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MARKSCHEME

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MATHEMATICS

Standard Level

Paper 2

Instructions to Examiners (red new, green check carefully)

Abbreviations

M   Marks awarded for attempting to use a correct **Method**; working must be seen.

(M)   Marks awarded for **Method**; may be implied by **correct** subsequent working.

A   Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding *M* marks.

(A)   Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R   Marks awarded for clear **Reasoning**.

N   Marks awarded for **correct** answers if **no** working shown.

AG   Answer given in the question and so no marks are awarded.

Using the markscheme

1   General

Mark according to RM assessor instructions and the document "**Mathematics SL: Guidance for e-marking May 2014**". It is **essential** that you read this document before you start marking. In particular, please note the following. Marks must be recorded using the annotation stamps, using the RM assessor tool. Please check that you are entering marks for the right question.

- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks. Do **not** use the ticks with numbers for anything else.
- If a part is completely wrong, stamp *A0* by the final answer.
- If a part gains anything else, all the working **must** have annotations stamped to show what marks are awarded.   This includes any zero marks.

All the marks will be added and recorded by RM assessor.

2   Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award *M0* followed by *A1*, as *A* mark(s) depend on the preceding *M* mark(s), if any. An exception to this rule is when work for *M1* is missing, as opposed to incorrect (see point 4).
- Where *M* and *A* marks are noted on the same line, *eg M1A1*, this usually means *M1* for an **attempt** to use an appropriate method (*eg* substitution into a formula) and *A1* for using the **correct** values.
- Where there are two or more *A* marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award *A0A1A1*.
- Where the markscheme specifies *(M2)*, *N3*, *etc*., do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further working.
- Most *M* marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.

3   *N* marks

*If **no** working shown, award N marks for **correct** answers – this includes acceptable answers (see accuracy booklet). In this case, ignore mark breakdown (**M, A, R**).*

- Do **not** award a mixture of *N* and other marks.
- There may be fewer *N* marks available than the total of *M*, *A* and *R* marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the *N* marks and the implied marks. There are times when all the marks are implied, but the *N* marks are not the full marks: this indicates that we want to see some of the working, without specifying what.
- For consistency within the markscheme, *N* marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the *N* marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the *N* marks for the correct answer.

4   Implied and must be seen marks

*Implied marks appear in **brackets** eg (M1).*
- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the *N* marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (*M1*) followed by *A1* for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (*M1*).

*Must be seen marks appear without **brackets** eg M1.*
- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to *M0* or *A0* for incorrect work) all subsequent marks may be awarded if appropriate.

5   Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further *A* marks can be awarded for work which uses the error, but *M* and *R* marks may be awarded if appropriate. (However, as noted above, if an *A* mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer *FT* marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin \theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).

- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final *A1*. Note that if the error occurs within the same subpart, the *FT* rules may result in further loss of marks.
- Where there are anticipated common errors, the *FT* answers are often noted on the markscheme, to help examiners. It should be stressed that these are not the only *FT* answers accepted, neither should *N* marks be awarded for these answers.

6   Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the *MR*, then use discretion to award fewer marks.
- If the *MR* leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin \theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7   Discretionary marks *(d)*
*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark*
explaining this decision.

8   Alternative methods
Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9   Alternative forms
*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10   Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (eg TI-89) are not allowed.

Calculator notation The mathematics SL guide says:
  Students must always use correct mathematical notation, not calculator notation.
Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

11   Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the " $k =$ ". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

*The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.*

12   Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked. The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets

13.   Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first *A1* is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, *FT* marks should be awarded if appropriate.

14.   Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures

Candidates should NO LONGER be penalized for an accuracy error (AP). Examiners should award marks according to the rules given in these instructions and the markscheme. Accuracy is not the same as correctness – an incorrect value does not achieve relevant A marks. It is only final answers which may lose marks for accuracy errors, not intermediate values. Please check work carefully for FT.

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (*eg* 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers.

Clarification of intermediate values accuracy instructions

Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award *A0* for the final answer. However, do not penalise inaccurate intermediate values that lead to an acceptable final answer.

