



MARKSCHEME

May 2014

MATHEMATICAL STUDIES

Standard Level

Paper 1

**Paper 1 Markscheme
Instructions to Examiners**

Notes: If in doubt about these instructions or any other marking issues, contact your team leader for clarification.

The number of marks for each question is 6.

1 Abbreviations

The markscheme may make use of the following abbreviations:

- M** Marks awarded for **Method**
- A** Marks awarded for an **Answer** or for **Accuracy**
- C** Marks awarded for **Correct** answers (irrespective of working shown)
- R** Marks awarded for clear **Reasoning**
- ft** Marks that can be awarded as **follow through** from previous results in the question

2 Method of Marking

- (a) All marking must be done in scoris using the mathematical studies annotations and in accordance with the current document for guidance in e-marking Mathematical Studies SL. It is essential that you read this document before you start marking.
- (b) If the candidate has full marks on a question use the **C6** annotation, if the candidate has made an attempt but scores zero marks use **C0**. If there is no attempt use the No response button. If a candidate does not score full or zero marks then full annotations **MUST** be shown.
- (c) In this paper, if the **correct answer is seen on the answer line the** maximum mark is awarded. **There is no need to check the working!** Award **C** marks and move on.
- (d) If the answer does not appear on the answer line, but the correct answer is seen in the working box with no subsequent working, award the maximum mark.
- (e) If the **answer is wrong**, marks should be awarded for the working according to the markscheme.
- (f) Working crossed out by the candidate should not be awarded any marks. Where candidates have written two solutions to a question, only the first solution should be marked.
- (g) A correct answer in the working box transcribed inaccurately to the answer line can receive full marks.
- (h) If correct working results in a correct answer **in the working box** but then further working is developed, full marks should **not** be awarded. In most such cases it will be a single final answer mark that is lost, however, a statement on the answer line should always be taken as the candidate's final decision on the answer **as long as it is unambiguous**. Accuracy of numerical answers is an exception to this rule – see Section 5.

Example: Factorise $x^2 - 5x - 6$

Markscheme	Candidates' Scripts	Marking
$(x-6)(x+1)$ (AI)(AI)	(i) Answer line: $(x+6)(x+1)$	(A0)(AI)
	(ii) Working box: $(x-6)(x+1)$ followed by $x=6$ and -1 , or just $6, -1$ in either working box or on answer line.	(AI) (A0)

3 Follow through (ft) Marks

Errors made at any step of a solution affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks can be awarded. Markschemes will indicate where it is appropriate to apply follow through in a question with '(ft)'.

- (a) Follow through applies only from one part of a question to a subsequent part of the question. Follow through does not apply within the same part.
- (b) If an answer resulting from follow through is extremely unrealistic (eg, negative distances or incorrect by large order of magnitude) then the final **A** mark should not be awarded.
- (c) If a question is transformed by an error into a **different, much simpler question** then follow through may not apply.
- (d) To award follow through marks for a question part, **there must be working present for that part**. An isolated follow through answer, without working is regarded as incorrect and receives no marks **even if it is approximately correct**.
- (e) The exception to the above would be in a question which is testing the candidate's use of the GDC, where working will not be expected. **The markscheme will clearly indicate where this applies**.
- (f) Inadvertent use of radians will be penalised the first time it occurs. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for the use of radians.

Example: Finding angles and lengths using trigonometry

Markscheme	Candidates' Scripts	Marking
(a) $\frac{\sin A}{3} = \frac{\sin 30}{4}$ (MI)(AI) $A = 22.0^\circ$ (22.0243...) (AI)	(a) $\frac{\sin A}{4} = \frac{\sin 30}{3}$ $A = 41.8^\circ$ <i>(Note: the 2nd (AI) here was not marked (ft) and cannot be awarded because there was an earlier error in the same question part.)</i>	(MI)(A0) <i>(use of sine rule but with wrong values)</i> (A0)
(b) $x = 7 \tan(22.0243\dots)$ (MI) $= 2.83$ (2.83163...) (AI)(ft)	(b) case (i) $x = 7 \tan 41.8^\circ$ $= 6.26$ but case (ii) 6.26	(MI) (AI)(ft) (C0) <i>since no working shown</i>

4 Using the Markscheme

(a) **A** marks are **dependent** on the preceding **M** mark being awarded, it is **not** possible to award **(M0)(A1)**. Once an **(M0)** has been awarded, all subsequent **A** marks are lost in that part of the question, even if calculations are performed correctly, until the next **M** mark.
The only exception will be for an answer where the accuracy is specified in the question – see section 5.

