20} = \frac{20}{2}\left(2(15)+(20-1)2\right)$ $\left(\text{or into } \frac{20}{2}(15+53)\right)$ ***M1***

$= 680$ (that is, 680 seats in total) ***A1*** ***N2***

QUESTION 4

(a) Using the chain rule *(M1)*

$f'(x) = \left(2\cos(5x-3)\right)5 \; \left(=10\cos(5x-3)\right)$ ***A1***

$f''(x) = -\left(10\sin(5x-3)\right)5$

$= -50\sin(5x-3)$ ***A1A1*** ***N2***

Note: Award ***A1*** for $\sin(5x-3)$, ***A1*** for –50.

(b) $\int f(x)\,\mathrm{d}x = -\frac{2}{5}\cos(5x-3)+c$ ***A1 A1*** ***N2***

Note: Award ***A1*** for $\cos(5x-3)$, ***A1*** for $-\frac{2}{5}$.

QUESTION 5

(a) $\overrightarrow{\text{OG}} = 5\boldsymbol{i} + 5\boldsymbol{j} + 5\boldsymbol{k}$ ***A2*** ***N2***

(b) $\overrightarrow{\text{BD}} = -5\boldsymbol{i} + 5\boldsymbol{k}$ ***A2*** ***N2***

(c) $\overrightarrow{\text{EB}} = 5\boldsymbol{i} + 5\boldsymbol{j} - 5\boldsymbol{k}$ ***A2*** ***N2***

Note: Award ***A0A2A2*** if the 5 is consistently omitted.

QUESTION 6

(a) $p = 30$ ***A2*** ***N2***

(b) **METHOD 1**

Period $= \dfrac{2\pi}{q}$ ***(M2)***

$= \dfrac{\pi}{2}$ ***(A1)***

$\Rightarrow q = 4$ ***A1*** ***N4***

METHOD 2

Horizontal stretch of scale factor $= \dfrac{1}{q}$ ***(M2)***

scale factor $= \dfrac{1}{4}$ ***(A1)***

$\Rightarrow q = 4$ ***A1*** ***N4***

QUESTION 7

(a) $a = \dfrac{\mathrm{d}v}{\mathrm{d}t}$ ***(M1)***

$= -10$ ***A1*** ***N2***

(b) $s = \int v dt$ ***(M1)***

$= 50t - 5t^2 + c$ ***A1***

$40 = 50(0) - 5(0) + c \Rightarrow c = 40$ ***A1***

$s = 50t - 5t^2 + 40$ ***A1*** ***N2***

Note:	Award ***(M1)*** and the first ***A1*** in part (b) if c is missing, but do **not** award the final 2 marks.

QUESTION 8

(a) $(f \circ g): x \mapsto 3(x+2) \ (= 3x+6)$ ***A2*** ***N2***

(b) **METHOD 1**

$f^{-1}(x) = \frac{x}{3} \quad g^{-1}(x) = x-2$ ***(M1)***

$f^{-1}(18) = \frac{18}{3}$ ***A1***

$g^{-1}(18) = 18-2$ ***A1***

$f^{-1}(18) + g^{-1}(18) = 6+16$ ***A1***

$f^{-1}(18) + g^{-1}(18) = 22$ ***AG*** ***N0***

METHOD 2

$3x = 18, \ x+2 = 18$ ***(M1)***

$x = 6, \ x = 16$ ***A1A1***

$f^{-1}(18) + g^{-1}(18) = 6+16$ ***A1***

$f^{-1}(18) + g^{-1}(18) = 22$ ***AG*** ***N0***

QUESTION 9

(a) $5\,000(1.063)^n$ *A1* *N1*

(b) Value $= \$\,5\,000(1.063)^5 \quad (= \$\,6\,786.3511\ldots)$
$= \$\,6\,790$ to 3 s.f. (Accept $\$\,6\,786$, or $\$\,6\,786.35$) *A1* *N1*

(c) (i) $5000(1.063)^n > 10000$ or $(1.063)^n > 2$ *A1* *N1*

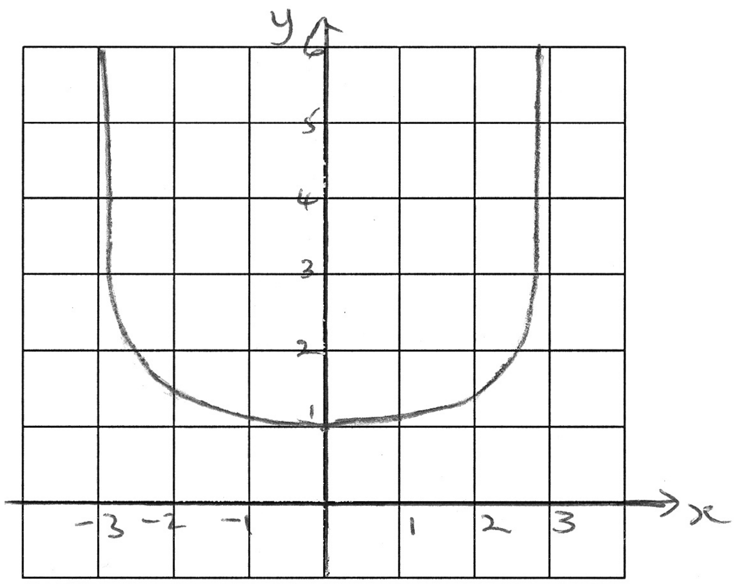
(ii) Attempting to solve the inequality $n\log(1.063) > \log 2$ *(M1)*
$n > 11.345...$ *(A1)*
12 years *A1* *N3*

Note: Candidates are likely to use TABLE or LIST on a GDC to find *n*. A good way of communicating this is suggested below.

