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**MATHEMATICS
STANDARD LEVEL
PAPER 1**

Candidate session number

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Thursday 3 May 2012 (afternoon)

Examination code

2	2	1	2	–	7	3	0	5

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the *Mathematics SL information booklet* is required for this paper.
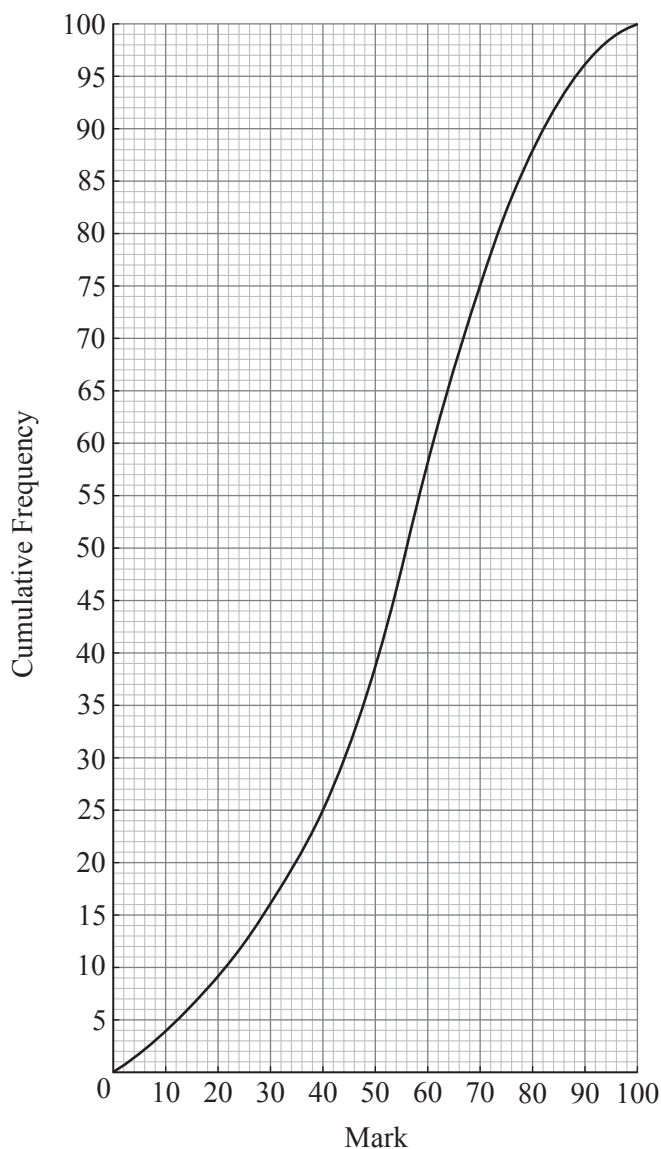- The maximum mark for this examination paper is *[90 marks]*.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 5]*

The cumulative frequency curve below represents the marks obtained by 100 students.

Mark

(This question continues on the following page)

(Question 1 continued)

(a)   Find the median mark. *[2 marks]*

(b)   Find the interquartile range. *[3 marks]*

Turn over

2. *[Maximum mark: 6]*

Let $f(x) = 2x - 1$ and $g(x) = 3x^2 + 2$.

