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International Baccalaureate®
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MATHEMATICS
STANDARD LEVEL
PAPER 1

Candidate session number		
0	0	

Wednesday 4 May 2011 (afternoon)

1 hour 30 minutes

Examination code							
2	2	1	1	–	7	3	0	5

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided.*

1. *[Maximum mark: 6]*

In an arithmetic sequence, $u_1 = 2$ and $u_3 = 8$.

(a) Find d. *[2 marks]*

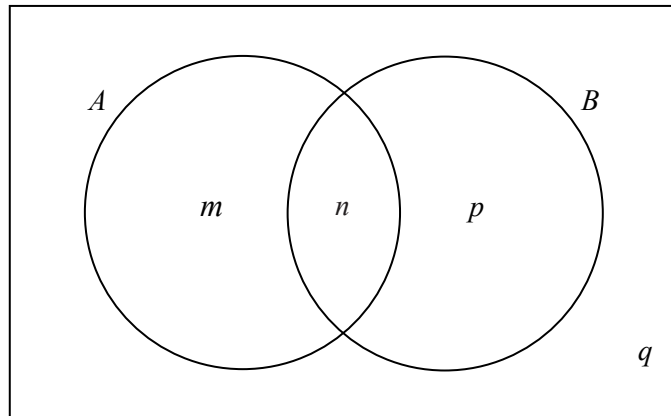(b) Find u_{20}. *[2 marks]*

(c) Find S_{20}. *[2 marks]*

2.   *[Maximum mark: 6]*

The Venn diagram below shows events A and B where $P(A) = 0.3$, $P(A \cup B) = 0.6$ and $P(A \cap B) = 0.1$. The values m, n, p and q are probabilities.

(a)   (i)   Write down the value of n.

      (ii)   Find the value of m, of p, and of q.   *[4 marks]*

(b)   Find $P(B')$.   *[2 marks]*

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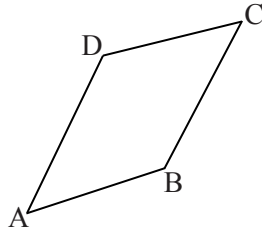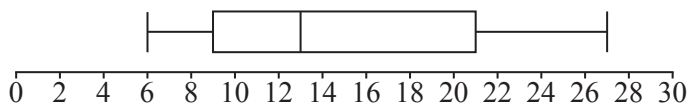**3.** *[Maximum mark: 7]*

The following diagram shows quadrilateral ABCD, with $\vec{AD} = \vec{BC}$, $\vec{AB} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\vec{AC} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$.

diagram not to scale

(a) Find $\vec{BC}$. *[2 marks]*

(b) Show that $\vec{BD} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$. *[2 marks]*

(c) Show that vectors $\vec{BD}$ and $\vec{AC}$ are perpendicular. *[3 marks]*

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4.   *[Maximum mark: 6]*

Let $h(x) = \dfrac{6x}{\cos x}$. Find $h'(0)$.

5. *[Maximum mark: 7]*

Let $f(x) = 3 \ln x$ and $g(x) = \ln 5x^3$.

(a) Express $g(x)$ in the form $f(x) + \ln a$, where $a \in \mathbb{Z}^+$. *[4 marks]*

(b) The graph of g is a transformation of the graph of f. Give a full geometric description of this transformation. *[3 marks]*

