International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MARKSCHEME

November 2013

MATHEMATICAL STUDIES

Standard Level

Paper 2

Paper 2 Markscheme
Instructions to Examiners

Notes: **If in doubt about these instructions or any other marking issues, contact your team leader for clarification.**

1   Abbreviations

M   Marks awarded for **Method**

A   Marks awarded for an **Answer** or for **Accuracy**

R   Marks awarded for clear **Reasoning**

G   Marks awarded for correct solutions obtained from a **Graphic Display Calculator**, when no working shown.

AG   **Answer Given** in the question and consequently, marks not awarded.

ft   Marks that can be awarded as **follow through** from previous results in the question.

2   Method of Marking

(a)   All marking must be done in scoris using the mathematical studies annotations and in accordance with the current document for guidance in e-marking Mathematical Studies SL. It is essential that you read this document before you start marking.

(b)   If a question part is completely correct use the number tick annotations to award full marks. If a part is completely wrong use the *A0* annotation, otherwise full annotations must be shown.

(c)   Working crossed out by the candidate should not be awarded any marks.

(d)   Where candidates have written two solutions to a question, only the first solution should be marked.

(e)   If correct working results in a correct answer but then further working is developed, full marks may **not** always be awarded. Full marks **will** be awarded if the candidate shows correct working leading to the correct answer. See also section 4(c).

Example: Calculate the gradient of the line passing through the points $(5, 3)$ and $(0, 9)$.

Markscheme	Candidates' Scripts	Marking
$\dfrac{9-3}{0-5}$ **(M1)** *Award **(M1)** for correct substitution in gradient formula* $= -\dfrac{6}{5}$ **(A1)**	(i) $\dfrac{9-3}{0-5} = -\dfrac{6}{5}$ Gradient is $= -\dfrac{6}{5}$ *(There is clear understanding of the gradient.)* $y = -\dfrac{6}{5}x + 9$ (ii) $\dfrac{9-3}{0-5} = -\dfrac{6}{5}$ $y = -\dfrac{6}{5}x + 9$ *(There is confusion about what is required.)*	**(M1)** **(A1)** **(M1)** **(A0)**

3   Follow-through (ft) Marks

Errors made at any step of a solution affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks can be awarded. Markschemes will indicate where it is appropriate to apply follow through in a question with **'(ft)'**.

(a)   Follow through applies only from one part of a question to a subsequent part of the question. Follow through does not apply within the same part.

(b)   If an answer resulting from follow through is extremely unrealistic (*e.g.* negative distances or incorrect by large order of magnitude) then the final *A* mark should not be awarded.

(c)   If a question is transformed by an error into a **different**, **much simpler question** then follow through may not apply.

(d)   To award follow through marks for a question part, **there must be working present for that part**. An isolated follow through answer, without working is regarded as incorrect and receives no marks **even if it is approximately correct**.

(e)   The exception to the above would be in a question which is testing the candidate's use of the GDC, where working will not be expected. **The markscheme will clearly indicate where this applies.**

(f)   Inadvertent use of radians will be penalised the first time it occurs. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for the use of radians.

Example: Finding angles and lengths using trigonometry

Markscheme	Candidates' Scripts	Marking
(a) $\dfrac{\sin A}{3} = \dfrac{\sin 30}{4}$ **(M1)(A1)** *Award* **(M1)** *for substitution in sine rule formula,* **(A1)** *for correct substitutions.*	(a) $\dfrac{\sin A}{4} = \dfrac{\sin 30}{3}$	**(M1)(A0)** *(use of sine rule but with wrong values)*
$A = 22.0°\ (22.0243\ldots)$ **(A1)(G2)**	$A = 41.8°$	**(A0)** *(**Note:** the 2nd **(A1)** here was not marked **(ft)** and cannot be awarded because there was an earlier error in the **same** question part.)*
(b) $x = 7\tan(22.0243\ldots°)$ **(M1)** $= 2.83\,(2.83163\ldots)$ **(A1)(ft)**	(b) case (i) $x = 7\tan 41.8°$ $= 6.26$ ***but*** case (ii) 6.26	**(M1)** **(A1)(ft)** **(G0)** *since no working shown*

4   Using the Markscheme

(a)   *A* marks are **dependent** on the preceding *M* mark being awarded, it is **not** possible to award *(M0)(A1)*. Once an *(M0)* has been awarded, all subsequent *A* marks are lost in that part of the question, even if calculations are performed correctly, until the next *M* mark.
The only exception to this will be for an answer where the accuracy is specified in the question – see section 5.

(b)   *A* marks are **dependent** on the *R* mark being awarded, it is **not** possible to award *(A1)(R0)*. Hence the *(A1)* cannot be awarded for an answer which is correct when no reason or the wrong reason is given.

(c)   In paper 2 candidates are expected to demonstrate their ability to communicate mathematics using appropriate working. Answers which are correct but not supported by adequate working will **not always receive full marks**, these unsupported answers are designated *G* in the mark scheme as an alternative to the full marks. Example *(M1)(A1)(A1)(G2)*.

Example: Using trigonometry to calculate an angle in a triangle.