(a) Find $f^{-1}(x)$. *[3 marks]*

(b) Find $(f \circ g)(1)$. *[3 marks]*

3.   *[Maximum mark: 6]*

The following diagram shows the graph of $f(x) = a\cos(bx)$, for $0 \leq x \leq 4$.
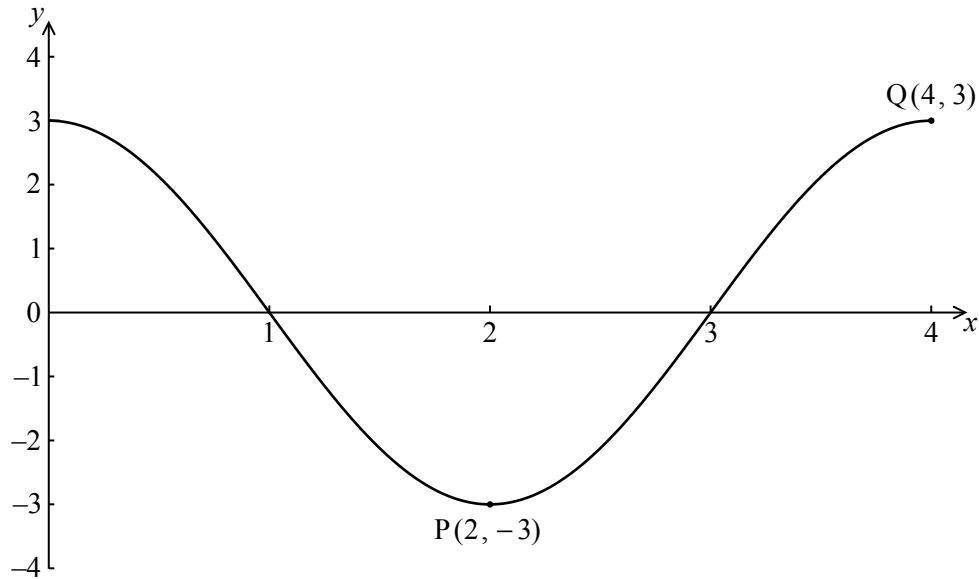
There is a minimum point at $P(2, -3)$ and a maximum point at $Q(4, 3)$.

(a)   (i)   Write down the value of a.

(ii)   Find the value of b.   *[3 marks]*

(b)   Write down the gradient of the curve at P.   *[1 mark]*

(c)   Write down the equation of the normal to the curve at P.   *[2 marks]*

Turn over

4. *[Maximum mark: 8]*

The random variable X has the following probability distribution, with $P(X > 1) = 0.5$.

x	0	1	2	3
$P(X = x)$	p	q	r	0.2

(a) Find the value of r. *[2 marks]*

(b) Given that $E(X) = 1.4$, find the value of p and of q. *[6 marks]*

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Answers written on this page
will not be marked.

