88147302

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

MATHEMATICS
STANDARD LEVEL
PAPER 2

Candidate session number

Thursday 13 November 2014 (morning)

1 hour 30 minutes

Examination code

8	8	1	4	–	7	3	0	2

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics SL*** *formula booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

13 pages

Please **do not** write on this page.

Answers written on this page
will not be marked.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 5]*

Let $f(x) = 2x + 3$ and $g(x) = x^3$.

(a) Find $(f \circ g)(x)$. *[2]*

(b) Solve the equation $(f \circ g)(x) = 0$. *[3]*

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2. *[Maximum mark: 6]*

The following table shows the Diploma score x and university entrance mark y for seven IB Diploma students.

Diploma score (x)	28	30	27	31	32	25	27
University entrance mark (y)	73.9	78.1	70.2	82.2	85.5	62.7	69.4

(a) Find the correlation coefficient. *[2]*

The relationship can be modelled by the regression line with equation $y = ax + b$.

(b) Write down the value of a and of b. *[2]*

Rita scored a total of 26 in her IB Diploma.

(c) Use your regression line to estimate Rita's university entrance mark. *[2]*

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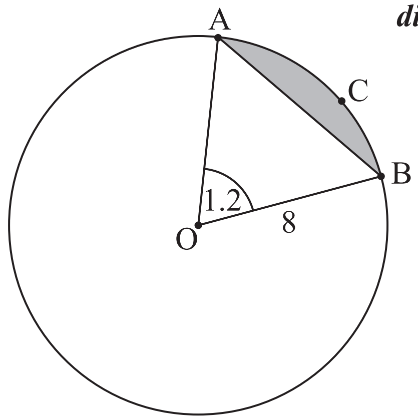
3. *[Maximum mark: 7]*

The following diagram shows a circle with centre O and radius 8 cm.

diagram not to scale



The points A, B and C are on the circumference of the circle, and $A\hat{O}B = 1.2$ radians.

(a) Find the length of arc ACB. *[2]*

(b) Find AB. *[3]*

(c) Hence, find the perimeter of the shaded segment ABC. *[2]*

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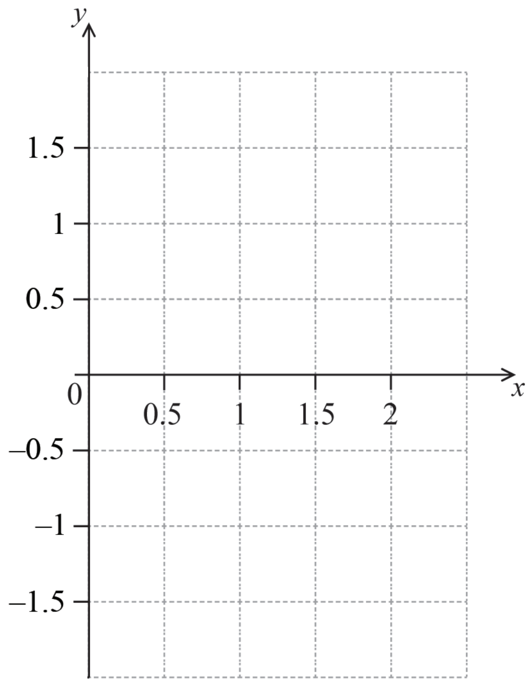
4. *[Maximum mark: 8]*

Let $f(x) = -x^4 + 2x^3 - 1$, for $0 \le x \le 2$.

(a) Sketch the graph of f on the following grid. *[3]*

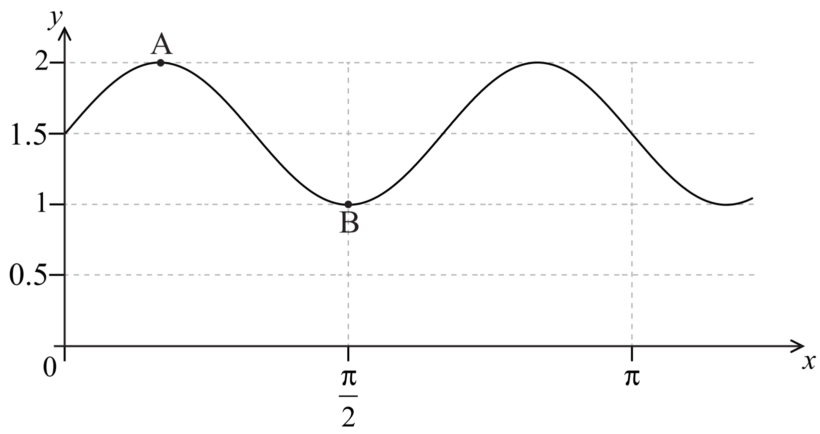


(b) Solve $f(x) = 0$. *[2]*

(c) The region enclosed by the graph of f and the x-axis is rotated 360° about the x-axis. Find the volume of the solid formed. *[3]*

5. *[Maximum mark: 7]*

The following diagram shows part of the graph of $y = p\sin(qx) + r$.



