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**MATHEMATICAL STUDIES
STANDARD LEVEL
PAPER 2**

Thursday 3 November 2011 (morning)

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- A graphic display calculator is required for this paper.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.

Please start each question on a new page. You are advised to show all working, where possible. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. [Maximum mark: 18]

The speed, s , in km h^{-1} , of 120 vehicles passing a point on the road was measured. The results are given below.

Speed, s (km h^{-1})	$0 < s \leq 50$	$50 < s \leq 60$	$60 < s \leq 70$	$70 < s \leq 80$	$80 < s \leq 90$	$90 < s \leq 100$
Number of vehicles	30	46	22	12	8	2

- (a) Write down the midpoint of the $60 < s \leq 70$ interval. [1 mark]
- (b) Use your graphic display calculator to find an estimate for
 - (i) the mean speed of the vehicles;
 - (ii) the standard deviation of the speeds of the vehicles. [3 marks]
- (c) Write down the number of vehicles whose speed is less than or equal to 60 km h^{-1} . [1 mark]

Consider the cumulative frequency table below.

Speed, s (km h^{-1})	$s \leq 50$	$s \leq 60$	$s \leq 70$	$s \leq 80$	$s \leq 90$	$s \leq 100$
Number of vehicles	30	a	b	110	c	120

- (d) Write down the value of a , of b and of c . [2 marks]
- (e) Draw a cumulative frequency graph for the information from the table. Use 1 cm to represent 10 km h^{-1} on the horizontal axis and 1 cm to represent 10 vehicles on the vertical axis. [4 marks]

(This question continues on the following page)

(Question 1 continued)

- (f) Use your cumulative frequency graph to estimate
- (i) the median speed of the vehicles;
 - (ii) the number of vehicles that are travelling at a speed less than or equal to 65 km h^{-1} .

[4 marks]

All drivers whose vehicle's speed is greater than one standard deviation above the speed limit of 50 km h^{-1} will be fined.

- (g) Use your graph to estimate the number of drivers who will be fined.

[3 marks]

2. [Maximum mark: 19]

Pam has collected data from a group of 400 IB Diploma students about the Mathematics course they studied and the language in which they were examined (English, Spanish or French). The summary of her data is given below.

	Mathematics HL	Mathematics SL	Mathematical Studies SL	Total
English	50	70	80	200
Spanish	30	50	30	110
French	20	30	40	90
Total	100	150	150	400

A student is chosen at random from the group.

- (a) Find the probability that the student
- (i) studied Mathematics HL;
 - (ii) was examined in French;
 - (iii) studied Mathematics HL and was examined in French;
 - (iv) did not study Mathematics SL and was not examined in English;
 - (v) studied Mathematical Studies SL given that the student was examined in Spanish.

[8 marks]

Pam believes that the Mathematics course a student chooses is independent of the language in which the student is examined.

- (b) Using your answers to parts (a) (i), (ii) and (iii) above, state whether there is any evidence for Pam’s belief. Give a reason for your answer.

[2 marks]

(This question continues on the following page)

(Question 2 continued)

Pam decides to test her belief using a Chi-squared test at the 5 % level of significance.

- (c) (i) State the null hypothesis for this test.
- (ii) Show that the expected number of Mathematical Studies SL students who took the examination in Spanish is 41.3, correct to 3 significant figures. *[3 marks]*
- (d) Write down
- (i) the Chi-squared calculated value;
- (ii) the number of degrees of freedom;
- (iii) the Chi-squared critical value. *[4 marks]*
- (e) State, giving a reason, whether there is sufficient evidence at the 5 % level of significance that Pam’s belief is correct. *[2 marks]*

3. [Maximum mark: 14]

The diagram shows triangle ABC in which $AB = 28 \text{ cm}$, $BC = 13 \text{ cm}$, $BD = 12 \text{ cm}$ and $AD = 20 \text{ cm}$.