Markscheme	Candidates' Scripts	Marking
(a) $\dfrac{\sin A}{3} = \dfrac{\sin 30}{4}$ *(M1)(A1)* *Award (M1) for substitution in sine rule formula, (A1) for correct substitutions.* $A = 22.0°$ (22.0243…) *(A1)(G2)*	(i) $\dfrac{\sin A}{3} = \dfrac{\sin 30}{4}$ $A = 22.0°$ (ii) $A = 22.0°$ ***Note: G** marks are used only if no working has been shown and the answer is correct.*	*(M1)(A1)* *(A1)* *(G2)*

(d)   **Alternative methods** may not always be included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method consistent with the markscheme.
Where alternative methods for complete questions are included in the markscheme, they are indicated by '**OR**' *etc*.

(e)   Unless the question specifies otherwise, accept **equivalent forms**. For example: $\dfrac{\sin\theta}{\cos\theta}$ for $\tan\theta$.
On the markscheme, these equivalent numerical or algebraic forms will sometimes be written in brackets after the required answer.
Where numerical answers are required as the final answer to a part of a question in the markscheme, the scheme will show, in order:
the 3 significant figure answer worked through from full calculator display;
the exact value (for example $\sqrt{3}$ if applicable);
the full calculator display in the form 2.83163… as in the example above.
Where answers are given to 3 significant figures and are then used in subsequent parts of the question leading to a **different** 3 significant figure answer, these solutions will also be given.

(f)   As this is an international examination, all valid **alternative forms of notation** should be accepted. Some examples of these are:

Decimal points: 1.7; 1'7; 1·7; 1,7 .

Different descriptions of an interval: $3 < x < 5$; (3, 5);] 3, 5 [.

Different forms of notation for set properties (*e.g.* complement): A' ; $\bar{A}$; A^c ; $U - A$; $(A$; $U \setminus A$.

Different forms of logic notation: $\neg p$; p' ; $\tilde{p}$; $\bar{p}$; $\sim p$.

$$p \Rightarrow q \, ; \; p \rightarrow q \, ; \; q \Leftarrow p \, .$$

(g)   Discretionary marks: There will be very rare occasions where the markscheme does not cover the work seen. In such cases the annotation DM should be used to indicate where an examiner has used discretion. Discretion should be used sparingly and if there is doubt and exception should be raised through scoris to the team leader.

There will be no whole paper penalty marks for accuracy AP, financial accuracy FP and units UP. Instead these skills will be assessed in particular questions and the marks applied according to the rules given in sections 5, 6 and 7 below.

5   Accuracy of Answers

Incorrect accuracy should be penalized once only in each question according to the rules below.

Unless otherwise stated in the question, all numerical answers should be given exactly or correct to 3 significant figures.

1.   If the candidate's unrounded answer is seen and would round to the required 3 sf answer, then award *(A1)* and ignore subsequent rounding.

2.   If the candidate's unrounded answer is **not** seen then award *(A1)* if the answer given is **correctly** rounded to 2 or more significant figures, otherwise *(A0)*.

Note: If the candidate's unrounded answer is **not** seen and the answer is given correct to 1 sf (correct or not), the answer will be considered wrong and will not count as incorrect accuracy. If this answer is used in subsequent parts, then working must be shown for further marks to be awarded.

3.   If a correct 2 sf answer is used in subsequent parts, then working **must** be shown for further marks to be awarded. (This treatment is the same as for following through from an incorrect answer.)

These 3 points (see numbers in superscript) have been summarized in the table below and illustrated in the examples following.

	If candidates final answer is given …				
	Exact or correct to 3 or more sf	**Incorrect to 3 sf**	Correct to 2 sf[3]	Incorrect to 2 sf	Correct or incorrect to 1 sf
Unrounded answer seen[1]	Award the final *(A1)* irrespective of correct or incorrect rounding				
Unrounded answer not seen[2]	*(A1)*	*(A0)*	*(A1)*	*(A0)*	*(A0)*
Treatment of subsequent parts	As per MS	Treat as follow through, only if working is seen[3]			

Examples:

Markscheme	Candidates' Scripts		Marking
9.43 (9.43398…) *(A1)*	(i)	9.43398… is seen followed by 9; 9.4; 9.43; 9.434 *etc*. (correctly rounded)	*(A1)*
	(ii)	9.43398… is seen followed by 9.433; 9.44 *etc*. (incorrectly rounded)	*(A1)*
	(iii)	9.4	*(A1)*
	(iv)	9	*(A0)* *(correct to 1 sf)*
	(v)	9.3	*(A0)* *(incorrectly rounded to 2 sf)*
	(vi)	9.44	*(A0)* *(incorrectly rounded to 3 sf)*

Markscheme	Candidates' Scripts		Marking
7.44 (7.43798…) *(A1)*	(i)	7.43798… is seen followed by 7; 7.4; 7.44; 7.438 *etc*. (correctly rounded)	*(A1)*
	(ii)	7.43798… is seen followed by 7.437; 7.43 *etc*. (incorrectly rounded)	*(A1)*
	(iii)	7.4	*(A1)*
	(iv)	7	*(A0)* *(correct to 1 sf)*
	(v)	7.5	*(A0)* *(incorrectly rounded to 2 sf)*
	(vi)	7.43	*(A0)* *(incorrectly rounded to 3 sf)*

Example: ABC is a right angled triangle with angle $ABC = 90°$, $AC = 32$ cm and $AB = 30$ cm.   Find (a) the length of BC, (b) The area of triangle ABC.