(b) **A** marks are **dependent** on the **R** mark being awarded, it is **not** possible to award **(A1)(R0)**. Hence the **(A1)** is not awarded for a correct answer if no reason or the wrong reason is given.

(c) **Alternative methods** may not always be included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method consistent with the markscheme.
Where alternative methods for complete questions are included in the markscheme, they are indicated by ‘**OR**’ *etc.*

(d) Unless the question specifies otherwise, accept **equivalent forms**. For example: $\frac{\sin \theta}{\cos \theta}$ for $\tan \theta$.
On the markscheme, these equivalent numerical or algebraic forms will sometimes be written in brackets after the required answer.

Where numerical answers are required as the final answer to a part of a question in the markscheme, the scheme will show, in order:

the 3 significant figure answer worked through from full calculator display;

the exact value (for example $\sqrt{3}$ if applicable);

the full calculator display in the form 2.83163... as in the example above.

Where answers are given to 3 significant figures and are then used in subsequent parts of the question leading to a **different** 3 significant figure answer, these solutions will also be given.

(e) As this is an international examination, all valid **alternative forms of notation** should be accepted. Some examples of these are:

Decimal points: 1.7; 1’7; 1·7; 1,7 .

Different descriptions of an interval: $3 < x < 5$; (3, 5);] 3, 5 [.

Different forms of notation for set properties (*e.g.* complement): A' ; \bar{A} ; A^c ; $U - A$; $(A; U \setminus A$.

Different forms of logic notation: $\neg p$; p' ; \tilde{p} ; \bar{p} ; $\sim p$.

$p \Rightarrow q$; $p \rightarrow q$; $q \Leftarrow p$.

(f) **Discretionary marks**: There will be very rare occasions where the markscheme does not cover the work seen. In such cases the annotation DM should be used to indicate where an examiner has used discretion. Discretion should be used sparingly and if there is doubt an exception should be raised through scoris to the team leader.

As with previous sessions there will be no whole paper penalty marks for accuracy AP, financial accuracy FP and units UP. Instead these skills will be assessed in particular questions and the marks applied according to the rules given in sections 5, 6 and 7 below.

5 Accuracy of Answers

Incorrect accuracy should be penalized once only in each question according to the rules below.

Unless otherwise stated in the question, all numerical answers should be given exactly or correct to 3 significant figures.

1. If the candidate’s unrounded answer is seen and would round to the required 3 sf answer, then award *(AI)* and ignore subsequent rounding.

Note: The unrounded answer may appear in either the working box or on the final answer line.

2. If the candidate’s unrounded answer is **not** seen then award *(AI)* if the answer given is **correctly** rounded to 2 or more significant figures, otherwise *(A0)*.

Note: If the candidate’s unrounded answer is **not** seen and the answer is given correct to 1 sf (correct or not), the answer will be considered wrong and will not count as incorrect accuracy. If this answer is used in subsequent parts, then working must be shown for further marks to be awarded.

3. If a correct 2 sf answer is used in subsequent parts, then working **must** be shown for further marks to be awarded. (This treatment is the same as for following through from an incorrect answer.)

These 3 points (see numbers in superscript) have been summarized in the table below and illustrated in the examples which follow.