Let $y = 1.063^x$ *(M1)*
When $x = 11$, $y = 1.9582$, when $x = 12$, $y = 2.0816$ *(A1)*
$x = 12$ *i.e.* 12 years *A1* *N3*

QUESTION 10

(a)



A1A1 *N2*

Note: Award ***A1*** for the general shape and ***A1*** for the y-intercept at 1.

(b) $x = 3$, $x = -3$ *A1A1* *N1N1*

(c) $y \geq 1$ *A2* *N2*

Note: Award ***N1*** for $y > 1$.

QUESTION 11

(a) Finding **correct** vectors, $\vec{AB} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ $\vec{AC} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$ ***A1A1***

Substituting correctly in the scalar product

$\vec{AB} \cdot \vec{AC} = 4(-3) + 3(1)$ ***A1***

$= -9$ ***AG*** ***N0***

(b) $|\vec{AB}| = 5$ $|\vec{AC}| = \sqrt{10}$ ***(A1)(A1)***

Attempting to use scalar product formula $\cos B\hat{A}C = \frac{-9}{5\sqrt{10}}$ ***M1***

$= -0.569$ (3 s.f.) ***AG*** ***N0***

QUESTION 12

(a) $X \sim B(100, 0.02)$

$E(X) = 100 \times 0.02 = 2$ ***A1*** ***N1***

(b) (i) $P(X = 3) = \binom{100}{3}(0.02)^3(0.98)^{97}$ ***(M1)***

$= 0.182$ ***A1*** ***N2***

(ii) **METHOD 1**

$P(X > 1) = 1 - P(X \le 1) = 1 - \left(P(X = 0) + P(X = 1)\right)$ ***M1***

$= 1 - \left((0.98)^{100} + 100(0.02)(0.98)^{99}\right)$ ***(M1)***

$= 0.597$ ***A1*** ***N2***

METHOD 2

$P(X > 1) = 1 - P(X \le 1)$ ***(M1)***

$= 1 - 0.40327$ ***(A1)***

$= 0.597$ ***A1*** ***N2***

Note: Award marks as follows for finding $P(X \ge 1)$, if working shown.

$P(X \ge 1)$ ***A0***

$= 1 - P(X \le 2) = 1 - 0.67668$ ***M1*(ft)**

$= 0.323$ ***A1*(ft)** ***N0***

QUESTION 13

(a) For a reasonable attempt to complete the square, (or expanding)

$3x^2 - 12x + 11 = 3(x^2 - 4x + 4) + 11 - 12$

$= 3(x-2)^2 - 1$ (Accept $h = 2, k = 1$) *A1A1* *N2*

(b) **METHOD 1**

Vertex shifted to $(2+3, -1+5) = (5, 4)$ *M1*

so the new function is $3(x-5)^2 + 4$ (Accept $p = 5, q = 4$) *A1A1* *N2*

METHOD 2

$g(x) = 3\big((x-3)-h\big)^2 + k + 5 = 3\big((x-3)-2\big)^2 - 1 + 5$ *M1*

$= 3(x-5)^2 + 4$ (Accept $p = 5, q = 4$) *A1A1* *N2*

QUESTION 14

(a) $\boldsymbol{A}^{-1} = \begin{pmatrix} 0.1 & 0.4 & 0.1 \\ -0.7 & 0.2 & 0.3 \\ -1.2 & 0.2 & 0.8 \end{pmatrix}$ *A2* *N2*

(b) For recognizing that the equations may be written as $\boldsymbol{A}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ *(M1)*

for attempting to calculate $\begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \end{pmatrix} = \boldsymbol{A}^{-1}\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ $\left(= \begin{pmatrix} 1.2 \\ 0.6 \\ 1.6 \end{pmatrix} \right)$ *M1*

$x = 1.2, y = 0.6, z = 1.6$ (Accept row or column vectors) *A2* *N3*

QUESTION 15

$X \sim \mathrm{N}(\mu, \sigma^2)$, $\mathrm{P}(X > 90) = 0.15$ **and** $\mathrm{P}(X < 40) = 0.12$ *(M1)*

Finding standardized values $1.036, -1.175$ *A1A1*

Setting up the equations $1.036 = \dfrac{90-\mu}{\sigma}, -1.175 = \dfrac{40-\mu}{\sigma}$ *(M1)*

$\mu = 66.6, \quad \sigma = 22.6$ *A1A1* *N2N2*

IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
PROGRAMA DEL DIPLOMA DEL BI

MATHEMATICS
STANDARD LEVEL
PAPER 2

SPECIMEN

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. All students should therefore be advised to show their working

1. *[Maximum mark: 16]*

The function f is defined by $f : x \mapsto -0.5x^2 + 2x + 2.5$.

