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**MATHEMATICAL STUDIES
STANDARD LEVEL
PAPER 2**

Thursday 13 November 2014 (morning)

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **Mathematical Studies SL formula booklet** is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is [90 marks].

Answer **all** questions in the answer booklet provided. Please start each question on a new page. You are advised to show all working, where possible. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, for example if graphs are used to find a solution, you should sketch these as part of your answer.

1. [Maximum mark: 21]

A biologist is studying the relationship between the number of chirps of the Snowy Tree cricket and the air temperature. He records the chirp rate, x , of a cricket, and the corresponding air temperature, T , in degrees Celsius.

The following table gives the recorded values.

Cricket’s chirp rate, x, (chirps per minute)	20	40	60	80	100	120
Temperature, T (°C)	8.0	12.8	15.0	18.2	20.0	21.1

- (a) Draw the scatter diagram for the above data. Use a scale of 2 cm for 20 chirps on the horizontal axis and 2 cm for 4 °C on the vertical axis. [4]
- (b) Use your graphic display calculator to write down the Pearson’s product–moment correlation coefficient, r , between x and T . [2]
- (c) Interpret the relationship between x and T using your value of r . [2]
- (d) Use your graphic display calculator to write down the equation of the regression line T on x . Give the equation in the form $T = ax + b$. [2]
- (e) Calculate the air temperature when the cricket’s chirp rate is 70. [2]
- (f) Given that $\bar{x} = 70$, draw the regression line T on x on your scatter diagram. [2]

A forest ranger uses her own formula for estimating the air temperature. She counts the number of chirps in 15 seconds, z , multiplies this number by 0.45 and then she adds 10.

- (g) Write down the formula that the forest ranger uses for estimating the temperature, T . Give the equation in the form $T = mz + n$. [1]

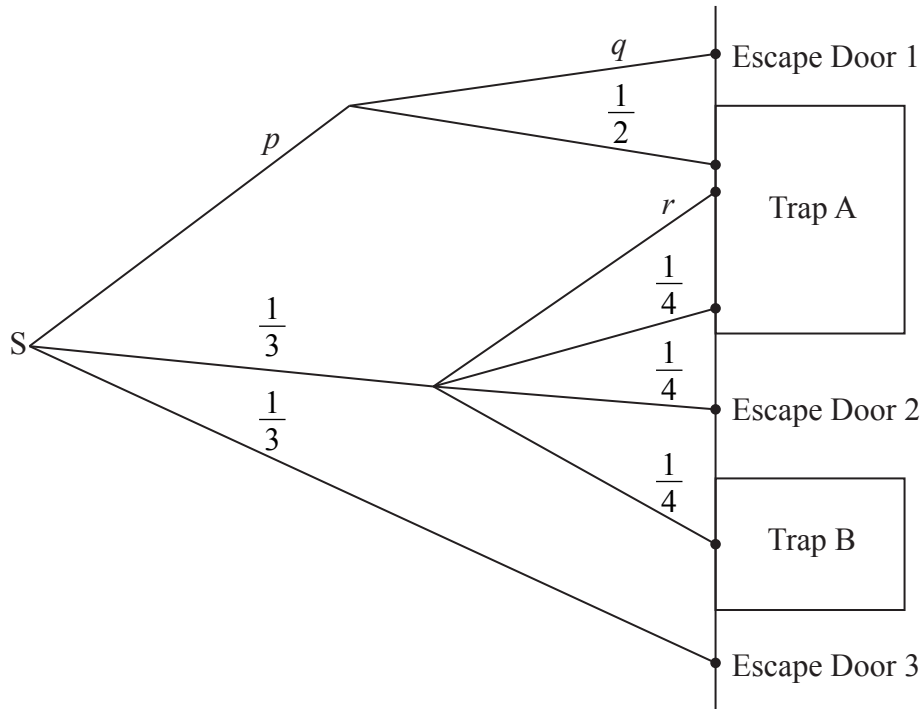
A cricket makes 20 chirps in **15** seconds.

- (h) For this chirp rate
 - (i) calculate an estimate for the temperature, T , **using the forest ranger’s formula**;
 - (ii) determine the actual temperature recorded by the biologist, **using the table above**;
 - (iii) calculate the percentage error in the forest ranger’s estimate for the temperature, compared to the actual temperature recorded by the biologist. [6]

2. [Maximum mark: 15]

Mike, the laboratory mouse, is placed at the starting point, S, of a maze. Some paths in the maze lead to Trap A, some to Trap B, and others to escape doors. Some paths have one and some have two sections. If his path forks, Mike randomly chooses a path **forward**.