Markscheme	Candidates' Scripts	Marking
(a) $BC = \sqrt{32^2 - 30^2}$ **(M1)** *Award **(M1)** for correct substitution in Pythagoras' formula* $= 11.1 \left(\sqrt{124}, 11.1355...\right)$ (cm) **(A1)** (b) $\text{Area} = \dfrac{1}{2} \times 30 \times 11.1355...$ **(M1)** *Award **(M1)** for correct substitution in area of triangle formula* $= 167 (167.032...) (\text{cm}^2)$ **(A1)(ft)**	(a) $BC = \sqrt{32^2 - 30^2}$ **(M1)** 11 (cm) **(A1)** *(2 sf answer only seen, but correct)* (b) case (i) $\text{Area} = \dfrac{1}{2} \times 30 \times 11$ **(M1)** *(working shown)* $= 165 (\text{cm}^2)$ **(A1)(ft)** case (ii) $= 165 (\text{cm}^2)$ **(M0)(A0)(ft)** *(No working shown, the answer 11 is treated as a ft, so no marks awarded here)*	

Certain answers obtained from the GDC are worth 2 marks and working will not be seen. In these cases only one mark should be lost for accuracy.

e.g. Chi-squared, correlation coefficient, mean

Markscheme	Candidates' Scripts	Marking
Chi-squared $7.68 (7.67543…)$ **(A2)**	(a) 7.7	**(G2)**
	(b) 7.67	**(G1)**
	(c) 7.6	**(G1)**
	(d) 8	**(G0)**
	(e) 7	**(G0)**
	(e) 7.66	**(G0)**

Regression line

Markscheme	Candidates' Scripts		Marking
$y = 0.888x + 13.5$ **(A2)** ($y = 0.887686\ldots x + 13.4895\ldots$) If an answer is not in the form of an equation award at most **(A1)(A0)**.	(a)	$y = 0.89x + 13$	**(G2)** *(both accepted)*
	(b)	$y = 0.88x + 13$	**(G1)** *(one rounding error)*
	(c)	$y = 0.88x + 14$	**(G1)** *(rounding error repeated)*
	(d) (i)	$y = 0.9x + 13$	
	(ii)	$y = 0.8x + 13$	**(G1)** *(1 sf not accepted)*
	(e)	$0.88x + 13$	**(G0)** *(one rounding error and not an equation)*

Maximum/minimum/points of intersection

Markscheme	Candidates' Scripts		Marking
$(2.06, 4.49)$ **(A1)(A1)** $(2.06020\ldots, 4.49253\ldots)$	(a)	$(2.1, 4.5)$	**(A1)(A1)** *(both accepted)*
	(b)	$(2.0, 4.4)$	**(A1)** *(same rounding error twice)*
	(c)	$(2.06, 4.4)$	**(A1)** *(one rounding error)*
	(d)	$(2, 4.4)$	**(A0)** *(1sf not accepted, one rounding error)*

Rounding of an exact answer to 3 significant figures **should be accepted if performed correctly**. Exact answers such as $\dfrac{1}{4}$ can be written as decimals to fewer than three significant figures if the result is still exact. Reduction of a fraction to its lowest terms is **not** essential, however where an answer simplifies to an integer this is expected.

Ratios of π and answers taking the form of square roots of integers or any rational power of an integer (*e.g.* $\sqrt{13}, 2^{\frac{2}{3}}, \sqrt[4]{5}$,) may be accepted as exact answers. All other powers (*e.g.* of non-integers) and values of transcendental functions such as sine and cosine must be evaluated.

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. In **all** such cases the final mark is not awarded if the rounding does not follow the instructions given in the question. A mark for specified accuracy can be regarded as a **(ft)** mark regardless of an immediately preceding **(M0)**.

6   Level of accuracy in finance questions

The accuracy level required for answers will be specified in all questions involving money. This will usually be either whole units or two decimal places. The first answer not given to the specified level of accuracy will not be awarded the final *A* mark. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for incorrect accuracy in a financial question.

Example: A financial question demands accuracy correct to 2 dp.

Markscheme	Candidates' Scripts	Marking
$231.62 (231.6189) *(A1)*	(i) 231.6	*(A0)*
	(ii) 232	*(A0)* (Correct rounding to incorrect level)
	(iii) 231.61	*(A0)*
	(iv) 232.00	*(A0)* (Parts (iii) and (iv) are both incorrect rounding to correct level)

7   Units in answers

There will be specific questions for which the units are required and this will be indicated clearly in the markscheme. The first correct answer with no units or incorrect units will not be awarded the final *A* mark. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for lack of units or incorrect units.

The units are considered only when the numerical answer is awarded *(A1)* under the accuracy rules given in Section 5.

Example:

Markscheme	Candidates' Scripts	Marking
(a) $37000 \, \text{m}^2$ *(A1)*	(a) $36000 \, \text{m}^2$	*(A0)* (Incorrect answer so units not considered)
(b) $3200 \, \text{m}^3$ *(A1)*	(b) $3200 \, \text{m}^2$	*(A0)* (Incorrect units)

If no method is shown and the answer is correct but with incorrect or missing units award G marks with a one mark penalty.