(a) Write down

(i) $f'(x)$;

(ii) $f'(0)$. *[2 marks]*

(b) Let N be the normal to the curve at the point where the graph intercepts the y-axis. Show that the equation of N may be written as $y = -0.5x + 2.5$. *[3 marks]*

Let $g : x \mapsto -0.5x + 2.5$.

(c) (i) Find the solutions of $f(x) = g(x)$.

(ii) Hence find the coordinates of the other point of intersection of the normal and the curve. *[6 marks]*

(d) Let R be the region enclosed between the curve and N.

(i) Write down an expression for the area of R.

(ii) Hence write down the area of R. *[5 marks]*

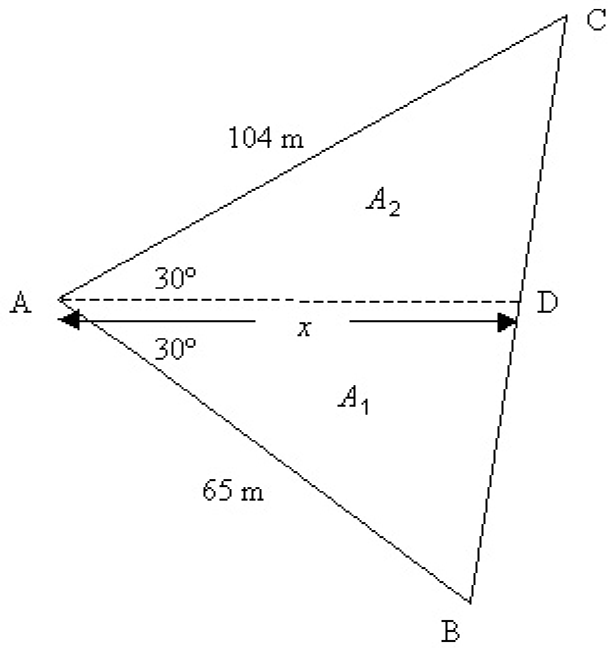
2. *[Maximum mark: 18]*

A farmer owns a triangular field ABC. One side of the triangle, [AC], is 104 m, a second side, [AB], is 65 m and the angle between these two sides is 60°.

(a) Use the cosine rule to calculate the length of the third side of the field. *[3 marks]*

(b) Given that $\sin 60^\circ = \frac{\sqrt{3}}{2}$, find the area of the field in the form $p\sqrt{3}$ where p is an integer. *[3 marks]*

Let D be a point on [BC] such that [AD] bisects the 60° angle. The farmer divides the field into two parts A_1 and A_2 by constructing a straight fence [AD] of length x metres, as shown on the diagram below.



(c) (i) Show that the area of A_1 is given by $\frac{65x}{4}$.

(ii) Find a similar expression for the area of A_2.

(iii) **Hence**, find the value of x in the form $q\sqrt{3}$, where q is an integer. *[7 marks]*

(d) (i) Explain why $\sin \hat{ADC} = \sin \hat{ADB}$.

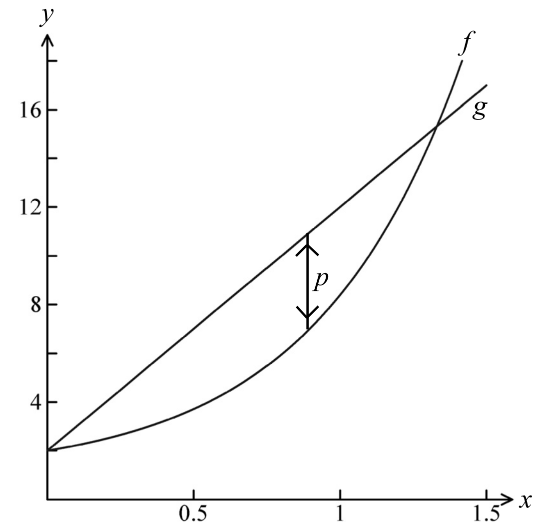
(ii) Use the result of part (i) and the sine rule to show that

$$\frac{BD}{DC} = \frac{5}{8}.$$ *[5 marks]*

3. *[Total mark: 22]*

Part A *[Maximum mark: 14]*

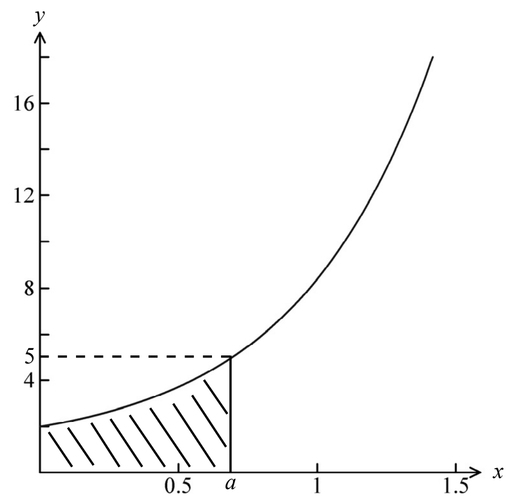
The diagram below shows the graphs of $f(x) = 1 + e^{2x}$, $g(x) = 10x + 2$, $0 \le x \le 1.5$



(a) (i) Write down an expression for the vertical distance p between the graphs of f and g.