All examiners must read this section carefully, as there are some changes (in red) since M13.

These instructions apply when answers need to be rounded, they do not apply to exact answers which have 3 or fewer figures. The answers will give a range of acceptable values, and any answer given to 3 or more sf that lies in this range will be accepted as well as answers given to the correct 2 sf (which will usually not be in the acceptable range). Answers which are given to 1 sf are not acceptable. There is also a change to the awarding of N marks for acceptable answers.

Where numerical answers are required as the **final** answer to a part of a question in the markscheme, the markscheme will show

     a truncated 6 sf value

     the exact value if applicable, the correct 3 sf answer and the range of acceptable values. This range includes both end values. Once an acceptable value is seen, ignore any subsequent values (even if rounded incorrectly).

Units (which are generally not required) will appear in brackets at the end.

Example

1.73205

$\sqrt{3}$ (exact), 1.73 [1.73, 1.74] (m)

Note that 1.73 is the correct 3 sf, 1.74 is incorrectly rounded but acceptable, 1.7 is the correct 2 sf value but 1.72 is wrong.

For subsequent parts, the markscheme will show the answers obtained from using unrounded values, and the answers from using previous **correct** 3 sf answers. Examiners will need to check the work carefully if candidates use any other acceptable answers. If other acceptable answers lead to an incorrect final answer (ie outside the range), do not award the final *A1*. This should not be considered as *FT*.

Intermediate values do **not** need to be given to the correct 3 sf.   If candidates work with fewer than 3 sf, or with incorrectly rounded values, this could lead to an incorrect answer, in which case award *A0* for the final answer. However, do not penalise intermediate inaccurate values that lead to an acceptable final answer.

In questions where the final answer gains *A2*, if other working shown, award *A1* for a correctly rounded 1 sf answer.

If there is **no** working shown, award the *N* marks for **any** acceptable answer, eg in the example above, if 1.73 achieves *N4*, then 1.74, 1.7, 1.7320 all achieve *N4,* but 2 achieves *N0* .

The following table shows what achieves the final mark if this is the **only** numerical answer seen, as long as there is other working.

	Correctly rounded	Incorrectly rounded
1sf	No	No
2sf	Yes	No
3sf	Yes	Yes (if in the acceptable range)
4 or more sf	Yes (if in the acceptable range)	Yes (if in the acceptable range)

Examples: The correct marking is given at the end of this section. Please decide what marks you would give each answer, and then check. Assume that working is shown unless otherwise indicated. If you disagree, please discuss with your team leader.

Example 1 (awards *A1* for final answer)

Markscheme
7.43798
7.44 [7.43, 7.44]   *A1*   *N3*

	Candidate's Script	Marking
(i)	7.43798 followed by anything	
(ii)	7.5	
(iii)	7.4	
(iv)	7.4 (with no working)	
(v)	7	
(vi)	7.438	
(vii)	7.43	
(viii)	7.43 (with no working)	
(ix)	7.437	
(x)	7.433	

Example 2 (awards *A2* for final answer)

Markscheme
8.43482
8.43 [8.43, 8.44]   *A2*   *N3*

	Candidate's Script	Marking
(i)	8.433016	
(ii)	8.44	
(iii)	8	
(iv)	8.42	
(v)	8.4 (with no working)	
(vi)	8 (with no working)	
(vii)	8.44 (with no working)	
(viii)	8.43 (with no working)	

Answers to the examples.

Example 1 (awards *A1* for final answer)

(i)	7.43798 followed by anything	*A1*
(ii)	7.5 (wrong)	*A0*
(iii)	7.4 (correct 2 sf)	*A1*
(iv)	7.4 (with no working)	*N3*
(v)	7 (1 sf)	*A0*
(vi)	7.438 (in acceptable range)	*A1*
(vii)	7.43 (acceptable 3 sf)	*A1*
(viii)	7.43 (with no working)	*N3*
(ix)	7.437 (in acceptable range)	*A1*
(x)	7.433 (in acceptable range)	*A1*

Example 2 (awards *A2* for final answer)