8   Graphic Display Calculators

Candidates will often be obtaining solutions directly from their calculators. They must use mathematical notation, not calculator notation. No method marks can be awarded for incorrect answers supported only by calculator notation. The comment 'I used my GDC' cannot receive a method mark.

1. (a)
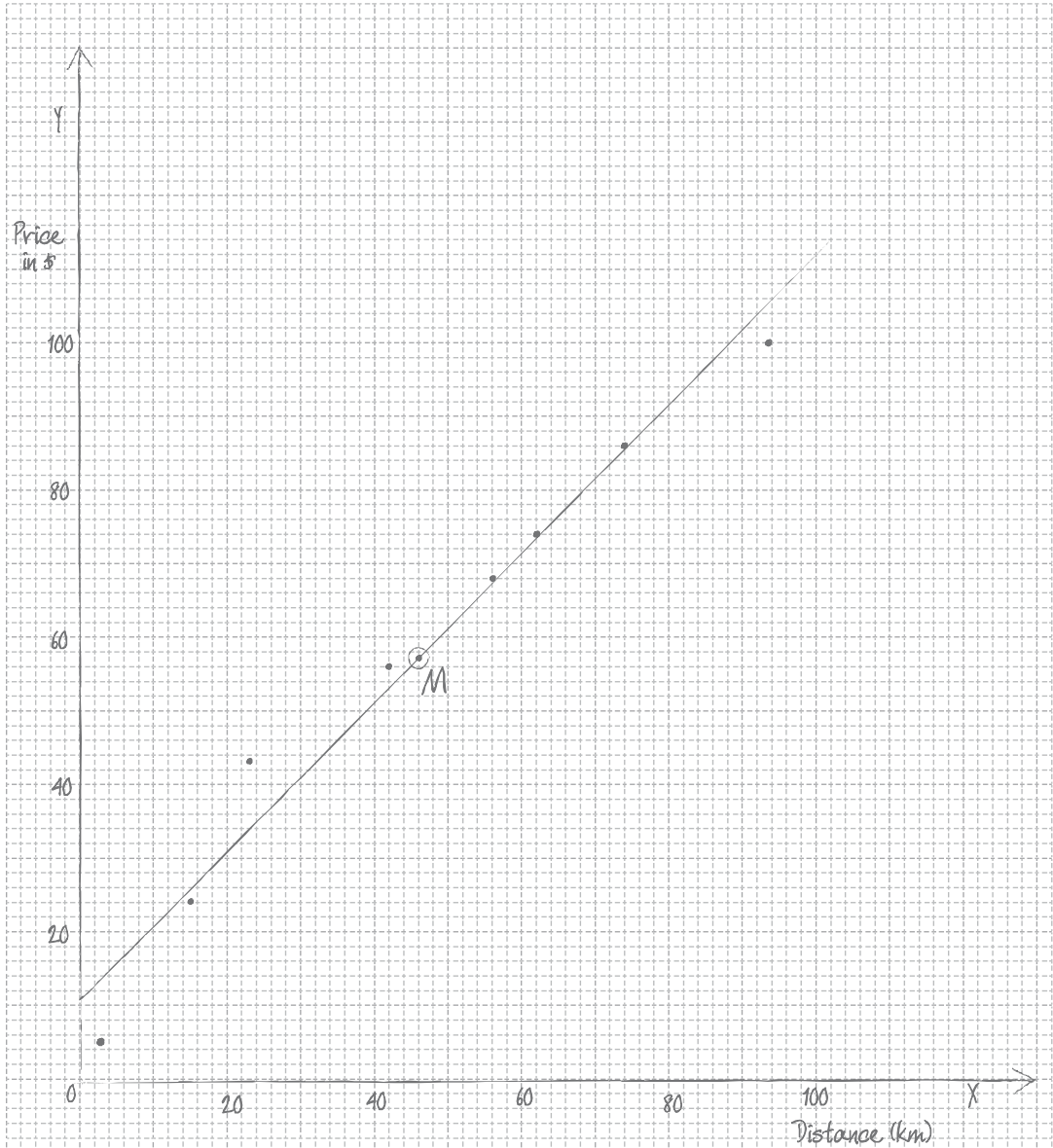
(A4)   *[4 marks]*

> **Notes:** Award *(A1)* for correct scale and labels (accept *x* and *y*).
> Award *(A3)* for 7 or 8 points plotted correctly.
> Award *(A2)* for 5 or 6 points plotted correctly.
> Award *(A1)* for 3 or 4 points plotted correctly.
> Award at most *(A1)(A2)* if points are joined up.
> If axes are reversed, award at most *(A0)(A3)*.
> If graph paper is not used, award at most *(A1)(A0)*.

(b)   (i) $(\bar{x} =)$ 46   *(G1)*

(ii) $(\bar{y} =)$ 57   *(G1)*   *[2 marks]*

(c) $M(46, 57)$ plotted and labelled on the scatter diagram   *(A1)*(**ft**)   *[1 mark]*

> **Notes:** Follow through from their part (b).
> Accept $(\bar{x}, \bar{y})$ as the label.

continued...