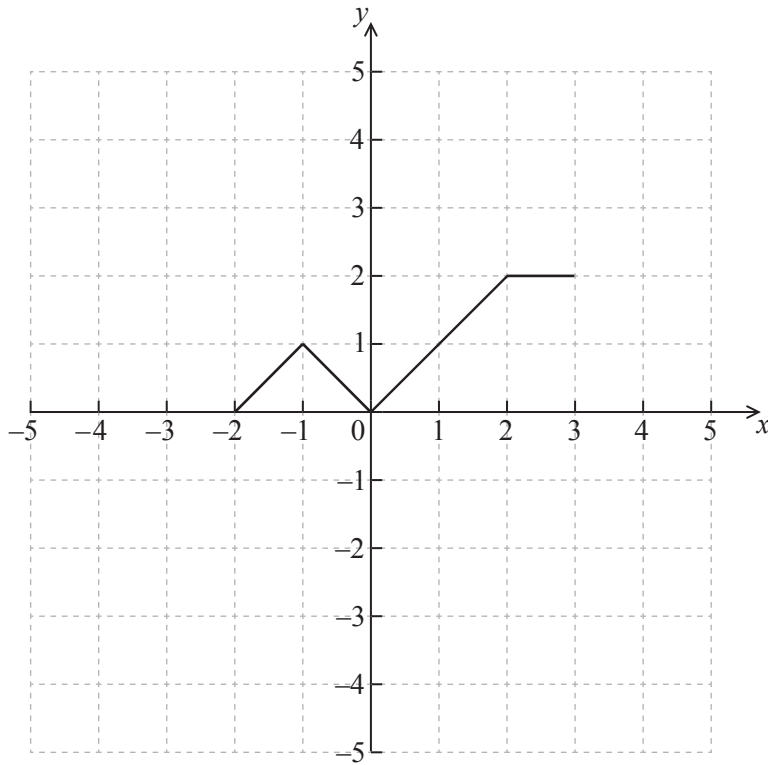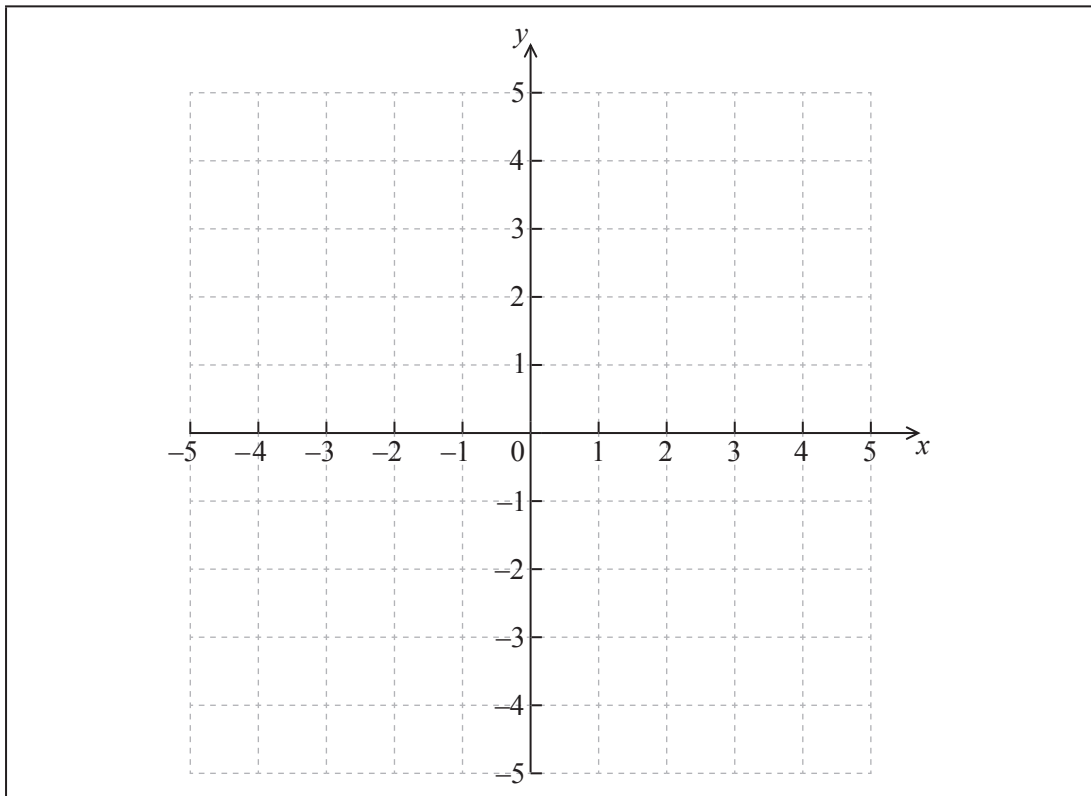**5.**   *[Maximum mark: 6]*

The diagram below shows the graph of a function $f(x)$, for $-2 \le x \le 3$.

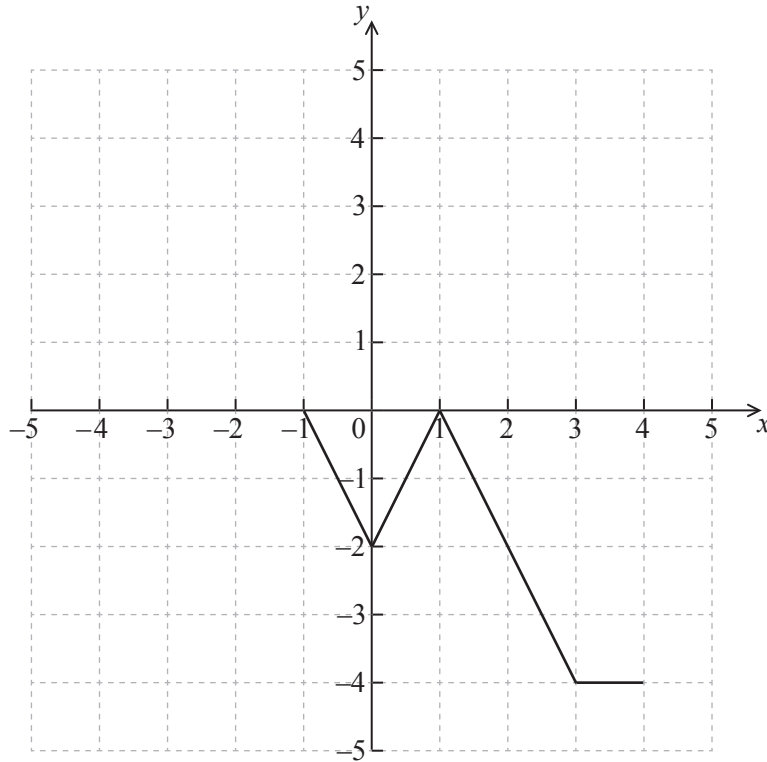(a)   Sketch the graph of $f(-x)$ on the grid below.   *[2 marks]*