	Candidate's Script	Marking
(i)	8.433016 (in acceptable range)	*A2*
(ii)	8.44	*A2*
(iii)	8 (1 sf, penalise 1 mark)	*A1*
(iv)	8.42 (outside acceptable range)	*A0*
(v)	8.4 (with no working)	*N3*
(vi)	8 (with no working)	*N0*
(vii)	8.44 (with no working)	*N3*
(viii)	8.43 (with no working)	*N3*

SECTION A

1. (a) attempt to form composite (in any order) **(M1)**

eg $f(x^3)$, $(2x+3)^3$

$(f \circ g)(x) = 2x^3 + 3$, $2(x)^3 + 3$ **A1** **N2**

[2 marks]

(b) evidence of appropriate approach **(M1)**

eg $2x^3 = -3$, sketch
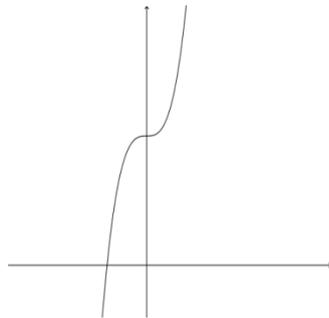
correct working **(A1)**

eg $x^3 = \dfrac{-3}{2}$, sketch
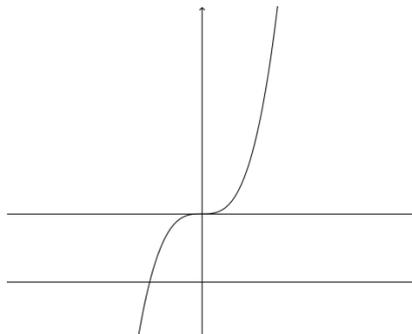
-1.14471

$x = \sqrt[3]{\dfrac{-3}{2}}$ (exact), -1.14 $[-1.15, -1.14]$ **A1** **N3**

[3 marks]

Total [5 marks]

2. (a)   evidence of set up   *(M1)*
        eg   correct value for r (or for a or b, seen in (b))

        0.996010
        $r = 0.996$ [0.996, 0.997]   *A1*   *N2*
        [2 marks]

    (b) $a = 3.15037$, $b = -15.4393$
        $a = 3.15$ [3.15, 3.16], $b = -15.4$ [$-15.5, -15.4$]   *A1A1*   *N2*
        [2 marks]

    (c)   substituting 26 into **their** equation   *(M1)*
        eg $y = 3.15(26) - 15.4$

        66.4704
        $66.5\, [66.4,\, 66.5]$ *A1*   *N2*
        [2 marks]

        Total [6 marks]

3. (a)   correct substitution into formula   *(A1)*
        eg $l = 1.2 \times 8$

        9.6 (cm)   *A1*   *N2*
        [2 marks]

    (b)   **METHOD 1**

        evidence of choosing cosine rule   *(M1)*
        eg $2r^2 - 2 \times r^2 \times \cos\left(\hat{\text{AOB}}\right)$

        correct substitution into right hand side   *(A1)*
        eg $8^2 + 8^2 - 2 \times 8 \times 8 \times \cos(1.2)$

        9.0342795
        $\text{AB} = 9.03$ [9.03, 9.04] (cm)   *A1*   *N2*

        METHOD 2

        evidence of choosing sine rule   *(M1)*
        eg $\dfrac{\text{AB}}{\sin\left(\hat{\text{AOB}}\right)} = \dfrac{\text{OB}}{\sin\left(\hat{\text{OAB}}\right)}$

        finding angle OAB or OBA (may be seen in substitution)   *(A1)*
        eg $\dfrac{\pi - 1.2}{2}$, 0.970796

        $\text{AB} = 9.03$ [9.03, 9.04] (cm)   *A1*   *N2*

        [3 marks]

Question 3 continued

(c)   correct working   **(A1)**
       eg $P = 9.6 + 9.03$

       18.6342
       18.6 [18.6, 18.7] (cm)   **A1   N2**
                                                                                     [2 marks]

                                                                                  Total [7 marks]