Question 1 continued

(d) (i) 0.986 (0.986322...)   *(G1)*

(ii) $y = 1.01x + 10.3$ ($y = 1.01431...x + 10.3412...$)   *(G1)(G1)*

> **Notes:** Award *(G1)* for 1.01*x*, *(G1)* for 10.3.
>   Award *(G1)(G0)* if not written in the form of an equation.

OR

$(y - 57) = 1.01(x - 46)$ $\left(y - 57 = 1.01431...(x - 46)\right)$ *(G1)(G1)*(ft)   *[3 marks]*

> **Note:** Award *(G1)* for 1.01, *(G1)* for their 57 and 46.

(e) straight line drawn on the scatter diagram   *(A1)*(ft)*(A1)*(ft)   *[2 marks]*

> **Notes:** The line must be straight for either of the two marks to be awarded.
>
> Award *(A1)*(ft) passing through their M plotted in (c).
> Award *(A1)*(ft) for correct *y*-intercept (between 9 and 12).
>
> Follow through from their *y*-intercept found in part (d).
> If part (d) is used, award *(A1)*(ft) for their intercept ($\pm$ 1).

(f) $y = 1.01431... \times 76 + 10.3412...$ *(M1)*

> **Note:** Award *(M1)* for substitution of 76 into their regression line.

$= 87.4295...$ *(A1)*(ft)

> **Note:** Follow through from part (d). If 3 sf values are used the value
>   is 87.06.

$87   *(A1)*(ft)*(G2)*   *[3 marks]*

> **Notes:** The final *(A1)* is awarded for their answer given correct to the
>   nearest dollar.
>   Method, followed by the answer of 87 earns *(M1)(G2)*. It is not
>   necessary to see the interim step.
>   Where the candidate uses their graph instead of the equation, and
>   arrives at an answer other than 87, award, at most, *(G1)*(ft).
>   If the candidate uses their graph and arrives at the required answer
>   of 87, award *(G2)*(ft).

Question 1 continued

(g) 76 is within the range of distances given in the data **OR** the correlation coefficient is close to 1. *(R1)* *[1 mark]*

> **Notes:** Award *(R1)* if **either** condition is given.
> Sufficient to indicate that 76 is 'within the data range' and the correlation is 'strong'.
> Allow r^2 close to 1.
> Do **not** accept "within the range of prices".

(h) Percentage error $= \dfrac{87-80}{80} \times 100$ *(M1)*

> **Note:** Award *(M1)* for correct substitution into formula.

8.75 % *(A1)(ft)(G2)* *[2 marks]*

> **Notes:** Follow through from their answer to part (f).
> Accept either the rounded or unrounded answer to part (f).
> If no integer value seen in part (f), follow through from their unrounded answer to part (f).
> Answer must be positive.

Total: [18 marks]

2. *Units are required in part (b).*

(a) $AC^2 = 30^2 + 24^2 - 2 \times 30 \times 24 \times \cos 35°$ *(M1)(A1)*

> **Note:** Award *(M1)* for substituted cosine rule formula, *(A1)* for correct substitutions.

$AC = 17.2$ cm $(17.2168\ldots)$ *(A1)(G2)* *[3 marks]*

> **Notes:** Use of radians gives 52.7002… Award *(M1)(A1)(A0)*.
> No marks awarded in this part of the question where candidates assume that angle $ACB = 90°$.

(b) Area of triangle $ABC = \dfrac{1}{2} \times 24 \times 30 \times \sin 35°$ *(M1)(A1)*

> **Notes:** Award *(M1)* for substitution into area formula, *(A1)* for correct substitutions.
> **Special Case:** Where a candidate has assumed that angle $ACB = 90°$ in part (a), award *(M1)(A1)* for a correct alternative substituted formula for the area of the triangle $\left(ie\ \dfrac{1}{2} \times \text{base} \times \text{height} \right)$.

$= 206$ cm^2 $(206.487\ldots$ cm$^2)$ *(A1)(G2)* *[3 marks]*

> **Notes:** Use of radians gives negative answer, $-154.145\ldots$ Award *(M1)(A1)(A0)*.
> **Special Case:** Award *(A1)***(ft)** where the candidate has arrived at an area which is correct to the standard rounding rules from their lengths (units required).

(c) $206.487\ldots \times 25 \times 2600$ *(M1)*

> **Note:** Award *(M1)* for multiplication of their answer to part (b) by 25 and 2600.

$13\,421\,688.61$ *(A1)*

> **Note:** Accept unrounded answer of $13\,390\,000$ for use of 206.

$13\,400\,000$ *(AG)* *[2 marks]*

> **Note:** The final *(A1)* cannot be awarded unless both the unrounded and rounded answers are seen.

continued...

Question 2 continued

(d) 1.34×10^7 *(A2)*   *[2 marks]*

> **Notes:** Award *(A2)* for the correct answer.
> Award *(A1)(A0)* for 1.34 and an incorrect index value.
> Award *(A0)(A0)* for any other combination (including answers such as 13.4×10^6).

(e) $2 \times 206.487\ldots + 24 \times 25 + 30 \times 25 + 17.2168\ldots \times 25$ *(M1)(M1)*

> **Note:** Award *(M1)* for multiplication of their answer to part (b) by 2 for area of two triangular ends, *(M1)* for three correct rectangle areas using 24, 30 and their 17.2.