	If candidates final answer is given ...				
	Exact or correct to 3 or more sf	Incorrect to 3 sf	Correct to 2 sf ³	Incorrect to 2 sf	Correct or incorrect to 1 sf
Unrounded answer seen ¹	Award the final <i>(AI)</i> irrespective of correct or incorrect rounding				
Unrounded answer not seen ²	<i>(AI)</i>	<i>(A0)</i>	<i>(AI)</i>	<i>(A0)</i>	<i>(A0)</i>
Treatment of subsequent parts	As per MS	Treat as follow through, only if working is seen. ³			

Examples:

Markscheme	Candidates' Scripts	Marking
9.43 (9.43398...) (AI)	(i) 9.43398... is seen in the working box followed by 9; 9.4; 9.43; 9.434 <i>etc.</i> (correctly rounded) (ii) 9.43398... is seen in the working box followed by 9.433; 9.44 <i>etc.</i> (incorrectly rounded) (iii) 9.4 (iv) 9 (v) 9.3 (vi) 9.44	(AI) (AI) (AI) (A0) (A0) (A0)

Markscheme	Candidates' Scripts	Marking
7.44 (7.43798...) (AI)	(i) 7.43798... is seen in the working box followed by 7; 7.4; 7.44; 7.438 <i>etc.</i> (correctly rounded) (ii) 7.43798... is seen in the working box followed by 7.437; 7.43 <i>etc.</i> (incorrectly rounded) (iii) 7.4 (iv) 7 (v) 7.5 (vi) 7.43	(AI) (AI) (AI) (A0) (A0) (A0)

Example: ABC is a right angled triangle with angle $ABC = 90^\circ$, $AC = 32$ cm and $AB = 30$ cm. Find (a) the length of BC, (b) The area of triangle ABC.

Markscheme	Candidates' Scripts	Marking
(a) $BC = \sqrt{32^2 - 30^2}$ (MI) Award (MI) for correct substitution in Pythagoras' formula $= 11.1 (\sqrt{124}, 11.1355\dots)$ (AI)	(a) $BC = \sqrt{32^2 - 30^2}$ 11 (cm) (2 sf answer only seen, but correct)	(MI) (AI)
(b) $\text{Area} = \frac{1}{2} \times 30 \times 11.1355\dots$ (MI) Award (MI) for correct substitution in area of triangle formula $= 167(167.032\dots)$ (AI)(ft)	(b) case (i) $\text{Area} = \frac{1}{2} \times 30 \times 11$ $= 165 \text{ (cm}^2\text{)}$ case (ii) $= 165 \text{ (cm}^2\text{)}$ (No working shown, the answer 11 is treated as a ft, so no marks awarded here)	(MI) (working shown) (AI)(ft) (M0)(A0)(ft)

Rounding of an exact answer to 3 significant figures **should be accepted if performed correctly**.

Exact answers such as $\frac{1}{4}$ can be written as decimals to fewer than 3 significant figures if the result is still exact. Reduction of a fraction to its lowest terms is **not** essential, however where an answer simplifies to an integer this is expected.

Ratios of π and answers taking the form of square roots of integers or any rational power of an integer (e.g. $\sqrt{13}, 2^{\frac{2}{3}}, \sqrt[4]{5}$,) may be accepted as exact answers. All other powers (eg, of non-integers) and values of transcendental functions such as sine and cosine must be evaluated.

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. In **all** such cases the final mark is not awarded if the rounding does not follow the instructions given in the question. A mark for specified accuracy can be regarded as a **(ft)** mark regardless of an immediately preceding **(M0)**.

Certain answers obtained from the GDC are worth 2 marks and working will not be seen. In these cases only one mark should be lost for accuracy.

eg, Chi-squared, correlation coefficient, mean

Markscheme	Candidates' Scripts	Marking
Chi-squared	(a) 7.7	(A2)
7.68 (7.67543...)	(b) 7.67	(A1)
(A2)	(c) 7.6	(A1)
	(d) 8	(A0)
	(e) 7	(A0)
	(e) 7.66	(A0)

Regression line

Markscheme	Candidates' Scripts	Marking
$y = 0.888x + 13.5$	(a) $y = 0.89x + 13$	(A2)
($y = 0.887686...x + 13.4895...$)		(both accepted)
If an answer is not in the form of an equation award at most (A1)(A0).	(b) $y = 0.88x + 13$	(A1)
		(one rounding error)
	(c) $y = 0.88x + 14$	(A1)
		(rounding error repeated)
	(d) (i) $y = 0.9x + 13$	
	(ii) $y = 0.8x + 13$	(A1)
		(1 sf not accepted)
	(e) $0.88x + 14$	(A0)
		(two rounding errors and not an equation)