4.   (a)
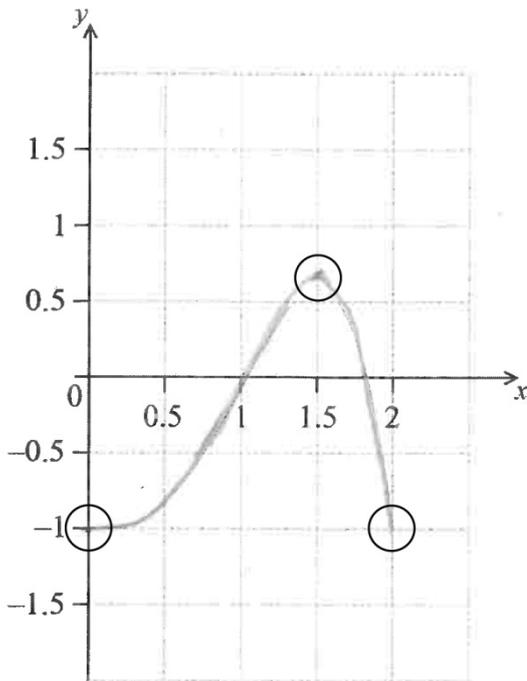
                                                                                  A1A1A1   N3
                                                                                     [3 marks]

> **Note:**   Award **A1** for both endpoints in circles,
>   **A1** for approximately correct shape (concave up to concave down).
>   Only if this **A1** for shape is awarded, award **A1** for maximum point in circle.

(b) $x = 1$ $x = 1.83928$
       $x = 1$ (exact) $x = 1.84$ [1.83, 1.84]   **A1A1   N2**
                                                                                     [2 marks]

(c)   attempt to substitute either (**FT**) limits or function into formula with f^2 **(M1)**
       (accept absence of π or d*x,* but do not accept any errors, including extra bits)
       eg $V = \pi \int_{1}^{1.84} f^2 \,, \int \left(-x^4 + 2x^3 - 1\right)^2 dx$

       0.636581
       $V = 0.637$ [0.636, 0.637]   **A2   N3**
                                                                                     [3 marks]

                                                                                  Total [8 marks]

5. (a) valid approach   *(M1)*

$$eg \quad \frac{2-1}{2}, \; 2-1.5$$

   $p = 0.5$ *A1   N2*

                                                       [2 marks]

(b) valid approach   *(M1)*

$$eg \quad \frac{1+2}{2}$$

   $r = 1.5$ *A1   N2*

                                                       [2 marks]

(c) **METHOD 1**

   valid approach (seen anywhere)   *M1*

$$eg \quad q = \frac{2\pi}{\text{period}}, \; \frac{2\pi}{\left(\frac{2\pi}{3}\right)}$$

   $\text{period} = \dfrac{2\pi}{3}$ (seen anywhere)   *(A1)*

   $q = 3$ *A1   N2*

   METHOD 2

   attempt to substitute one point and **their** values for p and r into y *M1*

$$eg \quad 2 = 0.5\sin\left(q\frac{\pi}{6}\right)+1.5, \; \frac{\pi}{2} = 0.5\sin(q1)+1.5$$

   correct equation in q *(A1)*

$$eg \quad q\frac{\pi}{6} = \frac{\pi}{2}, \; q\frac{\pi}{2} = \frac{3\pi}{2}$$

   $q = 3$ *A1   N2*

   METHOD 3

   valid reasoning comparing the graph with that of $\sin x$ *R1*
   eg   position of max/min, graph goes faster

   correct working   *(A1)*

$$eg \quad \text{max at } \frac{\pi}{6} \text{ not at } \frac{\pi}{2}, \text{ graph goes 3 times as fast}$$

   $q = 3$ *A1   N2*

                                                       [3 marks]

                                                   Total [7 marks]