      $2193.26\ldots$ *(A1)*

> **Note:** Accept 2192 for use of 3 sf answers.

      2190   *(AG)*   *[3 marks]*

> **Note:** The final *(A1)* cannot be awarded unless both the unrounded and rounded answers are seen.

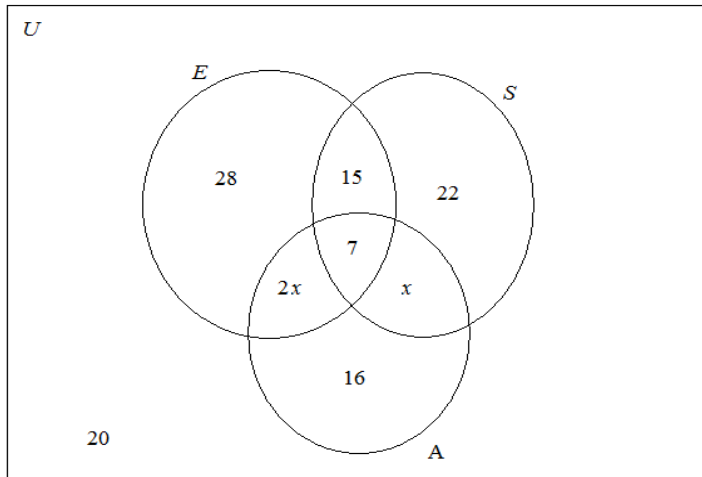(f) $\dfrac{2190 \times 2600}{22 \times 10000}$ *(M1)(M1)*

> **Notes:** Award *(M1)* for multiplication by 2600 and division by 22, *(M1)* for division by 10000.
> The use of 22 may be implied *ie* division by 2200 would be acceptable.

      25.9 litres (25.8818…)   *(A1)(G2)*   *[3 marks]*

> **Note:** Accept 26.

                                                                                                  Total: [16 marks]

3.   (a)

(A1)(A1)(A1)(A1)(A1)   *[5 marks]*

Notes: Award *(A1)* for rectangle and three labelled intersecting circles.
Award *(A1)* for 7 in correct place.
Award *(A1)* for 28, 22 and 16 in the correct places.
Award *(A1)* for 15, *x* and 2*x* in the correct places.
Award *(A1)* for 20 in the correct place.
Accept 4 and 8 instead of *x* and 2*x*.
Do not penalize if *U* is omitted from the diagram.

(b) $3x = 120 - (20 + 28 + 15 + 22 + 7 + 16)$ *(M1)*

Note: Award *(M1)* for setting up a correct equation involving *x*, the 120 and values from their diagram.

   $x = 4$ *(A1)(ft)(G2)*   *[2 marks]*

Note: Follow through from part (a). For the follow through to be awarded *x* must be a positive integer.

(c)   (Women who had visited) Europe **or** South America and (but had) **not** (visited) Asia   *(A1)(A1)*   *[2 marks]*

Notes: Award *(A1)* for "(visited) Europe **or** South America" (or both).
Award *(A1)* for "and (but) had **not** visited Asia".
E(urope) union *S*(outh America) intersected with not *A*(sia) earns no marks, *(A0)*.

(d)   20   *(A1)*   *[1 mark]*

Note: Award *(A0)* for the embedded answer of $n(20)$.

(e) $\dfrac{58}{120} \left(\dfrac{29}{60},\ 0.483,\ 48.3\% \right) (0.48333\ldots)$ *(A1)(ft)(A1)(G2)*   *[2 marks]*

Note: Award *(A1)(ft)* for numerator, follow through from their value of *x*, or their diagram, *(A1)* for denominator.

Question 3 continued

(f)　$\dfrac{15}{35}\left(\dfrac{3}{7},\ 0.429,\ 42.9\%\right)\ (0.428571\ldots)$　　　　　　　*(A1)*(ft)*(A1)*(ft)*(G2)*　　*[2 marks]*

> **Note:** Award *(A1)*(ft) for numerator, *(A1)*(ft) for denominator, follow through from their value of *x* or their diagram.

(g)　$\dfrac{48}{120}\times\dfrac{47}{119}$　　　　　　　　　　　*(A1)*(ft)*(M1)*

> **Notes:** Award *(A1)*(ft) for two correct fractions, follow through from their denominator in part (e), follow through the numerator from their answer to part (b) or from their diagram, *(M1)* for multiplication of their two fractions.

　　$=\dfrac{2256}{14280}\left(\dfrac{94}{595},\ 0.158,15.8\%\right)\ (0.157983...)$　　　　*(A1)*(ft)*(G2)*　　*[3 marks]*

> **Notes:** Award *(A1)(M1)(A1)* for correct fractions, correctly multiplied together with an answer of 0.16.
>
> Award *(A0)(M1)(A0)* for $\dfrac{48}{120}\times\dfrac{48}{120}=0.16$.
>
> Award *(G1)* for an answer of 0.16 with no working seen.

Total: [17 marks]

4.   (a) $\frac{3}{4}(-2)^4 - (-2)^3 - 9(-2)^2 + 20$ *(M1)*

> **Note:** Award *(M1)* for substituting $x = -2$ in the function.