(This question continues on the following page)

(Question 5 continued)

(b)   The graph of f is transformed to obtain the graph of g. The graph of g is shown below.

The function g can be written in the form $g(x) = af(x+b)$. Write down the value of a and of b. *[4 marks]*

Turn over

6. *[Maximum mark: 7]*

Consider the equation $x^2 + (k-1)x + 1 = 0$, where k is a real number.

Find the values of k for which the equation has two **equal** real solutions.

7. *[Maximum mark: 7]*

Given that $\left(1+\dfrac{2}{3}x\right)^{n}(3+nx)^{2}=9+84x+\ldots$, find the value of n.

Turn over

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions on the answer sheets provided. Please start each question on a new page.*

8.   *[Maximum mark: 15]*

The following diagram shows the graph of a quadratic function f, for $0 \leq x \leq 4$.
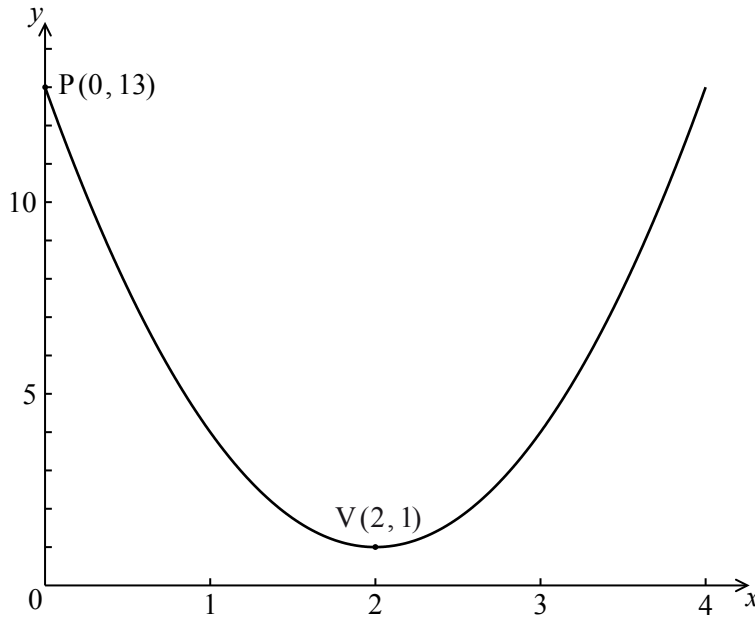
The graph passes through the point $P(0, 13)$, and its vertex is the point $V(2, 1)$.

(a)   The function can be written in the form $f(x) = a(x - h)^2 + k$.

   (i)   Write down the value of h and of k.