6. *[Maximum mark: 5]*

A scientist has 100 female fish and 100 male fish. She measures their lengths to the nearest cm. These are shown in the following box and whisker diagrams.
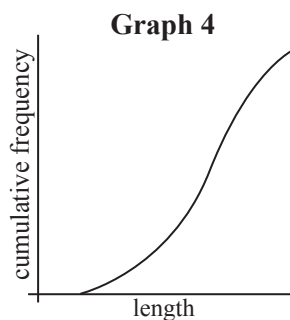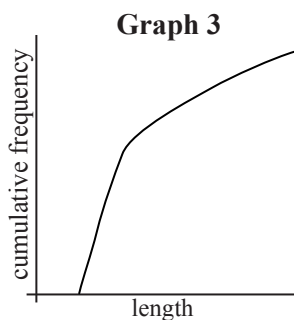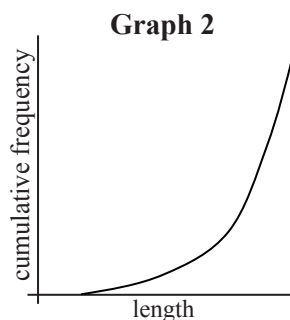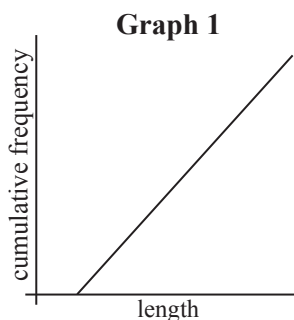
Female fish

0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30

Male fish

0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30
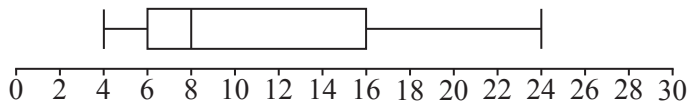
(a) Find the range of the lengths of **all** 200 fish. *[3 marks]*

(b) Four cumulative frequency graphs are shown below.

Graph 1

Graph 2

Graph 3

Graph 4

Which graph is the best representation of the lengths of the **female** fish? *[2 marks]*

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7.   *[Maximum mark: 8]*

Let $A = \begin{pmatrix} 3 & x \\ -2 & -3 \end{pmatrix}$.

(a)   Find the value of x for which A^{-1} does not exist.   *[3 marks]*

(b)   Given that $A = A^{-1}$, find x.   *[5 marks]*

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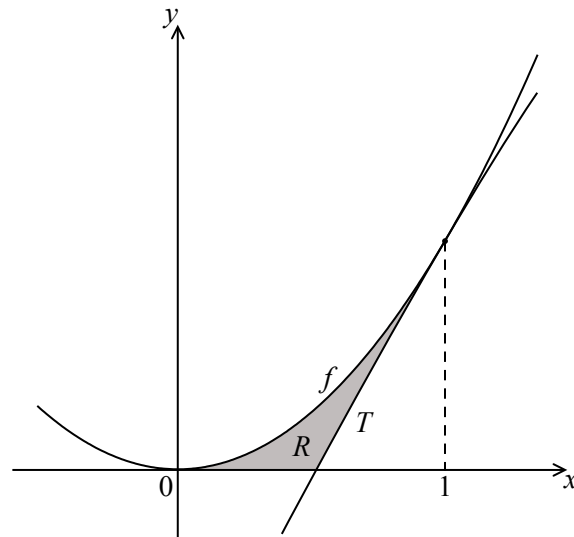..

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

SECTION B

*Answer **all** questions on the answer sheets provided. Please start each question on a new page.*

8.   *[Maximum mark: 16]*

The following diagram shows part of the graph of the function $f(x) = 2x^2$.

diagram not to scale

The line T is the tangent to the graph of f at $x = 1$.

(a)   Show that the equation of T is $y = 4x - 2$.   *[5 marks]*

(b)   Find the x-intercept of T.   *[2 marks]*

(c)   The shaded region R is enclosed by the graph of f, the line T, and the x-axis.

   (i)   Write down an expression for the area of R.