(ii) Given that p has a maximum value for $0 \le x \le 1.5$, find the value of x at which this occurs. *[6 marks]*

The graph of $y = f(x)$ only is shown in the diagram below. When $x = a$, $y = 5$.



(b) (i) Find $f^{-1}(x)$.

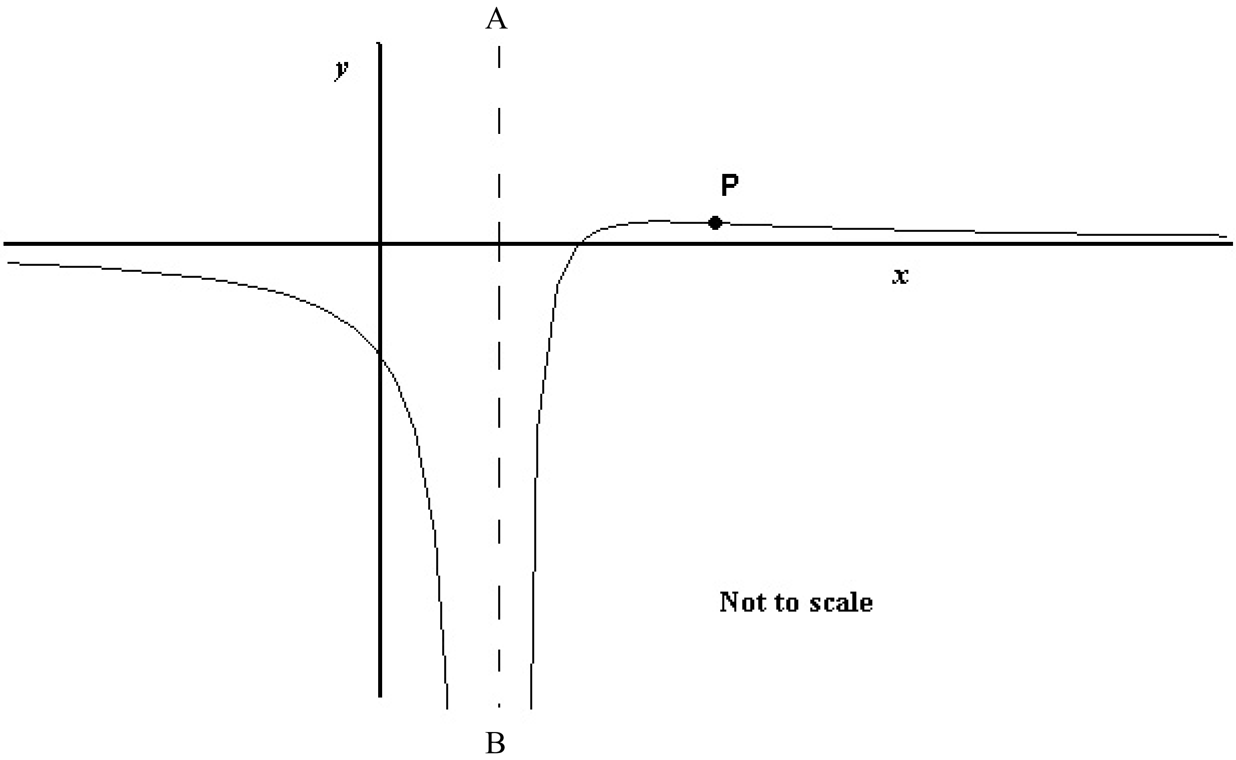
(ii) **Hence** show that $a = \ln 2$. *[5 marks]*

(c) The region shaded in the diagram is rotated through 360° about the x-axis. Write down an expression for the volume obtained. *[3 marks]*

Part B *[Maximum mark: 8]*

Consider the function $h : x \mapsto \frac{x-2}{(x-1)^2}, x \neq 1$.

A sketch of part of the graph of h is given below.



The line (AB) is a vertical asymptote. The point P is a point of inflexion.

(a) Write down the **equation** of the vertical asymptote. *[1 mark]*

(b) Find $h'(x)$, writing your answer in the form

$$\frac{a-x}{(x-1)^n}$$

where a and n are constants to be determined. *[4 marks]*

(c) Given that $h''(x) = \frac{2x-8}{(x-1)^4}$, calculate the coordinates of P. *[3 marks]*

4. *[Maximum mark: 19]*

Bag A contains 2 red balls and 3 green balls. Two balls are chosen at random from the bag without replacement. Let X denote the number of red balls chosen. The following table shows the probability distribution for X.