Maximum/minimum/points of intersection

Markscheme	Candidates' Scripts	Marking
(2.06, 4.49) (AI)(AI) (2.06020..., 4.49253...)	(a) (2.1, 4.5)	(AI)(AI) (both accepted)
	(b) (2.0, 4.4)	(AI) (same rounding error twice)
	(c) (2.06, 4.4)	(AI) (one rounding error)
	(d) (2, 4.4)	(A0) (1sf not accepted, one rounding error)

6 Level of accuracy in finance questions

The accuracy level required for answers will be specified in all questions involving money. This will usually be either whole units or two decimal places. The first answer not given to the specified level of accuracy will not be awarded the final **A** mark. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for incorrect accuracy in a financial question.

Example: A financial question demands accuracy correct to 2 dp.

Markscheme	Candidates' Scripts	Marking
\$231.62 (231.6189) (AI)	(i) 231.6	(A0)
	(ii) 232	(A0) <i>(Correct rounding to incorrect level)</i>
	(iii) 231.61	(A0)
	(iv) 232.00	(A0) <i>(Parts (iii) and (iv) are both incorrect rounding to correct level)</i>

7 Units in answers

There will be specific questions for which the units are required and this will be indicated clearly in the markscheme. The first correct answer with no units or incorrect units will not be awarded the final **A** mark. The markscheme will give clear instructions to ensure that only one or two marks per paper can be lost for lack of units or incorrect units.

The units are considered only when the numerical answer is awarded (AI) under the accuracy rules given in Section 5.

Markscheme	Candidates' Scripts	Marking
(a) 37000 m ² (AI)	(a) 36000 m ²	(A0) <i>(Incorrect answer so units not considered)</i>
(b) 3200 m ³ (AI)	(b) 3200 m ²	(A0) <i>(Incorrect units)</i>

If no method is shown and the answer is correct but with incorrect or missing units award the C marks with a one mark penalty.

8 Graphic Display Calculators

Candidates will often obtain solutions directly from their calculators. They must use mathematical notation, not calculator notation. No method marks can be awarded for incorrect answers supported only by calculator notation. The comment "I used my GDC" cannot receive a method mark.

1. (a) 1.5×10^8 (km) (A2) (C2)

Notes: Award (A2) for the correct answer.
Award (AI)(A0) for 1.5 and an incorrect index.
Award (A0)(A0) for answers of the form 15×10^7 .

- (b) $2\pi 1.5 \times 10^8$ (MI)
= 942000000(km) (942477796.1..., $3 \times 10^8 \pi$, 9.42×10^8) (AI)(ft) (C2)

Notes: Award (MI) for correct substitution into correct formula. Follow through from part (a).
Do not accept calculator notation 9.42E8
Do not accept use of $\frac{22}{7}$ or 3.14 for π .

- (c) 17×942000000 (MI)
= 1.60×10^{10} (km) ($1.60221... \times 10^{10}$, 1.6014×10^{10} , 16022122530, $(5.1 \times 10^9) \pi$) (AI)(ft) (C2)

Note: Follow through from part (b).

[6 marks]

2. (a) If Eva is losing weight then Eva is on a diet (AI)(AI) (C2)

Notes: Award (AI) for If... then...
For Spanish candidates, **only** accept “Si” and “entonces”.
For French candidates, **only** accept “Si” and “alors”.
For all 3 languages these words are from the subject guide.
Award (AI) for correct propositions in correct order.

- (b) If Eva is not on a diet then she is not losing weight (AI)(AI) (C2)

Notes: Award (AI) for “not on a diet” and “not losing weight” seen,
(AI) for complete correct answer.
No follow through from part (a).