The following tree diagram represents the maze, showing all possible paths, and the probability that Mike chooses a certain section of a path through the maze.



- (a) Write down the value of
 - (i) p ;
 - (ii) q ;
 - (iii) r . [3]
- (b) (i) Find the probability that Mike reaches Trap B.
- (ii) Find the probability that Mike reaches Trap A.
- (iii) Find the probability that Mike escapes from the maze. [7]

Sonya, a lab assistant, counts the number of paths that lead to traps or escape doors. She believes that the probability that Mike will be trapped is greater than the probability that he will escape.

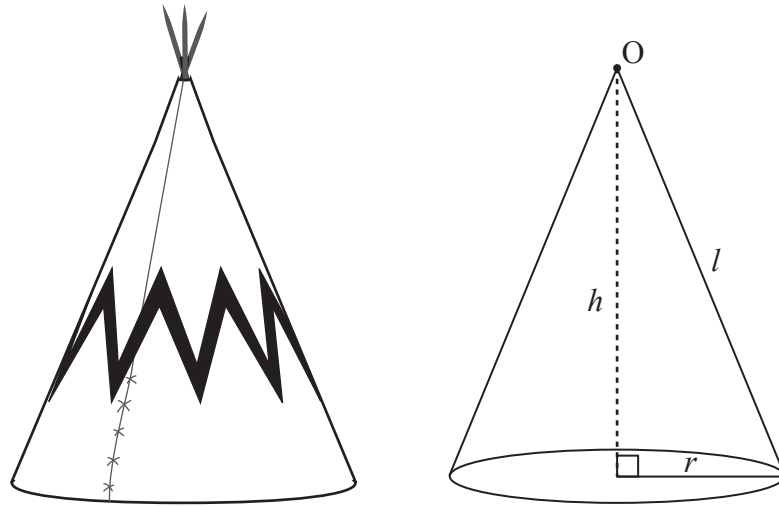
- (c) State whether Sonya is correct. Give a mathematical justification for your conclusion. [2]

During the first trial Mike escapes.

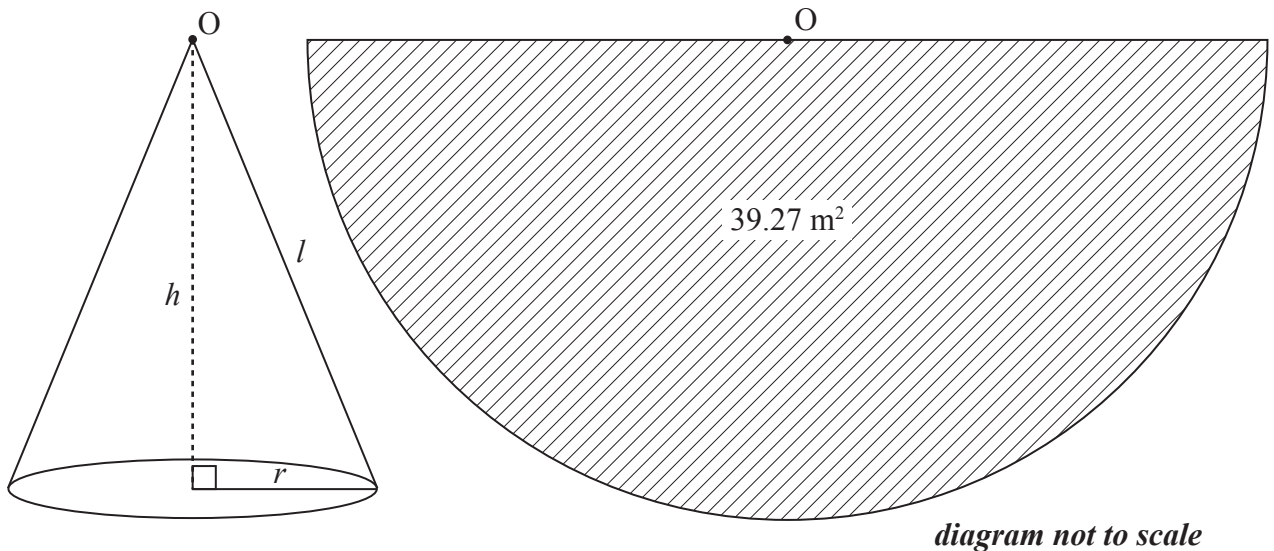
- (d) Given that Mike escaped, find the probability that he went directly from S to Escape Door 3. [3]

3. [Maximum mark: 16]

Tepees were traditionally used by nomadic tribes who lived on the Great Plains of North America. They are cone-shaped dwellings and can be modelled as a cone, with vertex O , shown below. The cone has radius, r , height, h , and slant height, l .