X	0	1	2
$\text{P}(X = x)$	$\frac{3}{10}$	$\frac{6}{10}$	$\frac{1}{10}$

(a) Calculate $\text{E}(X)$, the mean number of red balls chosen. *[3 marks]*

Bag B contains 4 red balls and 2 green balls. Two balls are chosen at random from bag B.

(b) (i) Draw a tree diagram to represent the above information, including the probability of each event.

(ii) Hence find the probability distribution for Y, where Y is the number of red balls chosen. *[8 marks]*

A standard die with six faces is rolled. If a 1 or 6 is obtained, two balls are chosen from bag A, otherwise two balls are chosen from bag B.

(c) Calculate the probability that two red balls are chosen. *[5 marks]*

(d) Given that two red balls are obtained, find the conditional probability that a 1 or 6 was rolled on the die. *[3 marks]*

5. *[Maximum mark: 15]*

In this question, distance is in kilometers, time is in hours.
A balloon is moving at a constant height with a speed of $18\,\text{km}\,\text{h}^{-1}$, in the direction of the vector $\begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$.

At time $t = 0$, the balloon is at point B with coordinates $(0, 0, 5)$.

(a) Show that the position vector $\boldsymbol{b}$ of the balloon at time t is given by

$$\boldsymbol{b} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} + t\begin{pmatrix} 10.8 \\ 14.4 \\ 0 \end{pmatrix}.$$

[6 marks]

At time $t = 0$, a helicopter goes to deliver a message to the balloon. The position vector $\boldsymbol{h}$ of the helicopter at time t is given by

$$\boldsymbol{h} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 49 \\ 32 \\ 0 \end{pmatrix} + t\begin{pmatrix} -48 \\ -24 \\ 6 \end{pmatrix}.$$

(b) (i) Write down the coordinates of the starting position of the helicopter.

(ii) Find the speed of the helicopter. *[4 marks]*

(c) The helicopter reaches the balloon at point R.

(i) Find the time the helicopter takes to reach the balloon.

(ii) Find the coordinates of R. *[5 marks]*

MARKSCHEME

SPECIMEN

MATHEMATICS

Standard Level

Paper 2

11 pages

Markscheme Instructions

A. Abbreviations

M Marks awarded for attempting to use a correct **Method:** the working must be seen.

(M) Marks awarded for **Method**: this may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**, usually dependent on preceding ***M*** marks: the working must be seen.

(A) Marks awarded for an **Answer** or for **Accuracy**: this may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**

N Marks awarded for **correct** answers, if **no** working (or no relevant working) shown: in general, these will not be all the marks for the question. Examiners should only award these ***N*** marks for correct answers where there is no working (or if there is working which earns no other marks).

B. Using the markscheme

Follow through (ft) marks: Only award **ft** marks when a candidate uses an incorrect answer in a subsequent **part**. Any exceptions to this will be noted on the markscheme. Follow through marks are now the exception rather than the rule within a question or part question. Follow through marks may only be awarded to work that is seen. Do **not** award ***N*** **(ft)** marks. If the question becomes much simpler then use discretion to award fewer marks.
If a candidate mis-reads data from the **question** apply follow-through.

Discretionary (d) marks: There will be rare occasions where the markscheme does not cover the work seen. In such cases, **(d)** should used to indicate where an examiner has used discretion. It must be accompanied by a brief note to explain the decision made.

It is important to understand the difference between "**implied**" marks, as indicated by the brackets, and marks which can only be awarded for work seen - no brackets. The implied marks can only be awarded if **correct** work is seen or implied in subsequent working. Normally this would be in the next line.

Where ***M1 A1*** are awarded on the same line, this usually means ***M1*** for an attempt to use an appropriate formula, ***A1*** for correct substitution.

As ***A*** marks are normally **dependent** on the preceding ***M*** mark being awarded, it is not possible to award ***M0 A1.***

As ***N*** marks are only awarded when there is no working, it is not possible to award a mixture of ***N*** and other marks.

Accept all correct alternative methods, even if not specified in the markscheme Where alternative methods for complete questions are included, they are indicated by **METHOD 1**, **METHOD 2**, *etc*. Other alternative (part) solutions, are indicated by **EITHER**….**OR**. Where possible, alignment will also be used to assist examiners to identify where these alternatives start and finish.

Unless the question specifies otherwise, accept **equivalent forms**. On the markscheme, these equivalent numerical or algebraic forms will generally be written in brackets after the required answer The markscheme indicate the required answer, by allocating full marks at that point. Once the correct answer is seen, ignore further working, unless it contradicts the answer.