6. valid approach to find the required term **(M1)**

eg $\dbinom{8}{r}\left(\dfrac{x^3}{2}\right)^{8-r}\left(\dfrac{p}{x}\right)^{r}$, $\dbinom{8}{0}\left(\dfrac{x^3}{2}\right)^{8}\left(\dfrac{p}{x}\right)^{0}+\dbinom{8}{1}\left(\dfrac{x^3}{2}\right)^{7}\left(\dfrac{p}{x}\right)^{1}+\ldots$, Pascal's triangle to required value

identifying constant term (may be indicated in expansion) **(A1)**

eg 7th term, $r=6$, $\left(\dfrac{1}{2}\right)^{2}$, $\dbinom{8}{6}$, $\left(\dfrac{x^3}{2}\right)^{2}\left(\dfrac{p}{x}\right)^{6}$

correct calculation (may be seen in expansion) **(A1)**

eg $\dbinom{8}{6}\left(\dfrac{x^3}{2}\right)^{2}\left(\dfrac{p}{x}\right)^{6}$, $\dfrac{8\times 7}{2}\times\dfrac{p^6}{2^2}$

setting up equation with **their** constant term equal to 5103 **M1**

eg $\dbinom{8}{6}\left(\dfrac{x^3}{2}\right)^{2}\left(\dfrac{p}{x}\right)^{6}=5103$, $p^6=\dfrac{5103}{7}$

$p=\pm 3$ **A1A1** **N3**

[6 marks]

7.　(a)　correct substitution of function and/or limits into formula　　　　*(A1)*
　　　　　(accept absence of d*t,* but do not accept any errors)

$$eg \quad \int_0^{\frac{\pi}{2}} v, \ \int \left| e^{\frac{1}{2}\cos t} -1 \right| dt, \ \int \left(e^{\frac{1}{2}\cos t} -1 \right)$$

　　　　　0.613747
　　　　　distance is 0.614 [0.613, 0.614] (m)　　　　　　　　　　　　　*A1*　　*N2*
　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　[2 marks]

　　(b)　**METHOD 1**

　　　　　valid attempt to find the distance travelled between $t = \dfrac{\pi}{2}$ and $t = 4$　　*(M1)*

$$eg \quad \int_{\frac{\pi}{2}}^4 \left(e^{\frac{1}{2}\cos t} -1 \right), \ \int_0^4 \left| e^{\frac{1}{2}\cos t} -1 \right| dt - 0.614$$

　　　　　distance is 0.719565　　　　　　　　　　　　　　　　　　　*A1*

　　　　　valid reason, referring to change of direction　(may be seen in explanation)　*R1*

　　　　　valid explanation comparing **their** distances　　　　　　　　　*R1*
　　　　　eg　　0.719565 > 0.614, distance moving back is more than distance
　　　　　　　　moving forward

> **Note:** Do not award the final *R1* unless the *A1* is awarded.

　　　　　particle passes through A again　　　　　　　　　　　　　　*AG*　　*N0*

　　　　　METHOD 2

　　　　　valid attempt to find displacement　　　　　　　　　　　　　*(M1)*

$$eg \quad \int_{\frac{\pi}{2}}^4 \left(e^{\frac{1}{2}\cos t} -1 \right), \ \int_0^4 \left(e^{\frac{1}{2}\cos t} -1 \right)$$

　　　　　correct displacement　　　　　　　　　　　　　　　　　　*A1*
　　　　　eg　　−0.719565, −0.105817

　　　　　recognising that displacement from 0 to $\dfrac{\pi}{2}$ is positive　　　　　*R1*

　　　　　eg　　displacement = distance from 0 to $\dfrac{\pi}{2}$

　　　　　valid explanation referring to positive and negative displacement　　*R1*
　　　　　eg　　0.719565 > 0.614, overall displacement is negative, since
　　　　　　　　displacement after $\dfrac{\pi}{2}$ is negative, then particle gone backwards
　　　　　　　　more than forwards

> **Note:** Do not award the final *R1* unless the *A1* and the first *R1* are awarded.

　　　　　particle passes through A again　　　　　　　　　　　　　　*AG*　　*N0*
　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　[4 marks]

Question 7 continued

Note: Special Case. If all working shown, and candidates seem to have misread the question, using $v = e^{\frac{1}{2}\cos t}$, award marks as follows:	