The point $A\left(\frac{\pi}{6}, 2\right)$ is a maximum point and the point $B\left(\frac{\pi}{2}, 1\right)$ is a minimum point. Find the value of

(a) p; *[2]*

(b) r; *[2]*

(c) q. *[3]*

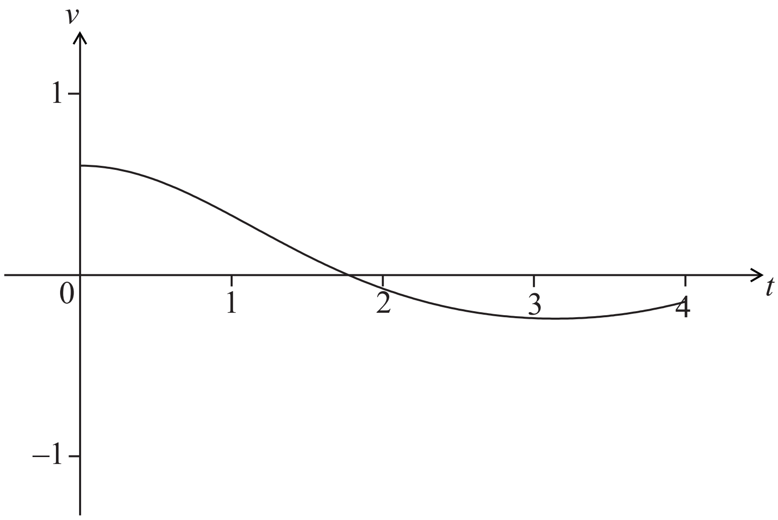
6. *[Maximum mark: 6]*

Consider the expansion of $\left(\frac{x^3}{2}+\frac{p}{x}\right)^8$. The constant term is 5103. Find the possible values of p.

7. *[Maximum mark: 6]*

A particle starts from point A and moves along a straight line. Its velocity, $v\,\text{m}\,\text{s}^{-1}$, after t seconds is given by $v(t) = e^{\frac{1}{2}\cos t} - 1$, for $0 \le t \le 4$. The particle is at rest when $t = \frac{\pi}{2}$.

The following diagram shows the graph of v.



(a) Find the distance travelled by the particle for $0 \le t \le \frac{\pi}{2}$. *[2]*

