



DIPLOMA PROGRAMME

MATHEMATICS SL

INFORMATION BOOKLET

For use by teachers and students, during the course and in the examinations

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Formulae

Presumed knowledge

Area of a parallelogram	$A = (b \times h)$, where b is the base, h is the height
Area of a triangle	$A = \frac{1}{2}(b \times h)$, where b is the base, h is the height
Area of a trapezium	$A = \frac{1}{2}(a + b)h$, where a and b are the parallel sides, h is the height
Area of a circle	$A = \pi r^2$, where r is the radius
Circumference of a circle	$C = 2\pi r$, where r is the radius
Volume of a pyramid	$V = \frac{1}{3}(\text{area of base} \times \text{vertical height})$
Volume of a cuboid	$V = l \times w \times h$, where l is the length, w is the width, h is the height
Volume of a cylinder	$V = \pi r^2 h$, where r is the radius, h is the height
Area of the curved surface of a cylinder	$A = 2\pi r h$, where r is the radius, h is the height
Volume of a sphere	$V = \frac{4}{3}\pi r^3$, where r is the radius
Volume of a cone	$V = \frac{1}{3}\pi r^2 h$, where r is the radius, h is the height
Distance between two points (x_1, y_1) and (x_2, y_2)	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
Coordinates of the midpoint of a line segment with endpoints (x_1, y_1) and (x_2, y_2)	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

Topic 1 – Algebra

1.1	<p>The n^{th} term of an arithmetic sequence</p> <p>The sum of n terms of an arithmetic sequence</p> <p>The n^{th} term of a geometric sequence</p> <p>The sum of n terms of a finite geometric sequence</p> <p>The sum of an infinite geometric sequence</p>	$u_n = u_1 + (n-1)d$ $S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$ $u_n = u_1 r^{n-1}$ $S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{u_1(1 - r^n)}{1 - r}, \quad r \neq 1$ $S = \frac{u_1}{1 - r}, \quad r < 1$
1.2	Exponents and logarithms	$a^x = b \Leftrightarrow x = \log_a b$ $a^x = e^{x \ln a}$ $\log_a a^x = x = a^{\log_a x}$ $\log_b a = \frac{\log_c a}{\log_c b}$
1.3	Binomial theorem	$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n$

Topic 2 – Functions and equations

2.5	Axis of symmetry of graph of a quadratic function	$f(x) = ax^2 + bx + c \Rightarrow \text{axis of symmetry } x = -\frac{b}{2a}$
2.6	<p>Solution of a quadratic equation</p> <p>Discriminant</p>	$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \quad a \neq 0$ $\Delta = b^2 - 4ac$

Topic 3—Circular functions and trigonometry

3.1	Length of an arc	$l = \theta r$, where θ is the angle measured in radians, r is the radius
	Area of a sector	$A = \frac{1}{2}\theta r^2$, where θ is the angle measured in radians, r is the radius
3.2	Identities	$\tan \theta = \frac{\sin \theta}{\cos \theta}$
		$\cos^2 \theta + \sin^2 \theta = 1$
3.3	Double angle formulae	$\sin 2\theta = 2 \sin \theta \cos \theta$
		$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$
3.6	Cosine rule	$c^2 = a^2 + b^2 - 2ab \cos C$; $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$
	Sine rule	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
	Area of a triangle	$A = \frac{1}{2}ab \sin C$, where a and b are adjacent sides, C is the included angle

Topic 4—Matrices

4.3	Determinant of a 2×2 matrix	$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow \det A = ad - bc$
	Inverse of a 2×2 matrix	$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow A^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$, $ad \neq bc$
	Determinant of a 3×3 matrix	$A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & k \end{pmatrix} \Rightarrow \det A = a \begin{vmatrix} e & f \\ h & k \end{vmatrix} - b \begin{vmatrix} d & f \\ g & k \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$