           $= 4$ *(A1)(G2)*   *[2 marks]*

> **Note:** If the coordinates $(-2, 4)$ are given as the answer award,
>   at most, *(M1)(A0)*. If no working shown award *(G1)*.
>   If $x = -2$, $y = 4$ seen then award full marks.

   (b) $3x^3 - 3x^2 - 18x$ *(A1)(A1)(A1)*   *[3 marks]*

> **Note:** Award *(A1)* for each correct term, award at most
>   *(A1)(A1)(A0)* if extra terms seen.

   (c) $f'(3) = 3 \times (3)^3 - 3 \times (3)^2 - 18 \times 3$ *(M1)*

> **Note:** Award *(M1)* for substitution in their $f'(x)$ of $x = 3$.

           $= 0$ *(A1)*

   OR

           $3x^3 - 3x^2 - 18x = 0$ *(M1)*

> **Note:** Award *(M1)* for equating their $f'(x)$ to zero.

           $x = 3$ *(A1)*

           $f'(x_1) = 3 \times (x_1)^3 - 3 \times (x_1)^2 - 18 \times x_1 < 0$ where $0 < x_1 < 3$ *(M1)*

> **Note:** Award *(M1)* for substituting a value of x_1 in the range
> $0 < x_1 < 3$ into their f' and showing it is negative
>   (decreasing).

           $f'(x_2) = 3 \times (x_2)^3 - 3 \times (x_2)^2 - 18 \times x_2 > 0$ where $x_2 > 3$ *(M1)*

> **Note:** Award *(M1)* for substituting a value of x_2 in the range
> $x_2 > 3$ into their f' and showing it is positive (increasing).

Question 4 continued

OR

With or without a sketch:
Showing $f(x_1) > f(3)$ where $x_1 < 3$ and x_1 is close to 3.   ***(M1)***

Showing $f(x_2) > f(3)$ where $x_2 > 3$ and x_2 is close to 3.   ***(M1)***

Note: If a sketch of $f(x)$ is drawn **in this part of the question and** $x = 3$ is identified as a stationary point on the curve, then (i) award, at most, ***(M1)(A1)(M1)(M0)*** if the stationary point has been found; (ii) award, at most, ***(M0)(A0)(M1)(M0)*** if the stationary point has not been previously found.

Since the gradients go from negative (decreasing) through zero to positive (increasing) it is a local minimum   ***(R1)(AG)***
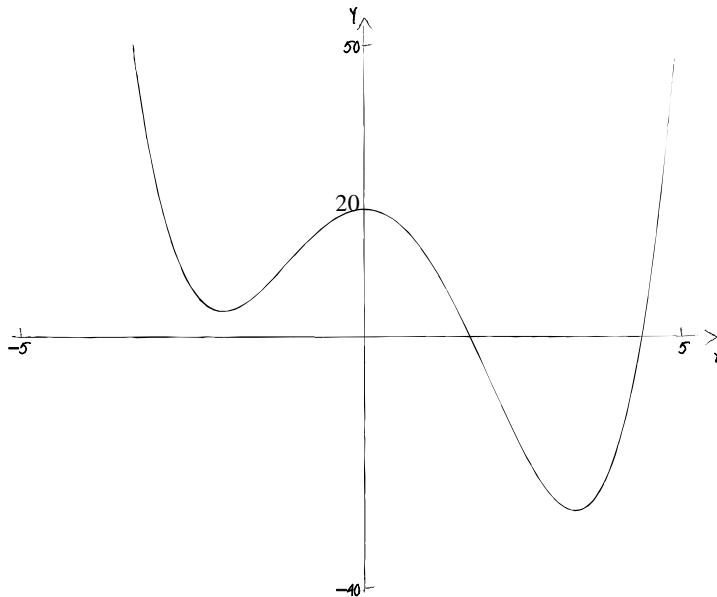
Note: Only award ***(R1)*** if the first two marks have been awarded *ie $f'(3)$* has been shown to be equal to 0.

[5 marks]

Question 4 continued

(d)

(A1)(A1)(A1)(A1)

Notes: Award *(A1)* for labelled axes and indication of scale on both axes.
Award *(A1)* for smooth curve with correct shape.
Award *(A1)* for local minima in 2nd and 4th quadrants.
Award *(A1)* for y intercept $(0, 20)$ seen and labelled. Accept 20 on y-axis.
Do **not** award the third *(A1)* mark if there is a turning point on the x-axis.
If the derivative function is sketched then award, at most, *(A1)(A0)(A0)(A0)*.
For a smooth curve (with correct shape) there should be **ONE** continuous thin line, no part of which is straight and no (one to many) mappings of x.

[4 marks]

(e) $(0, 20)$ *(G1)(G1)*

Note: If parentheses are omitted award *(G0)(G1)*.

　　OR

　　$x = 0, \ y = 20$ *(G1)(G1)*   *[2 marks]*

Note: If the derivative function is sketched in part (d), award *(G1)(ft)(G1)(ft)* for $(-1.12, 12.2)$.

(f) $f'(2) = 3(2)^3 - 3(2)^2 - 18(2)$ *(M1)*

Notes: Award *(M1)* for substituting $x = 2$ into their $f'(x)$.

