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MATHEMATICAL STUDIES
STANDARD LEVEL
PAPER 2

Friday 4 May 2012 (morning)

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **Mathematical Studies SL** *information booklet* is required for this paper.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is *[90 marks]*.

Please start each question on a new page. You are advised to show all working, where possible. Where an answer is incorrect, some marks may be given for correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1.   *[Maximum mark: 16]*

Beartown has three local newspapers: *The Art Journal*, *The Beartown News*, and *The Currier*.

A survey shows that

32 % of the town's population read *The Art Journal*,
46 % read *The Beartown News,*
54 % read *The Currier*,
3 % read *The Art Journal* and *The Beartown News* **only**,
8 % read *The Art Journal* and *The Currier* **only**,
12 % read *The Beartown News* and *The Currier* **only**, and
5 % of the population reads **all** three newspapers.

(a)   Draw a Venn diagram to represent this information. Label *A* the set that represents *The Art Journal* readers, *B* the set that represents *The Beartown News* readers, and *C* the set that represents *The Currier* readers.   *[4 marks]*

(b)   Find the percentage of the population that does not read any of the three newspapers.   *[2 marks]*

(c)   Find the percentage of the population that reads exactly one newspaper.   *[2 marks]*

(d)   Find the percentage of the population that reads *The Art Journal* or *The Beartown News* but not *The Currier*.   *[2 marks]*

A local radio station states that 83 % of the population reads either *The Beartown News* or *The Currier*.

(e)   Use your Venn diagram to decide whether the statement is true. Justify your answer.   *[2 marks]*

The population of Beartown is 120 000. The local radio station claimed that 34 000 of the town's citizens read at least two of the local newspapers.

(f)   Find the percentage error in this claim.   *[4 marks]*

2. *[Maximum mark: 16]*

The seniors from Gulf High School are required to participate in exactly one after-school sport. Data were gathered from a sample of 120 students regarding their choice of sport. The following data were recorded.

Gender	Sport			
	Football	**Tennis**	**Basketball**	**Total**
Male	17	8	10	35
Female	31	17	37	85
Total	48	25	47	120

A χ^2 test was carried out at the 5 % significance level to analyse the relationship between gender and choice of after-school sport.

(a)   Write down the null hypothesis, H_0, for this test. *[1 mark]*

(b)   Find the expected value of female footballers. *[2 marks]*

(c)   Write down the number of degrees of freedom. *[1 mark]*

(d)   Write down the critical value of χ^2, at the 5 % level of significance. *[1 mark]*

(e)   Use your graphic display calculator to determine the χ^2_{calc} value. *[2 marks]*

(f)   Determine whether H_0 should be accepted. Justify your answer. *[2 marks]*

One student is chosen at random from the 120 students.

(g)   Find the probability that this student

    (i)   is male;

    (ii)   plays tennis. *[2 marks]*
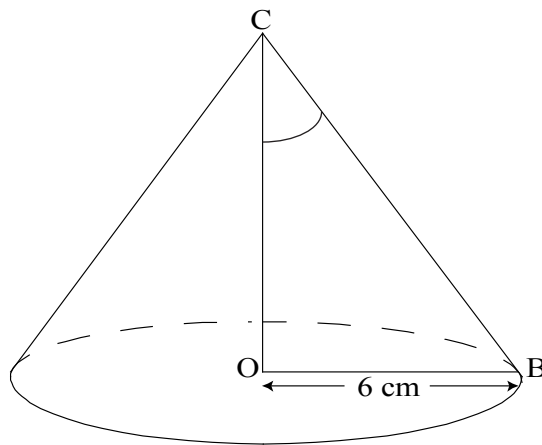
Two students are chosen at random from the 120 students.

(h)   Find the probability that

    (i)   both play football;

    (ii)   neither play basketball. *[5 marks]*

3.   *[Maximum mark: 18]*

A solid metal **cylinder** has a base radius of 4 cm and a height of 8 cm.

(a)   Find the area of the base of the cylinder.   *[2 marks]*

(b)   Show that the volume of the metal used in the cylinder is 402 cm^3, given correct to three significant figures.   *[2 marks]*

(c)   Find the total surface area of the cylinder.   *[3 marks]*

The cylinder was melted and recast into a solid cone, shown in the following diagram. The base radius OB is 6 cm.

(d)   Find the height, OC, of the cone.   *[3 marks]*

(e)   Find the size of angle BCO.   *[2 marks]*

(f)   Find the slant height, CB.   *[2 marks]*

(g)   Find the total surface area of the cone.   *[4 marks]*

4. *[Maximum mark: 20]*

Part A

The Green Park Amphitheatre was built in the form of a horseshoe and has 20 rows. The number of seats in each row increase by a fixed amount, d, compared to the number of seats in the previous row. The number of seats in the sixth row, u_6, is 100, and the number of seats in the tenth row, u_{10}, is 124. u_1 represents the number of seats in the first row.

(a)   (i)   Write an equation for u_6 in terms of d and u_1.

(ii)   Write an equation for u_{10} in terms of d and u_1.   *[2 marks]*

(b)   Write down the value of

(i) d;

(ii) u_1.   *[2 marks]*

(c)   Find the **total** number of seats in the amphitheatre.   *[3 marks]*

A few years later, a **second** level was added to increase the amphitheatre's capacity by another 1600 seats. Each row has four more seats than the previous row. The first row on this level has 70 seats.

(d)   Find the number of rows on the second level of the amphitheatre.   *[4 marks]*

(Question 4 continued)

Part B

Frank is at the amphitheatre and receives a text message at 12:00. Five minutes later he forwards the text message to three people. Five minutes later, those three people forward the text message to three new people. **Assume this pattern continues and each time the text message is sent to people who have not received it before.**

The number of new people who receive the text message forms a geometric sequence

$$1, 3, \ldots$$

(a)   Write down the next two terms of this geometric sequence. *[1 mark]*

(b)   Write down the common ratio of this geometric sequence. *[1 mark]*

(c)   Calculate the number of people who will receive the text message at 12:30. *[2 marks]*

(d)   Calculate the **total** number of people who will have received the text message by 12:30. *[2 marks]*

(e)   Calculate the exact time at which a total of 29 524 people will have received the text message. *[3 marks]*

5. *[Maximum mark: 20]*

Consider the function $f(x) = -\dfrac{1}{3}x^3 + \dfrac{5}{3}x^2 - x - 3$.

(a) Sketch the graph of $y = f(x)$ for $-3 \le x \le 6$ and $-10 \le y \le 10$ showing clearly the axes intercepts and local maximum and minimum points. Use a scale of 2 cm to represent 1 unit on the x-axis, and a scale of 1 cm to represent 1 unit on the y-axis. *[4 marks]*

(b) Find the value of $f(-1)$. *[2 marks]*

(c) Write down the coordinates of the y-intercept of the graph of $f(x)$. *[1 mark]*

(d) Find $f'(x)$. *[3 marks]*

(e) Show that $f'(-1) = -\dfrac{16}{3}$. *[1 mark]*

(f) Explain what $f'(-1)$ represents. *[2 marks]*

(g) Find the equation of the tangent to the graph of $f(x)$ at the point where x is -1. *[2 marks]*

(h) Sketch the tangent to the graph of $f(x)$ at $x = -1$ on your diagram for (a). *[2 marks]*

P and Q are points on the curve such that the tangents to the curve at these points are horizontal. The x-coordinate of P is a, and the x-coordinate of Q is b, $b > a$.

(i) Write down the value of

    (i) a;

    (ii) b. *[2 marks]*

(j) Describe the behaviour of $f(x)$ for $a < x < b$. *[1 mark]*