   (ii)   Find the area of R.   *[9 marks]*

Turn over

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

9.   *[Maximum mark: 15]*

The following diagram shows part of the graph of a quadratic function f.
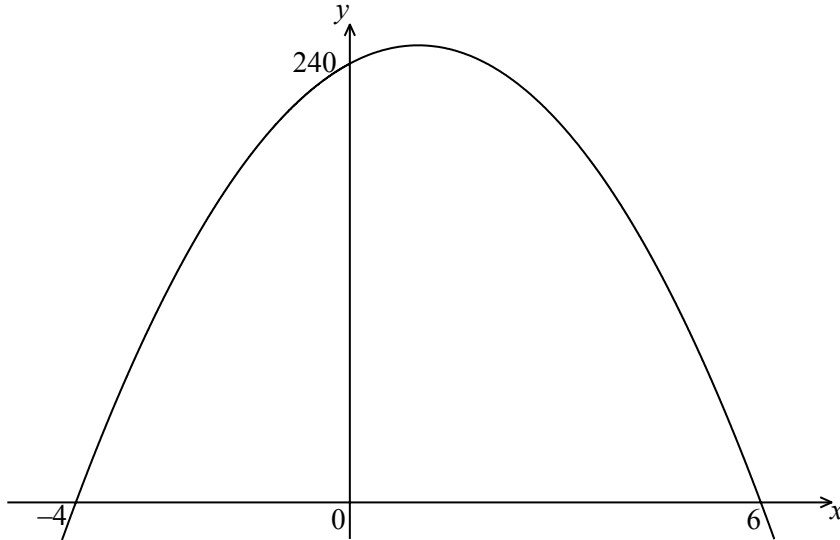
The x-intercepts are at $(-4, 0)$ and $(6, 0)$, and the y-intercept is at $(0, 240)$.

(a)   Write down $f(x)$ in the form $f(x) = -10(x - p)(x - q)$.   *[2 marks]*

(b)   Find another expression for $f(x)$ in the form $f(x) = -10(x - h)^2 + k$.   *[4 marks]*

(c)   Show that $f(x)$ can also be written in the form $f(x) = 240 + 20x - 10x^2$.   *[2 marks]*

A particle moves along a straight line so that its velocity, $v\ \mathrm{m\,s}^{-1}$, at time t seconds is given by $v = 240 + 20t - 10t^2$, for $0 \leq t \leq 6$.
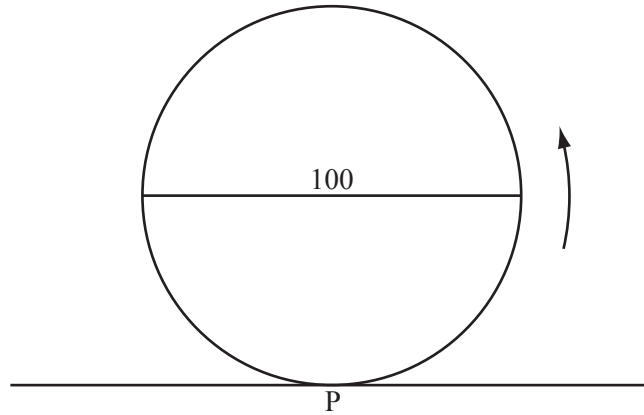
(d)   (i)   Find the value of t when the speed of the particle is greatest.

      (ii)   Find the acceleration of the particle when its speed is zero.   *[7 marks]*

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

10.   *[Maximum mark: 14]*

The following diagram represents a large Ferris wheel, with a diameter of 100 metres.

Let P be a point on the wheel.   The wheel starts with P at the lowest point, at ground level.   The wheel rotates at a constant rate, in an anticlockwise (counterclockwise) direction.   One revolution takes 20 minutes.

(a)   Write down the height of P above ground level after

   (i)   10 minutes;

   (ii)   15 minutes.   *[2 marks]*

Let $h(t)$ metres be the height of P above ground level after t minutes.   Some values of $h(t)$ are given in the table below.

t	$h(t)$
0	0.0
1	2.4
2	9.5
3	20.6
4	34.5
5	50.0

(b)   (i)   Show that $h(8) = 90.5$.

   (ii)   Find $h(21)$.   *[4 marks]*

(c)   **Sketch** the graph of h, for $0 \le t \le 40$.   *[3 marks]*

(d)   Given that h can be expressed in the form $h(t) = a\cos bt + c$, find a, b and c.   *[5 marks]*

Please **do not** write on this page.

Answers written on this page will
not be marked.