(a) correct substitution of function and/or limits into formula   ***A0MR***
(accept absence of d*t,* but do not accept any errors)

$$eg \quad \int_{0}^{\frac{\pi}{2}} \left(e^{\frac{1}{2}\cos t} \right), \int \left| e^{\frac{1}{2}\cos t} \right| dt , \int \left(e^{\frac{1}{2}\cos t} \right)$$

2.184544
distance is 2.18 [2.18, 2.19] (m)   ***A1***   ***N0***

(b) **METHOD 1**

valid attempt to find the distance travelled between $t = \dfrac{\pi}{2}$ and $t = 4$ ***M1***

$$eg \quad \int_{\frac{\pi}{2}}^{4} \left(e^{\frac{1}{2}\cos t} \right), \int_{0}^{4} \left| e^{\frac{1}{2}\cos t} \right| dt - 2.18$$

distance is 1.709638   ***A1***

reference to change of direction (may be seen in explanation)   ***R1***

reasoning/stating particle passes/does not pass through A again   ***R0***

METHOD 2

valid attempt to find displacement   ***M1***

$$eg \quad \int_{\frac{\pi}{2}}^{4} \left(e^{\frac{1}{2}\cos t} \right), \int_{0}^{4} \left(e^{\frac{1}{2}\cos t} \right)$$

correct displacement   ***A1***
eg   1.709638, 3.894182

recognising that displacement from 0 to $\dfrac{\pi}{2}$ is positive   ***R0***

reasoning/stating particle passes/does not pass through A again   ***R0***

With method 2, there is no valid reasoning about whether the particle
passes through A again or not, so they cannot gain the ***R*** marks.

Total [6 marks]

SECTION B

8. (a) recognizing that the median is at half the total frequency   **(M1)**

$$eg \quad \frac{2000}{2}$$

  $m = 2500$ (dollars)   **A1   N2**

  [2 marks]

  (b) (i) 500 families have a monthly income less than 2000   **A1   N1**

  (ii) correct cumulative frequency, 1850   **(A1)**

  subtracting **their** cumulative frequency from 2000   **(M1)**
  $eg \quad 2000 - 1850$

  150 families have a monthly income of more than 4000 dollars   **A1   N2**

> **Note:** If working shown, award **M1A1A1** for $128 + 22 = 150$, using the table.

  [4 marks]

  (c) correct calculation   **(A1)**
  $eg \quad 2000 - (436 + 64 + 765 + 28 + 122)$, $1850 - 500 - 765$

  $p = 585$ **A1   N2**

  [2 marks]

  (d) (i) correct working   **(A1)**
  $eg \quad 436 + 765 + 28$

  0.6145 (exact)
  $\frac{1229}{2000}$, 0.615 [0.614, 0.615]   **A1   N2**

  (ii) correct working/probability for number of families   **(A1)**

  $eg \quad 122 + 28$, $\frac{150}{2000}$, 0.075

  0.186666
  $\frac{28}{150}\left(=\frac{14}{75}\right)$, 0.187 [0.186, 0.187]   **A1   N2**

  [4 marks]

  (e) evidence of using correct mid-interval values (1500, 3000, 4500)   **(A1)**

  attempt to substitute into $\dfrac{\sum fx}{\sum f}$ **(M1)**

  $eg \quad \dfrac{1500 \times 64 + 3000 \times p + 4500 \times 122}{64 + 585 + 122}$

  3112.84
  3110 [3110, 3120] (dollars)   **A1   N2**

  [3 marks]

  Total [15 marks]

9.   (a)   (i)   valid approach   *(M1)*

eg $r = \dfrac{u_2}{u_1}$, $\dfrac{4}{4.2}$

$r = 1.05$ (exact)   *A1*   *N2*

(ii)   attempt to substitute into formula, with **their** r *(M1)*
eg 4×1.05^n , $4 \times 1.05 \times 1.05 \ldots$

correct substitution   *(A1)*
eg 4×1.05^4 , $4 \times 1.05 \times 1.05 \times 1.05 \times 1.05$