A model tepee is displayed at a Great Plains exhibition. The curved surface area of this tepee is covered by a piece of canvas that is 39.27 m^2 , and has the shape of a semicircle, as shown in the following diagram.



(This question continues on the following page)

(Question 3 continued)

- (a) Show that the slant height, l , is 5 m, correct to the nearest metre. [2]
- (b) (i) Find the circumference of the base of the cone.
- (ii) Find the radius, r , of the base.
- (iii) Find the height, h . [6]

A company designs cone-shaped tents to resemble the traditional tepees.

These cone-shaped tents come in a range of sizes such that the sum of the diameter and the height is equal to **9.33 m**.

- (c) Write down an expression for the height, h , in terms of the radius, r , of these cone-shaped tents. [1]
- (d) Show that the volume of the tent, V , can be written as

$$V = 3.11\pi r^2 - \frac{2}{3}\pi r^3. \quad [1]$$

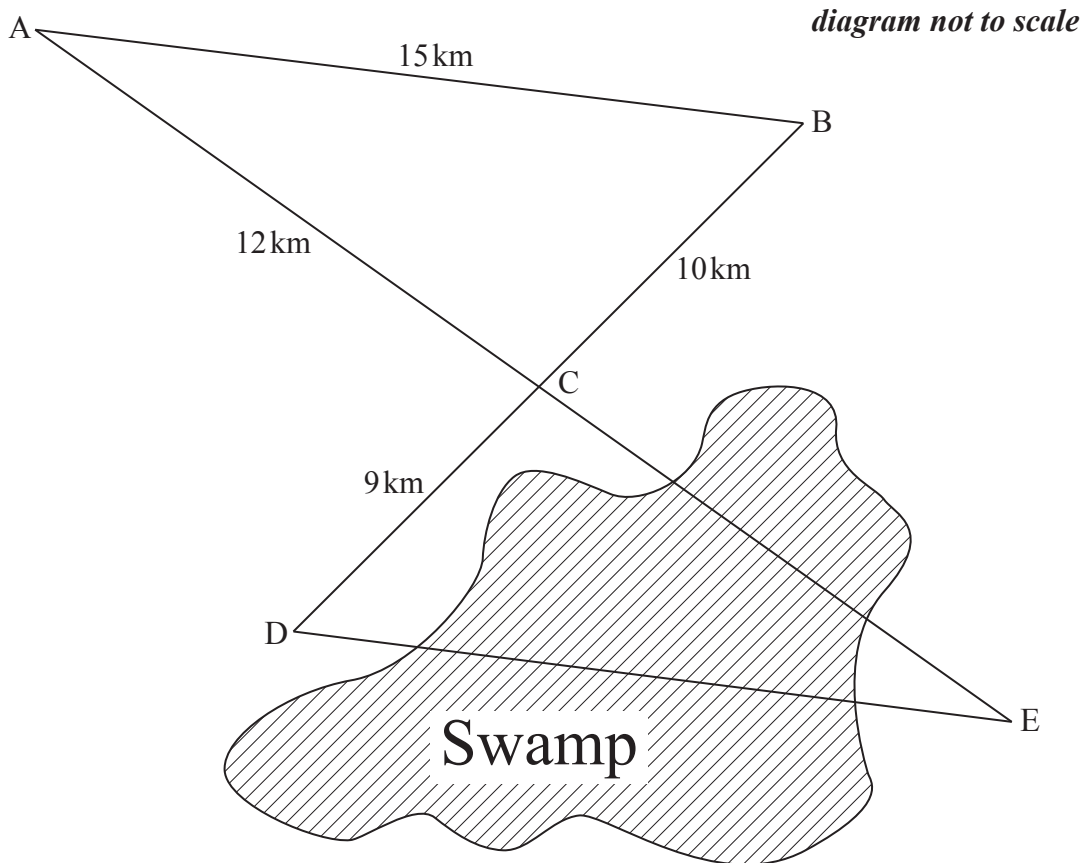
- (e) Find $\frac{dV}{dr}$. [2]
- (f) (i) Determine the **exact** value of r for which the volume is a maximum.
- (ii) Find the maximum volume. [4]

4. [Maximum mark: 13]

A surveyor has to calculate the area of a triangular piece of land, DCE.
The lengths of CE and DE cannot be directly measured because they go through a swamp.