Topic 5—Vectors

5.1	<p>Magnitude of a vector</p> <p>Distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2)</p> <p>Coordinates of the midpoint of a line segment with endpoints (x_1, y_1, z_1), (x_2, y_2, z_2)</p>	$ \mathbf{v} = \sqrt{v_1^2 + v_2^2 + v_3^2}, \text{ where } \mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$ $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$
5.2	<p>Scalar product</p> <p>Angle between two vectors</p>	$\mathbf{v} \cdot \mathbf{w} = \mathbf{v} \mathbf{w} \cos \theta, \text{ where } \theta \text{ is the angle between } \mathbf{v} \text{ and } \mathbf{w}$ $\mathbf{v} \cdot \mathbf{w} = v_1 w_1 + v_2 w_2 + v_3 w_3, \text{ where } \mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}, \mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$ $\cos \theta = \frac{v_1 w_1 + v_2 w_2 + v_3 w_3}{ \mathbf{v} \mathbf{w} }$
5.3	<p>Vector representation (equation) of a line</p>	$\mathbf{r} = \mathbf{a} + t\mathbf{b}$

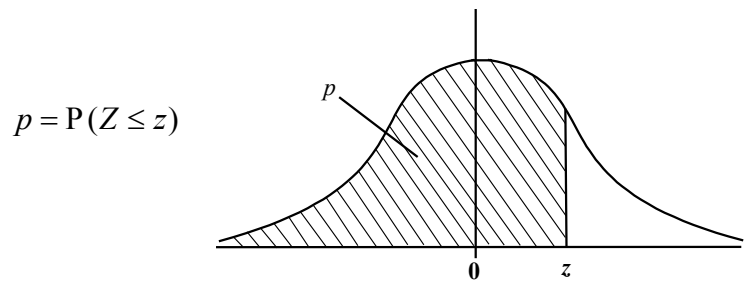
Topic 6—Statistics and probability

<p>6.3</p> <p>Population parameters</p> <p>Mean μ</p> <p>Variance σ^2</p> <p>Standard deviation σ</p> <p>Sample statistics</p> <p>Mean \bar{x}</p> <p>Variance s_n^2</p> <p>Standard deviation s_n</p>		<p>Let $n = \sum_{i=1}^k f_i$.</p> $\mu = \frac{\sum_{i=1}^k f_i x_i}{n}$ $\sigma^2 = \frac{\sum_{i=1}^k f_i (x_i - \mu)^2}{n}$ $\sigma = \sqrt{\frac{\sum_{i=1}^k f_i (x_i - \mu)^2}{n}}$ $\bar{x} = \frac{\sum_{i=1}^k f_i x_i}{n}$ $s_n^2 = \frac{\sum_{i=1}^k f_i (x_i - \bar{x})^2}{n}$ $s_n = \sqrt{\frac{\sum_{i=1}^k f_i (x_i - \bar{x})^2}{n}}$
<p>6.5</p> <p>Probability of an event A</p> <p>Complementary events</p>		$P(A) = \frac{n(A)}{n(U)}$ $P(A) + P(A') = 1$
<p>6.6</p> <p>Combined events</p> <p>Mutually exclusive events</p>		$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $P(A \cup B) = P(A) + P(B)$
<p>6.7</p> <p>Conditional probability</p> <p>Independent events</p>		$P(A B) = \frac{P(A \cap B)}{P(B)}$ $P(A \cap B) = P(A)P(B)$
<p>6.9</p> <p>Expected value of a discrete random variable X</p>		$E(X) = \mu = \sum_x x P(X = x)$
<p>6.10</p> <p>Binomial distribution</p> <p>Mean</p>		$X \sim B(n, p) \Rightarrow P(X = r) = \binom{n}{r} p^r (1-p)^{n-r}, r = 0, 1, \dots, n$ $E(X) = np$
<p>6.11</p> <p>Standardized normal variable</p>		$z = \frac{x - \mu}{\sigma}$