Brackets will also be used for what could be described as the well-expressed answer, but which candidates may not write in examinations. Examiners need to be aware that the marks for answers should be awarded for the form preceding the brackets eg in differentiating $f(x) = 2\sin(5x-3)$, the markscheme says

$$f'(x) = \left(2\cos(5x-3)\right)5 \quad \left(=10\cos(5x-3)\right) \qquad \textbf{\textit{A1}}$$

This means that the ***A1*** is awarded for seeing $\left(2\cos(5x-3)\right)5$, although we would normally write the answer as $10\cos(5x-3)$.

As this is an international examination, all **alternative forms of notation** should be accepted.

Where the markscheme specifies ***M2*, *A3***, etc, for an answer do NOT split the marks unless otherwise instructed.

Do **not** award full marks for a correct answer, all working must be checked.

Candidates should be penalized **once IN THE PAPER** for an accuracy error (**AP).** There are two types of accuracy error:

- **Rounding errors:** only applies to final answers not to intermediate steps.
- **Level of accuracy**: when this is not specified in the question the general rule is *unless otherwise stated in the question all numerical answers must be given exactly or to three significant figure*s.

QUESTION 1

(a) (i) $f'(x) = -x + 2$ *A1*

(ii) $f'(0) = 2$ *A1*

[2 marks]

(b) Gradient of tangent at y-intercept $= f'(0) = 2$

$\Rightarrow$ gradient of normal $= -\frac{1}{2}\ (= -0.5)$ *A1*

Finding y-intercept is 2.5 *A1*

Therefore, equation of the normal is

$y - 2.5 = -\frac{1}{2}(x - 0) \quad (y - 2.5 = -0.5x)$ *M1*

$y = -0.5x + 2.5$ *(AG)* *N0*

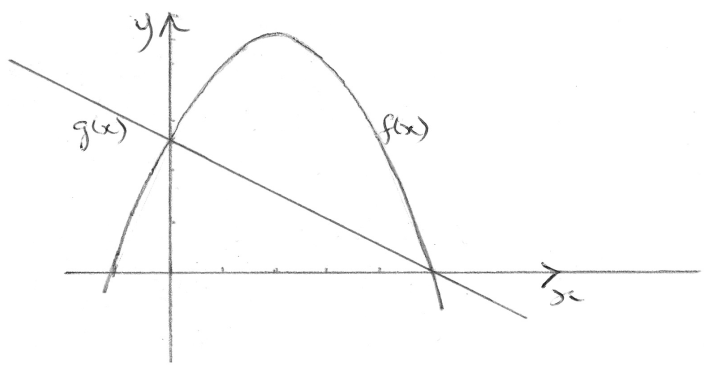
[3 marks]

(c) (i) **EITHER**

solving $-0.5x^2 + 2x + 2.5 = -0.5x + 2.5$ *(M1)A1*

$\Rightarrow x = 0$ or $x = 5$ *A1* *N2*

OR



M1

Curves intersect at $x = 0, x = 5$ *(A1)*

So solutions to $f(x) = g(x)$ are $x = 0, x = 5$ *A1* *N2*

OR

$\Rightarrow 0.5x^2 - 2.5x = 0$ *(A1)*

$\Rightarrow -0.5x(x - 5) = 0$ *M1*

$\Rightarrow x = 0$ or $x = 5$ *A1* *N2*

(ii) Curve and normal intersect when $x = 0$ or $x = 5$ *(M2)*

Other point is when $x = 5$

$\Rightarrow y = -0.5(5) + 2.5 = 0$ (so other point $(5, 0)$ *A1* *N2*

[6 marks]

continued…

Question 1 continued

(d) (i) Area $=\int_0^5 \left(f(x)-g(x)\right)\mathrm{d}x \left(\text{or} \int_0^5 (-0.5x^2+2x+2.5)\,\mathrm{d}x-\frac{1}{2}\times 5\times 2.5\right)$ ***A1A1A1*** ***N3***

Note:	Award ***A1*** for the integral, ***A1*** for **both** correct limits on the integral, and ***A1*** for the difference.

(ii) Area = Area under curve – area under line $(A = A_1 - A_2)$ ***(M1)***

$A_1=\frac{50}{3},\ A_2=\frac{25}{4}$

Area $=\frac{50}{3}-\frac{25}{4}=\frac{125}{12}$ (or 10.4 (3 s.f.)) ***A1*** ***N2***

[5 marks]

Total [16 marks]

QUESTION 2

(a) using the cosine rule $a^2 = b^2 + c^2 - 2bc\cos\hat{A}$ ***(M1)***

substituting correctly $BC^2 = 65^2 + 104^2 - 2(65)(104)\cos 60^\circ$ ***A1***

$= 4\,225 + 10\,816 - 6\,760 = 8\,281$

$\Rightarrow BC = 91\text{ m}$ ***A1*** ***N2***

[3 marks]

(b) finding the area, using $\frac{1}{2}bc\sin\hat{A}$ ***(M1)***

substituting correctly, area $= \frac{1}{2}(65)(104)\sin 60^\circ$ ***A1***

$= 1\,690\sqrt{3}$ (Accept $p = 1\,690$) ***A1*** ***N2***

[3 marks]