AB, DE, BD and AE are straight paths. Paths AE and DB intersect at point C.
The length of AB is 15 km, BC is 10 km, AC is 12 km, and DC is 9 km.

The following diagram shows the surveyor's information.



- (a) (i) Find the size of angle ACB.
- (ii) Show that the size of angle DCE is 85.5° , correct to one decimal place. [4]

The surveyor measures the size of angle CDE to be twice that of angle DEC.

- (b) (i) Using angle $DCE = 85.5^\circ$, find the size of angle DEC.
- (ii) Find the length of DE. [5]
- (c) Calculate the area of triangle DEC. [4]

5. [Maximum mark: 14]

In a game, n small pumpkins are placed 1 metre apart in a straight line. Players start 3 metres before the first pumpkin.



Each player **collects** a single pumpkin by picking it up and bringing it back to the start. The nearest pumpkin is collected first. The player then collects the next nearest pumpkin and the game continues in this way until the signal is given for the end.

Sirma runs to get each pumpkin and brings it back to the start.

- (a) Write down the distance, a_1 , in metres that she has to run in order to **collect** the first pumpkin. [1]

The distances she runs to **collect** each pumpkin form a sequence a_1, a_2, a_3, \dots .

- (b) (i) Find a_2 .
- (ii) Find a_3 . [2]
- (c) Write down the common difference, d , of the sequence. [1]

The final pumpkin Sirma **collected** was 24 metres from the start.

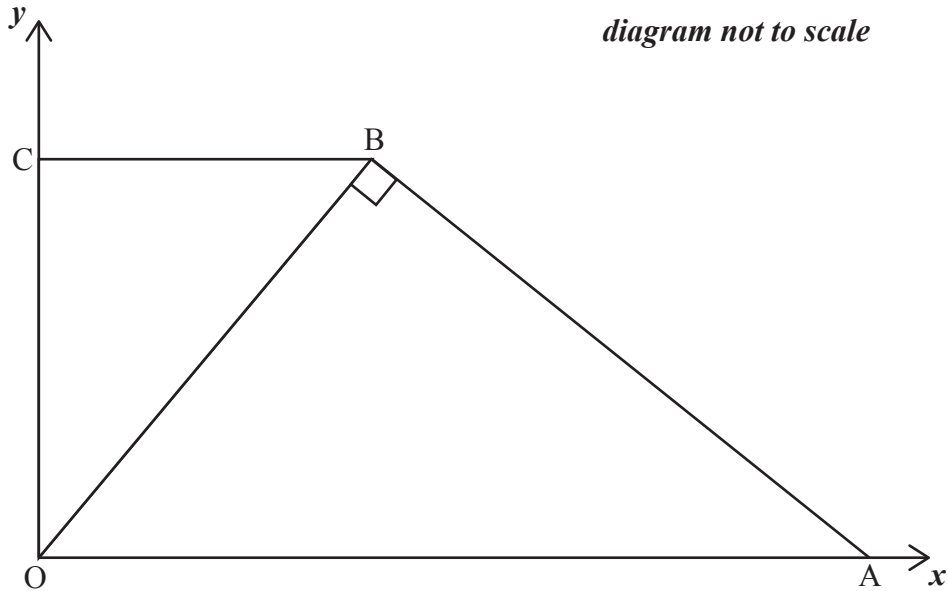
- (d) (i) Find the total number of pumpkins that Sirma **collected**.
- (ii) Find the total distance that Sirma ran to **collect** these pumpkins. [5]

Peter also plays the game. When the signal is given for the end of the game he has run 940 metres.

- (e) Calculate the total number of pumpkins that Peter **collected**. [3]
- (f) Calculate Peter's distance from the start when the signal is given. [2]

6. [Maximum mark: 11]

The following diagram shows two triangles, OBC and OBA, on a set of axes. Point C lies on the y -axis, and O is the origin.



The equation of the line BC is $y = 4$.

- (a) Write down the coordinates of point C. [1]

The x -coordinate of point B is a .

- (b) (i) Write down the coordinates of point B;
 (ii) Write down the gradient of the line OB. [2]

Point A lies on the x -axis and the line AB is perpendicular to line OB.

- (c) (i) Write down the gradient of line AB.
 (ii) Show that the equation of the line AB is $4y + ax - a^2 - 16 = 0$. [3]

The area of triangle AOB is **three times** the area of triangle OBC.

- (d) Find an expression, **in terms of a** , for
 (i) the area of triangle OBC;
 (ii) the x -coordinate of point A. [3]
- (e) Calculate the value of a . [2]