　　　　$= -24$ *(A1)(ft)(G2)*   *[2 marks]*

continued...

Question 4 continued

(g) (i)   Gradient of perpendicular $= \dfrac{1}{24}$ (0.0417, 0.041666…)   *(A1)(ft)(G1)*

> **Note:** Follow through from part (f).

(ii) $y + 12 = \dfrac{1}{24}(x - 2)$ *(M1)(M1)*

> **Note:** Award *(M1)* for correct substitution of $(2, -12)$, *(M1)* for correct substitution of their perpendicular gradient into equation of line.

OR

$-12 = \dfrac{1}{24} \times 2 + d$ *(M1)*

$d = -\dfrac{145}{12}$

$y = \dfrac{1}{24}x - \dfrac{145}{12}$ *(M1)*

> **Note:** Award *(M1)* for correct substitution of $(2, -12)$ and gradient into equation of a straight line, *(M1)* for correct substitution of the perpendicular gradient and correct substitution of d into equation of line.

$b = -24,\, c = -290$ *(A1)(ft)(A1)(ft)(G3)*   *[5 marks]*

> **Note:** Follow through from parts (f) and g(i).
> To award **(ft)** marks, b and c must be integers.
> Where candidate has used 0.042 from g(i), award *(A1)(ft)* for -288.

Total: [23 marks]

5. *The first answer not given to two decimal places is not awarded the final (A1). Incorrect rounding is not penalized thereafter.*

(a) $37\,500 \times 0.7234$ *(M1)*
      $= 27127.50$ *(A1)(G2)* *[2 marks]*

(b) 6947.50 *(A1)(ft)(G1)* *[1 mark]*

Note: Follow through from part (a) irrespective of whether working is seen.

(c) $\dfrac{6947.50 \times 4.5 \times 4}{100} + 6947.50$ *(M1)(M1)*

Note: Award *(M1)* for their correctly substituted simple interest formula, *(M1)* for addition of their part (b).

   $= 8198.05$ *(A1)(ft)(G2)* *[3 marks]*

Note: Follow through from part (b).

(d) 27127.50×0.91 *(A1)(M1)*

Note: Award *(A1)* for 0.91 seen or equivalent, *(M1)* for their 27 127.50 multiplied by 0.91

   OR

   $27127.50 - 0.09 \times 27127.50$ *(A1)(M1)*

Note: Award *(A1)* for 0.09×27127.50 seen, and *(M1)* for $27127.50 - 0.09 \times 27127.50$.

   $= 24\,686.03$ *(A1)(ft)(G2)* *[3 marks]*

Note: Follow through from part (a).

Question 5 continued

(e) $27127.50 \times \left(1 - \dfrac{9}{100}\right)^{4}$ *(M1)(A1)*(ft)

Notes: Award *(M1)* for substituted compound interest formula,
            (A1)(ft) for correct substitution.
            Follow through from part (a).

OR

$27127.50 \times (0.91)^{4}$ *(M1)(A1)*(ft)

Notes: Award *(M1)* for substituted geometric sequence
            formula, *(A1)*(ft) for correct substitution.
            Follow through from part (a).

OR (lists (i))

$24686.03,\ 22464.28\ldots,\ 20442.50\ldots,\ 18602.67\ldots$ *(M1)(A1)*(ft)

Notes: Award *(M1)* for at least the 2nd term correct (calculated
            from their (a) $\times\ 0.91$). Award *(A1)*(ft) for four correct
            terms (rounded or unrounded).
            Follow through from part (a).
            Accept list containing the last three terms only (24686.03
            may be implied).

OR (lists(ii))

$27127.50 - (2441.47... + 2221.74... + 2021.79... + 1839.82...)$ *(M1)(A1)*(ft)

Notes: Award *(M1)* for subtraction of four terms from 27127.50.
            Award *(A1)* for four correct terms (rounded or unrounded).
            Follow through from part (a).

$= 18\,602.67$ *(A1)*
$= 18\,600$ *(AG)*   *[3 marks]*

Note: The final *(A1)* is not awarded unless both the unrounded and
            rounded answers are seen.

Question 5 continued

(f) $\dfrac{18\,600\ +\ 8198.05}{0.8694} - 30\,500$ **(M1)(M1)(M1)**

Note:	Award *(M1)* for their answer to part (c) added to $18\,600$, *(M1)* for $\dfrac{18600+(any\ value)}{0.8694}$, *(M1)* for the *difference* between $\dfrac{18600+(any\ value)}{0.8694}$ and $30\,500$.

OR

$\dfrac{18\,600\ +\ 8198.05 - 30\,500 \times 0.8694}{0.8694}$ **(M1)(M1)(M1)**

Note:	Award *(M1)* for their answer to part (c) added to $18\,600$, *(M1)* for *difference* between (their answer to part (c) added to $18\,600$) and $(30\,500 \times 0.8694)$, *(M1)* for dividing the resultant value by 0.8694. If the value for the exchange rate used is 0.7234, then award, at most, *(M1)(M0)(M1)*.

$= 323.61$ **(A1)(ft)(G3)** *[4 marks]*

Note:	Follow through from their part (c). Award *(A1)*(**ft**) for final answer provided it is positive, and dependant on all three method marks.

Total: [16 marks]