(b) Explain why the particle passes through A again. *[4]*

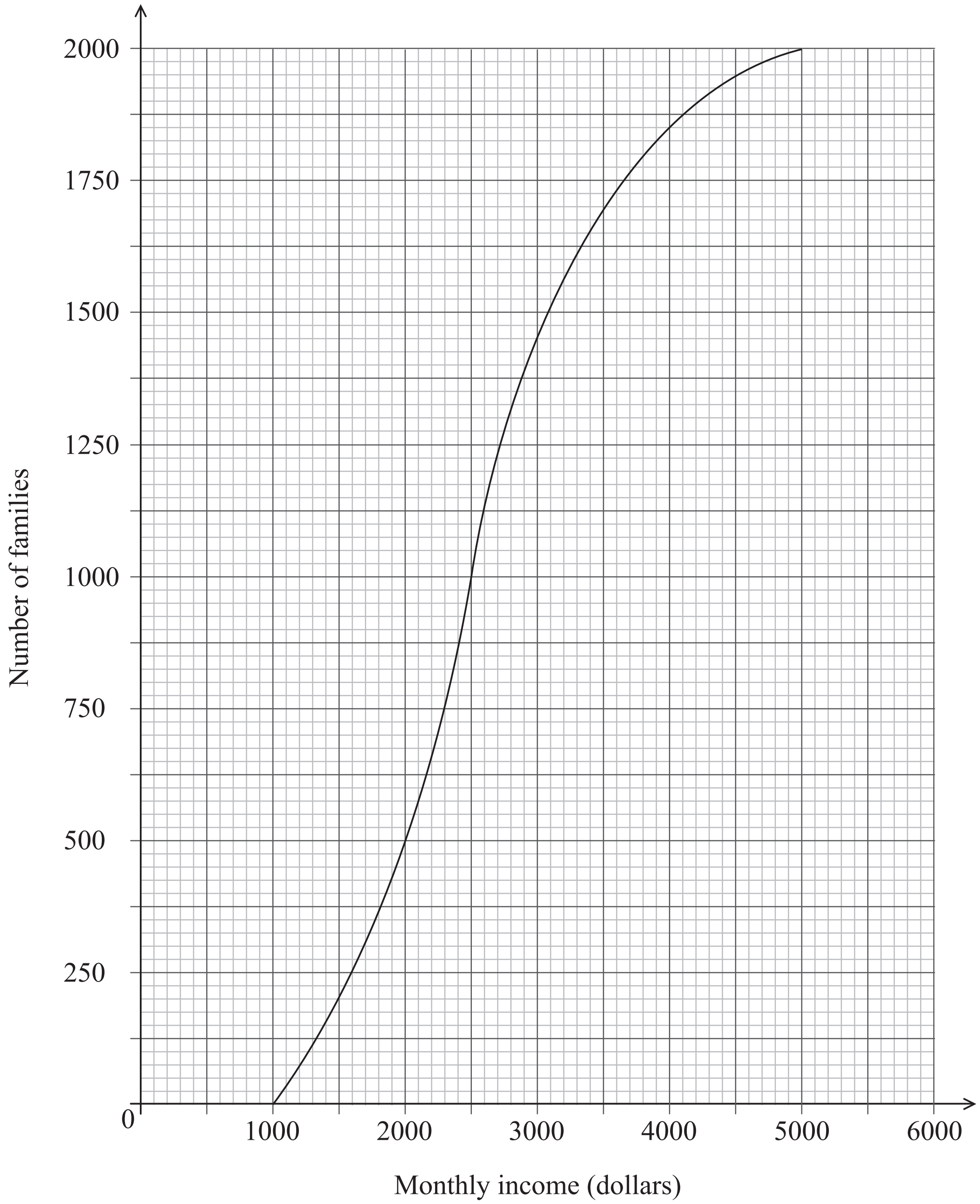
*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

8. *[Maximum mark: 15]*

The following cumulative frequency graph shows the monthly income, I dollars, of 2000 families.



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*Do **NOT** write solutions on this page.*

(Question 8 continued)

(a) Find the median monthly income. *[2]*

(b) (i) Write down the number of families who have a monthly income of 2000 dollars or less.

(ii) Find the number of families who have a monthly income of more than 4000 dollars. *[4]*

The 2000 families live in two different types of housing. The following table gives information about the number of families living in each type of housing and their monthly income I.

	$1000 < I \le 2000$	$2000 < I \le 4000$	$4000 < I \le 5000$
Apartment	436	765	28
Villa	64	p	122

(c) Find the value of p. *[2]*

(d) A family is chosen at random.

(i) Find the probability that this family lives in an apartment.

(ii) Find the probability that this family lives in an apartment, given that its monthly income is greater than 4000 dollars. *[4]*

(e) Estimate the mean monthly income for families living in a villa. *[3]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 14]*

The first two terms of a geometric sequence u_n are $u_1 = 4$ and $u_2 = 4.2$.

(a) (i) Find the common ratio.

(ii) Hence or otherwise, find u_5 . *[5]*

Another sequence v_n is defined by $v_n = an^k$, where $a, k \in \mathbb{R}$, and $n \in \mathbb{Z}^+$, such that $v_1 = 0.05$ and $v_2 = 0.25$.

(b) (i) Find the value of a .

(ii) Find the value of k . *[5]*

(c) Find the smallest value of n for which $v_n > u_n$. *[4]*

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 16]*

The weights of fish in a lake are normally distributed with a mean of 760 g and standard deviation σ. It is known that 78.87 % of the fish have weights between 705 g and 815 g.

(a) (i) Write down the probability that a fish weighs more than 760 g.

(ii) Find the probability that a fish weighs less than 815 g. *[4]*

(b) (i) Write down the standardized value for 815 g.

(ii) Hence or otherwise, find σ. *[4]*

A fishing contest takes place in the lake. Small fish, called tiddlers, are thrown back into the lake. The maximum weight of a tiddler is 1.5 standard deviations below the mean.

(c) Find the maximum weight of a tiddler. *[2]*

(d) A fish is caught at random. Find the probability that it is a tiddler. *[2]*

(e) 25 % of the fish in the lake are salmon. 10 % of the salmon are tiddlers. Given that a fish caught at random is a tiddler, find the probability that it is a salmon. *[4]*

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