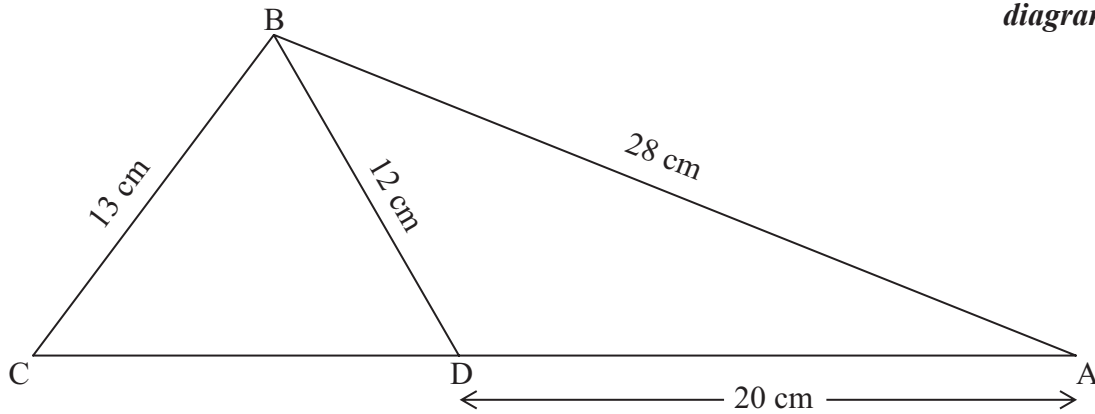
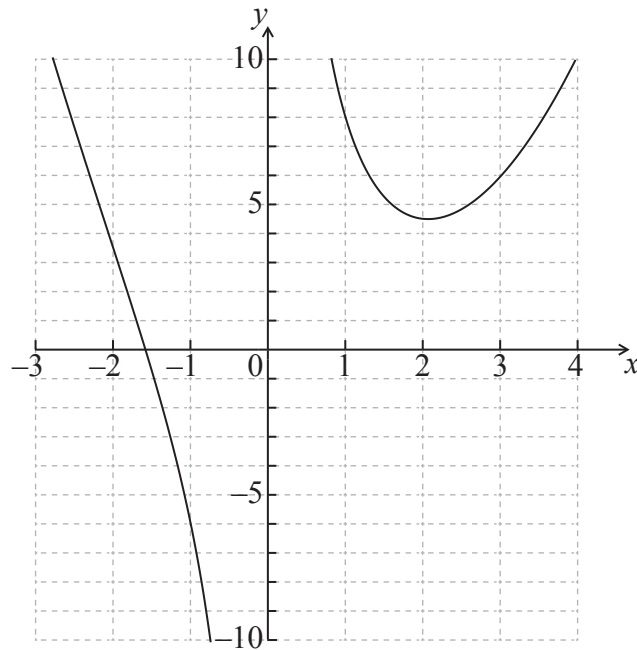


diagram not to scale

- (a) Calculate the size of angle ADB. [3 marks]
- (b) Find the area of triangle ADB. [3 marks]
- (c) Calculate the size of angle BCD. [4 marks]
- (d) Show that the triangle ABC is not right angled. [4 marks]

4. [Maximum mark: 19]

The diagram shows part of the graph of $f(x) = x^2 - 2x + \frac{9}{x}$, where $x \neq 0$.