(c) (i) $A_1 = \left(\frac{1}{2}\right)(65)(x)\sin 30^\circ$ ***A1***

$= \frac{65x}{4}$ ***AG*** ***N0***

(ii) $A_2 = \left(\frac{1}{2}\right)(104)(x)\sin 30^\circ$ ***M1***

$= 26x$ ***A1*** ***N1***

(iii) stating $A_1 + A_2 = A$ or substituting $\frac{65x}{4} + 26x = 1\,690\sqrt{3}$ ***(M1)***

simplifying $\frac{169x}{4} = 1\,690\sqrt{3}$ ***A1***

$x = \frac{4 \times 1690\sqrt{3}}{169}$ ***A1***

$\Rightarrow x = 40\sqrt{3}$ (Accept $q = 40$) ***A1*** ***N2***

[7 marks]

(d) (i) Recognizing that supplementary angles have equal sines

e.g. $A\hat{D}C = 180^\circ - A\hat{D}B \Rightarrow \sin A\hat{D}C = \sin A\hat{D}B$ ***R1***

(ii) using sin rule in ΔADB and ΔACD ***(M1)***

substituting correctly $\frac{BD}{\sin 30^\circ} = \frac{65}{\sin A\hat{D}B} \Rightarrow \frac{BD}{65} = \frac{\sin 30^\circ}{\sin A\hat{D}B}$ ***A1***

and $\frac{DC}{\sin 30^\circ} = \frac{104}{\sin A\hat{D}C} \Rightarrow \frac{DC}{104} = \frac{\sin 30^\circ}{\sin A\hat{D}C}$ ***M1***

since $\sin A\hat{D}B = \sin A\hat{D}C$

$\frac{BD}{65} = \frac{DC}{104} \Rightarrow \frac{BD}{DC} = \frac{65}{104}$ ***A1***

$\Rightarrow \frac{BD}{DC} = \frac{5}{8}$ ***AG*** ***N0***

[5 marks]

Total [18 marks]

QUESTION 3

Part A

(a) (i) $p = (10x + 2) - (1 + e^{2x})$ ***A2*** ***N2***

Note: Award ***A1*** for $(1 + e^{2x}) - (10x + 2)$.

(ii) $\frac{dp}{dx} = 10 - 2e^{2x}$ ***A1A1***

$\frac{dp}{dx} = 0 \quad (10 - 2e^{2x} = 0)$ ***M1***

$x = \frac{\ln 5}{2} \ (= 0.805)$ ***A1*** ***N4***

[6 marks]

(b) (i) **METHOD 1**

$x = 1 + e^{2y}$ ***M1***

$\ln(x-1) = 2y$ ***A1***

$f^{-1}(x) = \frac{\ln(x-1)}{2} \left(\text{Allow } y = \frac{\ln(x-1)}{2}\right).$ ***A1*** ***N2***

METHOD 2

$y - 1 = e^{2x}$ ***A1***

$\frac{\ln(y-1)}{2} = x$ ***M1***

$f^{-1}(x) = \frac{\ln(x-1)}{2} \left(\text{Allow } y = \frac{\ln(x-1)}{2}\right)$ ***A1*** ***N2***

(ii) $a = \frac{\ln(5-1)}{2} \left(= \frac{1}{2}\ln 2^2\right)$ ***M1***

$= \frac{1}{2} \times 2\ln 2$ ***A1***

$= \ln 2$ ***AG*** ***N0***

[5 marks]

(c) Using $V = \int_a^b \pi y^2 \, dx$ ***(M1)***

$\text{Volume} = \int_0^{\ln 2} \pi(1 + e^{2x})^2 \, dx \quad \left(\text{or} \int_0^{0.805} \pi(1 + e^{2x})^2 \, dx\right)$ ***A2*** ***N3***

[3 marks]

Sub-total [14 marks]

continued…

Question 3 continued

Part B

(a) $x = 1$ ***A1*** ***N1***

[1 mark]

(b) Using quotient rule ***(M1)***

Substituting correctly $g'(x) = \frac{(x-1)^2(1)-(x-2)[2(x-1)]}{(x-1)^4}$ ***A1***

$= \frac{(x-1)-(2x-4)}{(x-1)^3}$ ***(A1)***

$= \frac{3-x}{(x-1)^3}$ (Accept $a = 3$, $n = 3$) ***A1*** ***N2***

[4 marks]

(c) Recognizing at point of inflexion $g''(x) = 0$ ***M1***

$x = 4$ ***A1***

Finding corresponding y-value $= \frac{2}{9} = 0.222$ *i.e.* $\text{P}\left(4, \frac{2}{9}\right)$ ***A1*** ***N2***

[3 marks]

Sub-total [8 marks]

Total [22 marks]