   (ii)   Show that $a = 3$.   *[4 marks]*

(b)   Find $f(x)$, giving your answer in the form $Ax^2 + Bx + C$.   *[3 marks]*

(c)   Calculate the area enclosed by the graph of f, the x-axis, and the lines $x = 2$ and $x = 4$.   *[8 marks]*
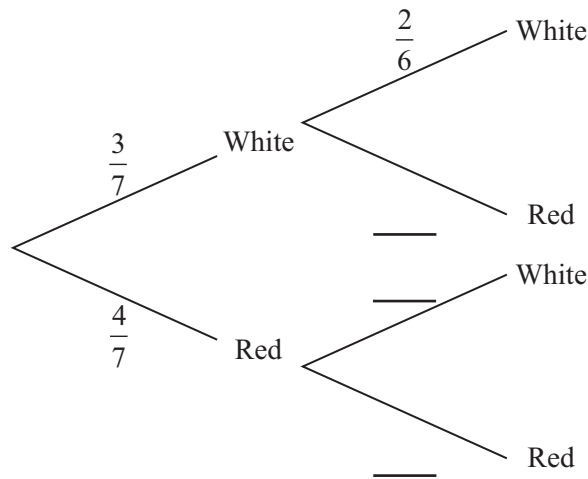
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9. *[Maximum mark: 14]*

Bag A contains three white balls and four red balls. Two balls are chosen at random without replacement.

(a) (i) **Copy** and complete the following tree diagram. *(Do **not** write on this page.)*

(ii) Find the probability that two white balls are chosen. *[5 marks]*

Bag B contains four white balls and three red balls. When two balls are chosen at random without replacement from bag B, the probability that they are both white is $\frac{2}{7}$.

A standard die is rolled. If 1 or 2 is obtained, two balls are chosen without replacement from bag A, otherwise they are chosen from bag B.

(b) Find the probability that the two balls are white. *[5 marks]*

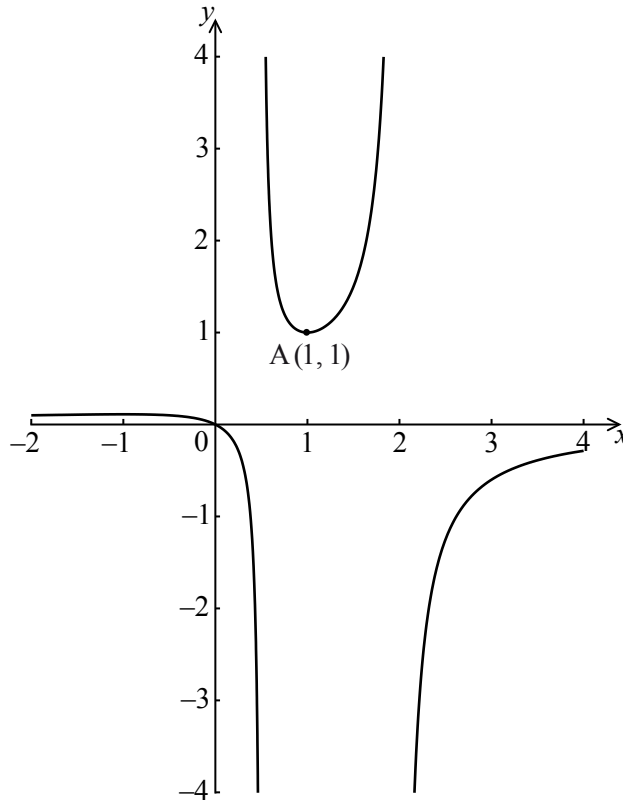(c) Given that both balls are white, find the probability that they were chosen from bag A. *[4 marks]*

Turn over

*Do **NOT** write solutions on this page.*

10.   *[Maximum mark: 16]*

Let $f(x) = \dfrac{x}{-2x^2 + 5x - 2}$ for $-2 \le x \le 4$, $x \ne \dfrac{1}{2}$, $x \ne 2$. The graph of f is given below.

The graph of f has a local minimum at $A(1, 1)$ and a local maximum at B.

(a)   Use the quotient rule to show that $f'(x) = \dfrac{2x^2 - 2}{(-2x^2 + 5x - 2)^2}$.   *[6 marks]*

(b)   Hence find the coordinates of B.   *[7 marks]*

(c)   Given that the line $y = k$ does not meet the graph of f, find the possible values
       of k.   *[3 marks]*

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Answers written on this page
will not be marked.