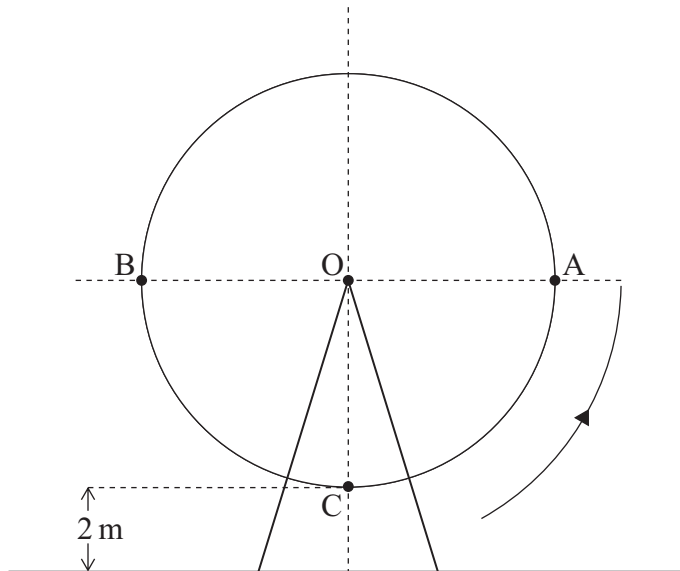


- (a) Write down
 - (i) the equation of the vertical asymptote to the graph of $y = f(x)$;
 - (ii) the solution to the equation $f(x) = 0$;
 - (iii) the coordinates of the local minimum point. [5 marks]
 - (b) Find $f'(x)$. [4 marks]
 - (c) Show that $f'(x)$ can be written as $f'(x) = \frac{2x^3 - 2x^2 - 9}{x^2}$. [2 marks]
 - (d) Find the gradient of the tangent to $y = f(x)$ at the point A(1, 8). [2 marks]
- The line, L , passes through the point A and is perpendicular to the tangent at A.
- (e) Write down the gradient of L . [1 mark]
 - (f) Find the equation of L . Give your answer in the form $y = mx + c$. [3 marks]
- L also intersects the graph of $y = f(x)$ at points B and C.
- (g) Write down the **x-coordinate** of B and of C. [2 marks]

5. [Maximum mark: 20]

The diagram shows a Ferris wheel that moves with constant speed and completes a rotation every 40 seconds. The wheel has a radius of 12 m and its lowest point is 2 m above the ground.

diagram not to scale



Initially, a seat C is vertically below the centre of the wheel, O. It then rotates in an anticlockwise (counterclockwise) direction.

(a) Write down

(i) the height of O above the ground;

(ii) the maximum height above the ground reached by C.

[2 marks]

In a revolution, C reaches points A and B, which are at the same height above the ground as the centre of the wheel.

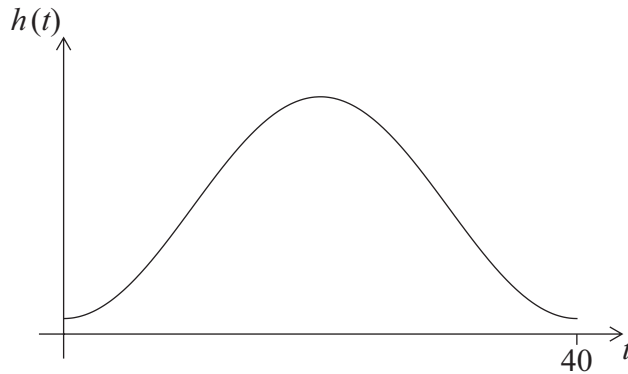
(b) Write down the number of seconds taken for C to first reach A and then B.

[2 marks]

(This question continues on the following page)

(Question 5 continued)

The sketch below shows the graph of the function, $h(t)$, for the height above ground of C, where h is measured in metres and t is the time in seconds, $0 \leq t \leq 40$.



- (c) **Copy** the sketch and show the results of part (a) and part (b) on your diagram. Label the points clearly with their coordinates. [4 marks]

The height of C above ground can be modelled by the function, $h(t) = a \cos(bt) + c$, where bt is measured in degrees and t is the time in seconds.

- (d) Find the value of
- (i) a ;
 - (ii) b ;
 - (iii) c . [5 marks]

C **first** reaches a height of 20 m above the ground after T seconds.

- (e) (i) Sketch a clearly labelled diagram of the wheel to show the position of C.
- (ii) Find the angle that C has rotated through to reach this position.
- (iii) Find the value of T . [7 marks]
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