Topic 7—Calculus

7.1	Derivative of $f(x)$	$y = f(x) \Rightarrow \frac{dy}{dx} = f'(x) = \lim_{h \rightarrow 0} \left(\frac{f(x+h) - f(x)}{h} \right)$
	Derivative of x^n	$f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$
	Derivative of $\sin x$	$f(x) = \sin x \Rightarrow f'(x) = \cos x$
	Derivative of $\cos x$	$f(x) = \cos x \Rightarrow f'(x) = -\sin x$
	Derivative of $\tan x$	$f(x) = \tan x \Rightarrow f'(x) = \frac{1}{\cos^2 x}$
	Derivative of e^x	$f(x) = e^x \Rightarrow f'(x) = e^x$
	Derivative of $\ln x$	$f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$
7.2	Chain rule	$y = g(u)$, where $u = f(x) \Rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
	Product rule	$y = uv \Rightarrow \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$
	Quotient rule	$y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
7.4	Standard integrals	$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$ $\int \frac{1}{x} dx = \ln x + C, \quad x > 0$ $\int \sin x dx = -\cos x + C$ $\int \cos x dx = \sin x + C$ $\int e^x dx = e^x + C$
7.5	Area under a curve	$A = \int_a^b y dx$
	Volume of revolution (rotation)	$V = \int_a^b \pi y^2 dx$

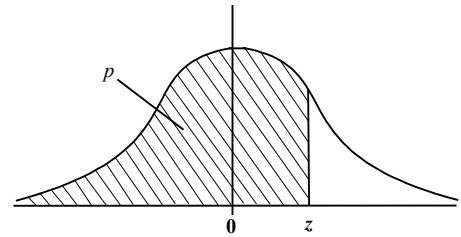
Area under the standard normal curve (topic 6.1 I)



<i>z</i>	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8079	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9773	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9892	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9983	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9988	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998

Inverse normal probabilities (topic 6.1 I)

$$p = P(Z \leq z)$$



p	0	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.50	0.0000	0.0025	0.0050	0.0075	0.0100	0.0125	0.0150	0.0176	0.0201	0.0226
0.51	0.0251	0.0276	0.0301	0.0326	0.0351	0.0376	0.0401	0.0426	0.0451	0.0476
0.52	0.0502	0.0527	0.0552	0.0577	0.0602	0.0627	0.0652	0.0677	0.0702	0.0728
0.53	0.0753	0.0778	0.0803	0.0828	0.0853	0.0878	0.0904	0.0929	0.0954	0.0979
0.54	0.1004	0.1030	0.1055	0.1080	0.1105	0.1130	0.1156	0.1181	0.1206	0.1231
0.55	0.1257	0.1282	0.1307	0.1332	0.1358	0.1383	0.1408	0.1434	0.1459	0.1484
0.56	0.1510	0.1535	0.1560	0.1586	0.1611	0.1637	0.1662	0.1687	0.1713	0.1738
0.57	0.1764	0.1789	0.1815	0.1840	0.1866	0.1891	0.1917	0.1942	0.1968	0.1993
0.58	0.2019	0.2045	0.2070	0.2096	0.2121	0.2147	0.2173	0.2198	0.2224	0.2250
0.59	0.2275	0.2301	0.2327	0.2353	0.2379	0.2404	0.2430	0.2456	0.2482	0.2508
0.60	0.2534	0.2559	0.2585	0.2611	0.2637	0.2663	0.2689	0.2715	0.2741	0.2767
0.61	0.2793	0.2819	0.2845	0.2872	0.2898	0.2924	0.2950	0.2976	0.3002	0.3029
0.62	0.3055	0.3081	0.3107	0.3134	0.3160	0.3186	0.3213	0.3239	0.3266	0.3292
0.63	0.3319	0.3345	0.3372	0.3398	0.3425	0.3451	0.3478	0.3505	0.3531	0.3558
0.64	0.3585	0.3611	0.3638	0.3665	0.3692	0.3719	0.3745	0.3772	0.3799	0.3826
0.65	0.3853	0.3880	0.3907	0.3934	0.3961	0.3989	0.4016	0.4043	0.4070	0.4097
0.66	0.4125	0.4152	0.4179	0.4207	0.4234	0.4262	0.4289	0.4316	0.4344	0.4372
0.67	0.4399	0.4427	0.4454	0.4482	0.4510	0.4538	0.4565	0.4593	0.4621	0.4649
0.68	0.4677	0.4705	0.4733	0.4761	0.4789	0.4817	0.4845	0.4874	0.4902	0.4930
0.69	0.4959	0.4987	0.5015	0.5044	0.5072	0.5101	0.5129	0.5158	0.5187	0.5215
0.70	0.5244	0.5273	0.5302	0.5331	0.5359	0.5388	0.5417	0.5446	0.5476	0.5505
0.71	0.5534	0.5563	0.5592	0.5622	0.5651	0.5681	0.5710	0.5740	0.5769	0.5799
0.72	0.5828	0.5858	0.5888	0.5918	0.5948	0.5978	0.6008	0.6038	0.6068	0.6098
0.73	0.6128	0.6158	0.6189	0.6219	0.6250	0.6280	0.6311	0.6341	0.6372	0.6403
0.74	0.6434	0.6464	0.6495	0.6526	0.6557	0.6588	0.6620	0.6651	0.6682	0.6714
0.75	0.6745	0.6776	0.6808	0.6840	0.6871	0.6903	0.6935	0.6967	0.6999	0.7031