$u_5 = 4.862025 \,(\text{exact})$, 4.86 $[4.86, 4.87]$ *A1*   *N2*
                                                                              [5 marks]

(b)   (i)   attempt to substitute $n = 1$ *(M1)*
eg $0.05 = a \times 1^k$

$a = 0.05$ *A1*   *N2*

(ii)   correct substitution of $n = 2$ into v_2 *A1*
eg $0.25 = a \times 2^k$

correct work   *(A1)*
eg   finding intersection point, $k = \log_2\left(\dfrac{0.25}{0.05}\right)$, $\dfrac{\log 5}{\log 2}$

2.32192
$k = \log_2 5$ (exact), 2.32 $[2.32, 2.33]$ *A1*   *N2*
                                                                              [5 marks]

(c)   correct expression for u_n *(A1)*
eg $4 \times 1.05^{n-1}$

EITHER

correct substitution into inequality (accept equation)   *(A1)*
eg $0.05 \times n^k > 4 \times 1.05^{n-1}$

valid approach to solve inequality (accept equation)   *(M1)*
eg   finding point of intersection, $n = 7.57994$ (7.59508 from 2.32)

$n = 8$ (must be an integer)   *A1*   *N2*

OR

table of values
when $n = 7$, $u_7 = 5.3604$, $v_7 = 4.5836$ *A1*
when $n = 8$, $u_8 = 5.6284$, $v_8 = 6.2496$ *A1*
$n = 8$ (must be an integer)   *A1*   *N2*
                                                                              [4 marks]

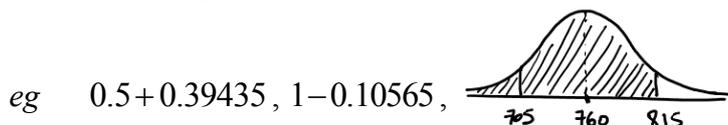   *Total [14 marks]*

10. | **Note:** There may be slight differences in answers, depending on which values candidates carry through in subsequent parts. Accept answers that are consistent with their working.

(a) (i) $P(X > 760) = 0.5$ (exact), [0.499, 0.500]   ***A1***   ***N1***

(ii) evidence of valid approach   ***(M1)***

recognising symmetry, $\dfrac{0.7887}{2}$, $1 - P(W < 815)$, $\dfrac{21.13}{2} + 78.87\%$

correct working   ***(A1)***

eg $0.5 + 0.39435$, $1 - 0.10565$,

0.89435 (exact), 0.894 [0.894, 0.895]   ***A1***   ***N2***

[4 marks]

(b) (i)   1.24999
$z = 1.25$ [1.24, 1.25]   ***A1***   ***N1***

(ii) evidence of appropriate approach   ***(M1)***

eg $\sigma = \dfrac{x - \mu}{1.25}$, $\dfrac{815 - 760}{\sigma}$

correct substitution   ***(A1)***

eg $1.25 = \dfrac{815 - 760}{\sigma}$, $\dfrac{815 - 760}{1.24999}$

44.0003
$\sigma = 44.0$ [44.0, 44.1] (g)   ***A1***   ***N2***

[4 marks]

(c) correct working   ***(A1)***
eg $760 - 1.5 \times 44$

693.999
694 [693, 694] (g)   ***A1***   ***N2***

[2 marks]

(d) 0.0668056
$P(X < 694) = 0.0668$ [0.0668, 0.0669]   ***A2***   ***N2***

[2 marks]

continued…

Question 10 continued

(e) recognizing conditional probability (seen anywhere)   **(M1)**

     eg $P(A|B)$, $\dfrac{0.025}{0.0668}$

appropriate approach involving conditional probability   **(M1)**

     eg $P(S|T) = \dfrac{P(S \text{ and } T)}{P(T)}$,

correct working

     eg $P(\text{salmon and tiddler}) = 0.25 \times 0.1$, $\dfrac{0.25 \times 0.1}{0.0668}$ **(A1)**

0.374220
0.374 [0.374, 0.375]   *A1*   *N2*

                                                                [4 marks]

                                                                  Total [16 marks]