- (c) The statements are logically equivalent (AI)(ft)
The contrapositive is always logically equivalent to the original statement (RI)(ft)

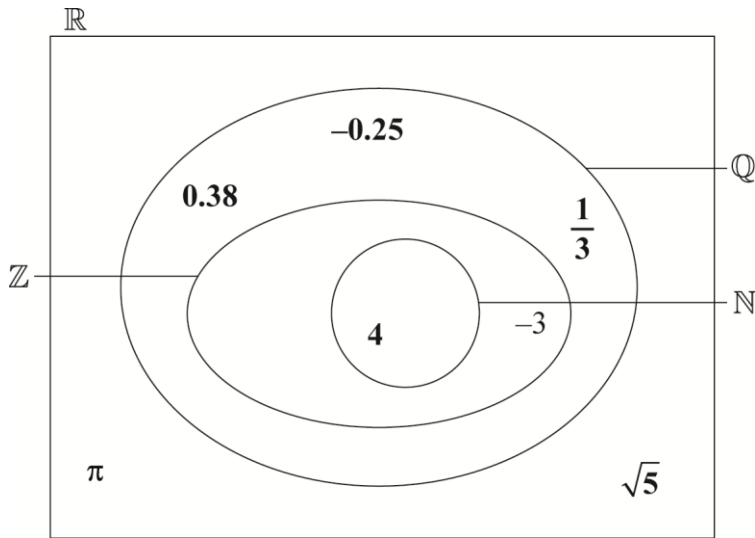
OR

- A correct truth table showing the equivalence (RI)(ft) (C2)

Note: Follow through from their answers to part (a) and part (b).

[6 marks]

3.



(AI)(AI)(AI)(AI)(AI) (C6)

Note: Award (AI) for each number correctly placed.
Award (A0) for any entry in more than one region.

[6 marks]

4. *The first time a correct answer has incorrect or missing units, the final (A1) is not awarded.*

(a) $\frac{4}{3}\pi(1)^3$ (MI)

Note: Award (MI) for correct substitution into correct formula.

$= 4.19 \left(4.18879\dots, \frac{4}{3}\pi \right) \text{ cm}^3$ (A1) (C2)

(b) $83.8 \left(83.7758\dots, \frac{80}{3}\pi \right) \text{ cm}^3$ (A1)(ft) (C1)

Note: Follow through from their answer to part (a).

(c) $10 \times 8 \times 2$ (MI)

Note: Award (MI) for correct substitution into correct formula.

$= 160 \text{ cm}^3$ (A1) (C2)

(d) $76.2 \left(76.2241\dots, \left(160 - \frac{80}{3}\pi \right) \right) \text{ cm}^3$ (A1)(ft) (C1)

Note: Follow through from their part (b) and their part (c).

[6 marks]

- 5. (a) 43 (mm) (AI) (CI)
- (b) 10 (mm) (AI) (CI)
- (c) $48 - 20$ (AI)
 $= 28$ (AI) (C2)

Note: Award (AI) for identifying correct quartiles, (AI) for correct subtraction of the quartiles.

- (d) (i) 20 (days) (AI)
 - (ii) 60 (days) (AI) (C2)
- [6 marks]**

- 6. (a) $2x(x - 4)$ or $2x^2 - 8x$ (AI) (CI)

Note: Award (A0) for $x - 4 \times 2x$.

- (b) $2x(x - 4) = 10$ (MI)

Note: Award (MI) for equating their answer in part (a) to 10.

$x^2 - 4x - 5 = 0$ (MI)

OR

Sketch of $y = 2x^2 - 8x$ and $y = 10$ (MI)

OR

Using GDC solver $x = 5$ and $x = -1$ (MI)

OR

$2(x + 1)(x - 5)$ (MI)

$x = 5$ (m) (AI)(ft) (C3)

Notes: Follow through from their answer to part (a).
 Award at most (MI)(MI)(A0) if both 5 and -1 are given as final answer.
 Final (AI)(ft) is awarded for choosing only the positive solution(s).

- (c) $2 \times 5 = 10$ (m) (AI)(ft)
 $5 - 4 = 1$ (m) (AI)(ft) (C2)

Note: Follow through from their answer to part (b).
 Do not accept negative answers.

[6 marks]

7. (a) $-\frac{80}{940} \left(-0.0851, -0.085106\dots, -\frac{4}{47} \right)$ (AI) (CI)

(b) $-0.0851 (-0.085106\dots) < -\frac{1}{12} (-0.083333\dots)$ (AI)(ft) (CI)

Notes: Accept “less than” in place of inequality.
Award (A0) if incorrect inequality seen.
Follow through from part (a).