Inverse normal probabilities (topic 6.11, continued)

p	0	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.76	0.7063	0.7095	0.7128	0.7160	0.7192	0.7225	0.7257	0.7290	0.7323	0.7356
0.77	0.7389	0.7421	0.7455	0.7488	0.7521	0.7554	0.7588	0.7621	0.7655	0.7688
0.78	0.7722	0.7756	0.7790	0.7824	0.7858	0.7892	0.7926	0.7961	0.7995	0.8030
0.79	0.8064	0.8099	0.8134	0.8169	0.8204	0.8239	0.8274	0.8310	0.8345	0.8381
0.80	0.8416	0.8452	0.8488	0.8524	0.8560	0.8596	0.8633	0.8669	0.8706	0.8742
0.81	0.8779	0.8816	0.8853	0.8890	0.8927	0.8965	0.9002	0.9040	0.9078	0.9116
0.82	0.9154	0.9192	0.9230	0.9269	0.9307	0.9346	0.9385	0.9424	0.9463	0.9502
0.83	0.9542	0.9581	0.9621	0.9661	0.9701	0.9741	0.9782	0.9822	0.9863	0.9904
0.84	0.9945	0.9986	1.0027	1.0069	1.0110	1.0152	1.0194	1.0237	1.0279	1.0322
0.85	1.0364	1.0407	1.0451	1.0494	1.0537	1.0581	1.0625	1.0669	1.0714	1.0758
0.86	1.0803	1.0848	1.0894	1.0939	1.0985	1.1031	1.1077	1.1123	1.1170	1.1217
0.87	1.1264	1.1311	1.1359	1.1407	1.1455	1.1504	1.1552	1.1601	1.1651	1.1700
0.88	1.1750	1.1800	1.1850	1.1901	1.1952	1.2004	1.2055	1.2107	1.2160	1.2212
0.89	1.2265	1.2319	1.2372	1.2426	1.2481	1.2536	1.2591	1.2646	1.2702	1.2759
0.90	1.2816	1.2873	1.2930	1.2988	1.3047	1.3106	1.3165	1.3225	1.3285	1.3346
0.91	1.3408	1.3469	1.3532	1.3595	1.3658	1.3722	1.3787	1.3852	1.3917	1.3984
0.92	1.4051	1.4118	1.4187	1.4255	1.4325	1.4395	1.4466	1.4538	1.4611	1.4684
0.93	1.4758	1.4833	1.4909	1.4985	1.5063	1.5141	1.5220	1.5301	1.5382	1.5464
0.94	1.5548	1.5632	1.5718	1.5805	1.5893	1.5982	1.6073	1.6164	1.6258	1.6352
0.95	1.6449	1.6546	1.6646	1.6747	1.6849	1.6954	1.7060	1.7169	1.7279	1.7392
0.96	1.7507	1.7624	1.7744	1.7866	1.7991	1.8119	1.8250	1.8384	1.8522	1.8663
0.97	1.8808	1.8957	1.9110	1.9268	1.9431	1.9600	1.9774	1.9954	2.0141	2.0335
0.98	2.0538	2.0749	2.0969	2.1201	2.1444	2.1701	2.1973	2.2262	2.2571	2.2904
0.99	2.3264	2.3656	2.4089	2.4573	2.5121	2.5758	2.6521	2.7478	2.8782	3.0902