QUESTION 4

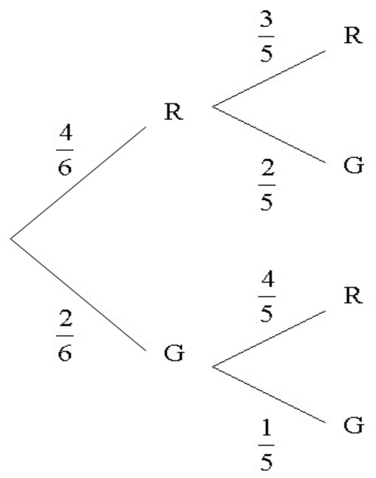
(a) Using $\mathrm{E}(X) = \sum_{0}^{2} x\,\mathrm{P}(X = x)$ ***(M1)***

Substituting correctly $\mathrm{E}(X) = 0 \times \frac{3}{10} + 1 \times \frac{6}{10} + 2 \times \frac{1}{10}$ ***A1***

$= \frac{8}{10}$ (0.8) ***A1*** ***N2***

[3 marks]

(b) (i)



A1A1A1 ***N3***

Note: Award ***A1*** for each complementary pair of probabilities, *i.e.* $\frac{4}{6}$ and $\frac{2}{6}$, $\frac{3}{5}$ and $\frac{2}{5}$, $\frac{4}{5}$ and $\frac{1}{5}$.

(ii) $\mathrm{P}(Y = 0) = \frac{2}{5} \times \frac{1}{5} = \frac{2}{30}$ ***A1***

$\mathrm{P}(Y = 1) = \mathrm{P}(RG) + \mathrm{P}(GR)$ $\left(= \frac{4}{6} \times \frac{2}{5} + \frac{2}{6} \times \frac{4}{5} \right)$ ***M1***

$= \frac{16}{30}$ ***A1***

$\mathrm{P}(Y = 2) = \frac{4}{6} \times \frac{3}{5} = \frac{12}{30}$ ***(A1)***

For forming a distribution ***M1***

y	0	1	2
$\mathrm{P}(Y = y)$	$\frac{2}{30}$	$\frac{16}{30}$	$\frac{12}{30}$

N4

[8 marks]

continued…

Question 4 continued

(c) $\text{P}(\text{Bag A}) = \frac{2}{6} \quad \left(= \frac{1}{3}\right)$ ***(A1)***

$\text{P}(\text{Bag B}) = \frac{4}{6} \quad \left(= \frac{2}{3}\right)$ ***(A1)***

For summing $\text{P}(A \cap RR)$ and $\text{P}(B \cap RR)$ ***(M1)***

Substituting correctly $\text{P}(RR) = \frac{1}{3} \times \frac{1}{10} + \frac{2}{3} \times \frac{12}{30}$ ***A1***

$= \frac{27}{90} \left(\frac{3}{10}, 0.3\right)$ ***A1*** ***N3***

[5 marks]

(d) For recognising that $\text{P}(1 \text{ or } 6 \mid RR) = \text{P}(A \mid RR) = \frac{\text{P}(A \cap RR)}{\text{P}(RR)}$ ***(M1)***

$= \frac{1}{30} \div \frac{27}{90}$ ***A1***

$= \frac{3}{27} \quad \left(\frac{1}{9}, 0.111\right)$ ***A1*** ***N2***

[3 marks]

Total [19 marks]

QUESTION 5

(a) Attempting to find unit vector $(\boldsymbol{e_b})$ in the direction of $\boldsymbol{b}$ ***(M1)***

Correct values $= \frac{1}{\sqrt{3^2+4^2+0^2}}\begin{pmatrix}3\\4\\0\end{pmatrix}$ ***A1***

$= \begin{pmatrix}0.6\\0.8\\0\end{pmatrix}$ ***A1***

Finding direction vector for $\boldsymbol{b}$, $\boldsymbol{v_b} = 18 \times \boldsymbol{e_b}$ ***(M1)***

$\boldsymbol{b} = \begin{pmatrix}10.8\\14.4\\0\end{pmatrix}$ ***A1***

Using vector representation $\boldsymbol{b} = \boldsymbol{b}_0 + t\boldsymbol{v_b}$ ***(M1)***

$= \begin{pmatrix}0\\0\\5\end{pmatrix} + t\begin{pmatrix}10.8\\14.4\\0\end{pmatrix}$ ***AG*** ***N0***

[6 marks]

(b) (i) $t = 0 \Rightarrow (49, 32, 0)$ ***A1*** ***N1***

(ii) Finding magnitude of velocity vector ***(M1)***

Substituting correctly $v_h = \sqrt{(-48)^2 + (-24)^2 + 6^2}$ ***A1***

$= 54\ (\text{km h}^{-1})$ ***A1*** ***N2***

[4 marks]

(c) (i) At R, $\begin{pmatrix}10.8t\\14.4t\\5\end{pmatrix} = \begin{pmatrix}49-48t\\32-24t\\6t\end{pmatrix}$ ***A1***

$t = \frac{5}{6}\ (= 0.833)\,(\text{hours})$ ***A1*** ***N1***

(ii) For substituting $t = \frac{5}{6}$ into expression for $\boldsymbol{b}$ or $\boldsymbol{h}$ ***(M1)***

$(9, 12, 5)$ ***A2*** ***N3***

[5 marks]

Total [15 marks]