(c) (i) ramp B is safe (AI)

the gradient of ramp B is $-\frac{1}{12}$ (RI)

Notes: Award (RI) for “the gradient of ramp B is $-\frac{1}{12}$ ” seen.
Do not award (AI)(R0).

(ii) $2x = 1920$ (MI)

Note: Accept alternative methods.

960 (cm) (AI) (C4)
[6 marks]

8. (a) $\frac{30}{100} \times \frac{48}{100} \times 100$ **OR** $\frac{30 \times 48}{100}$ (M1)

Note: Award (M1) for correct substitution into correct formula.

$= 14.4 \left(\frac{72}{5} \right)$ (A1) (C2)

(b) 13.0 (12.9554...) (A2) (C2)

Note: Award (A1)(A0) for 12.9.

(c) the null hypothesis is not accepted (A1)(ft)
 $\chi^2_{calc} > \chi^2_{crit}$ **OR** $13.0 > 7.82$ (R1)

OR
the null hypothesis is not accepted (A1)(ft)
 $p\text{-value } (0.0047) (0.00473391...) < 0.05$ (R1) (C2)

Notes: Follow through from their answer to part (b).
Do not award (A1)(ft)(R0).

[6 marks]

9. (a) $\frac{100000}{129}$ (MI)
= 775 (GBP) (AI) (C2)
- (b) (i) $\frac{30200}{239}$ (MI)
1GBP=126 JPY (AI)

Note: Accept 126 (JPY).
Award (MI) for $\frac{239}{30200}$.
Award (A0) for 1JPY = 0GBP

- (ii) No, the part (b)(i) rate is not better value than the part (a) rate. (AI)(ft)
30200 < 30831 (RI)
- OR**
- No, the part (b)(i) rate is not better value than the part (a) rate. (AI)(ft)
129 > 126 (RI) (C4)

Note: Accept “part (a) rate is better” for the (AI)(ft).
Follow through from part (b)(i).
A numerical comparison must be seen to award (RI).

[6 marks]

10. (a) $\frac{350}{\tan 20^\circ}$ (MI)
= 961.617... (AI)
= 962 (m) (AI)(ft) (C3)

Notes: Award (MI) for correct substitution into correct formula, (AI) for correct answer, (AI)(ft) for correct rounding to the nearest metre.
Award (M0)(A0)(A0) for 961 without working.

(b) $961.617... - 250 = 711.617...$ (AI)(ft)
 $\tan^{-1}\left(\frac{350}{711.617...}\right)$ (MI)
= 26.2° (26.1896...) (AI)(ft) (C3)

Notes: Accept 26.1774... from use of 3 sf answer 962 from part (a). Follow through from their answer to part (a).
Accept alternative methods.

[6 marks]

11. (a) $1.2 \text{ (mg l}^{-1}\text{)}$

(A1) (C1)

(b) $1.2 \times (0.87)^3$

(M1)

Note: Award (M1) for correct substitution into given formula.

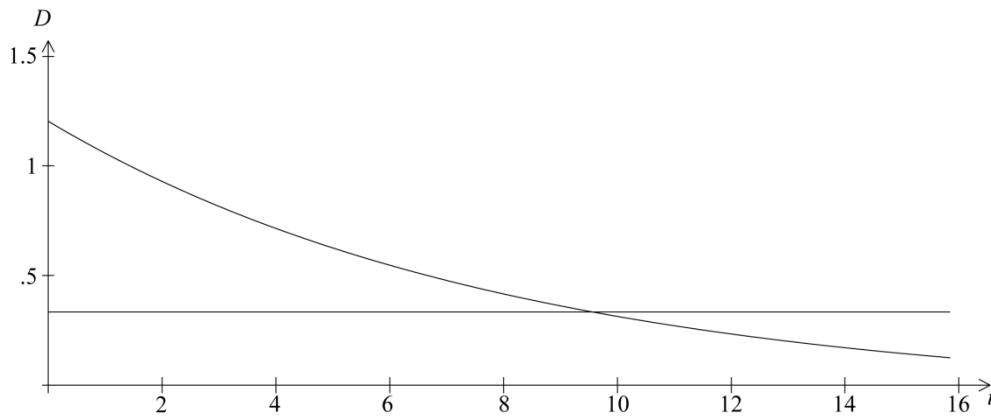
$= 0.790 \text{ (mg l}^{-1}\text{)}$ (0.790203...)

(A1) (C2)

(c) $1.2 \times 0.87^t = 0.333$

(M1)

Note: Award (M1) for setting up the equation.



(M1)

Notes: Some indication of scale is to be shown, for example the window used on the calculator.
Accept alternative methods.

9.21(hours) (9.20519..., 9 hours 12 minutes, 9:12)

(A1) (C3)

[6 marks]

12. (a) $t = -20.1n + 205$
 $t = (-20.1046\dots)n + (204.755\dots)$ (AI)(AI) (C2)

Notes: Award (AI) for -20.1 and 205 seen,
(AI) for an equation involving t and n .

- (b) -0.941 ($-0.941366\dots$) (A2) (C2)

Notes: Award (A0)(AI) for $+0.941$.

- (c) $-20.1046\dots \times 4 + 204.755\dots$ (M1)

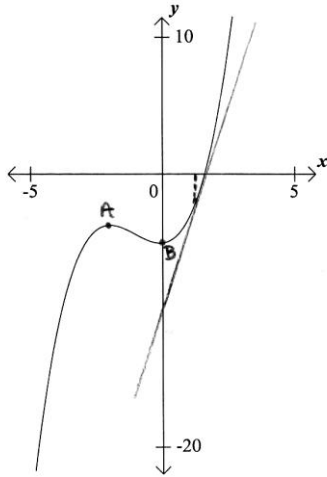
Note: Award (M1) for substitution into their regression equation.

- 124 (minutes) ($124.337\dots$) (AI)(ft) (C2)

Notes: Follow through from their regression equation found in part (a).
Accept 125 (minutes) (124.6).

[6 marks]

13.



- (a) correct label on graph (A1) (C1)
- (b) correct label on graph (A1) (C1)
- (c) $-1.33 < x < 0$ $\left(-\frac{4}{3} < x < 0 \right)$ (A1) (C1)
- (d) tangent drawn at $x = 1$ on graph (A1) (C1)
- (e) $y = 7x - 9$ (A1)(A1) (C2)

Notes: Award (A1) for 7, (A1) for -9.
If answer not given as an equation award at most (A1)(A0).

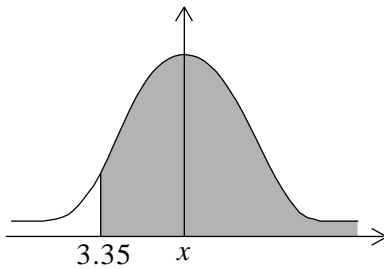
[6 marks]

14. (a) $0.5 \left(50\%, \frac{50}{100}, \frac{1}{2} \right)$ (AI) (C1)

(b) 0.954 (0.954499..., 95.4 %, 95.4499...%) (AI) (C1)

Note: Accept 95% or 0.95.

(c)

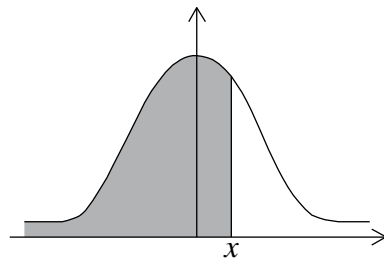


(M1)

Note: Accept alternative methods.

0.631 (0.630558..., 63.1%, 63.0558...%) (AI) (C2)

(d)



(M1)

Note: Accept alternative methods.

3.50 (3.50091...) (AI) (C2)

[6 marks]

15. (a) $6x^2 - 5 - \frac{4}{x^2}$ (AI)(AI)(AI)(AI) (C4)

Note: Award (AI) for $6x^2$, (AI) for -5 , (AI) for -4 , (AI) for x^{-2} or $\frac{1}{x^2}$.
Award at most (AI)(AI)(AI)(A0) if additional terms are seen.

(b) (1.15, 3.77) ((1.15469..., 3.76980...)) (AI)(AI) (C2)

Notes: Award (AI)(AI) for “ $x = 1.15$ and $y = 3.77$ ”.
Award at most (A0)(AI)(ft) if parentheses are omitted.

[6 marks]