



22117405

**MATHEMATICAL STUDIES
STANDARD LEVEL
PAPER 1**

Wednesday 4 May 2011 (afternoon)

1 hour 30 minutes

Candidate session number

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Examination code

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Write your answers in the boxes provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.



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Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Write your answers in the answer boxes provided. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

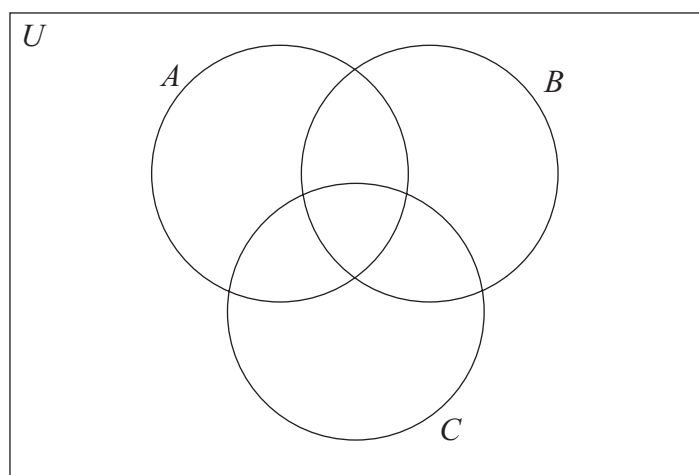
1. U is the set of all the **positive** integers less than or equal to 12.
 A , B and C are subsets of U .

$$A = \{1, 2, 3, 4, 6, 12\}$$

$$B = \{\text{odd integers}\}$$

$$C = \{5, 6, 8\}$$

- (a) Write down the number of elements in $A \cap C$. [1 mark]
- (b) List the elements of B . [1 mark]
- (c) Complete the following Venn diagram with **all** the elements of U . [4 marks]



Working:

Answers:

- (a)
- (b)



2. 31 pupils in a class were asked to estimate the number of sweets in a jar. The following stem and leaf diagram gives their estimates.

Stem	Leaf	Key: 4 7 represents 47 sweets
4	2, 4, 7, 8, 9	
5	1, 1, 2, 3, 8, 9	
6	0, 2, 2, 4, 6, 6, 7, 8, 8	
7	0, 0, 1, 3, 4, 5, 5, 7	
8	1, 2, 2	

(a) For the pupils' estimates, write down

(i) the median;

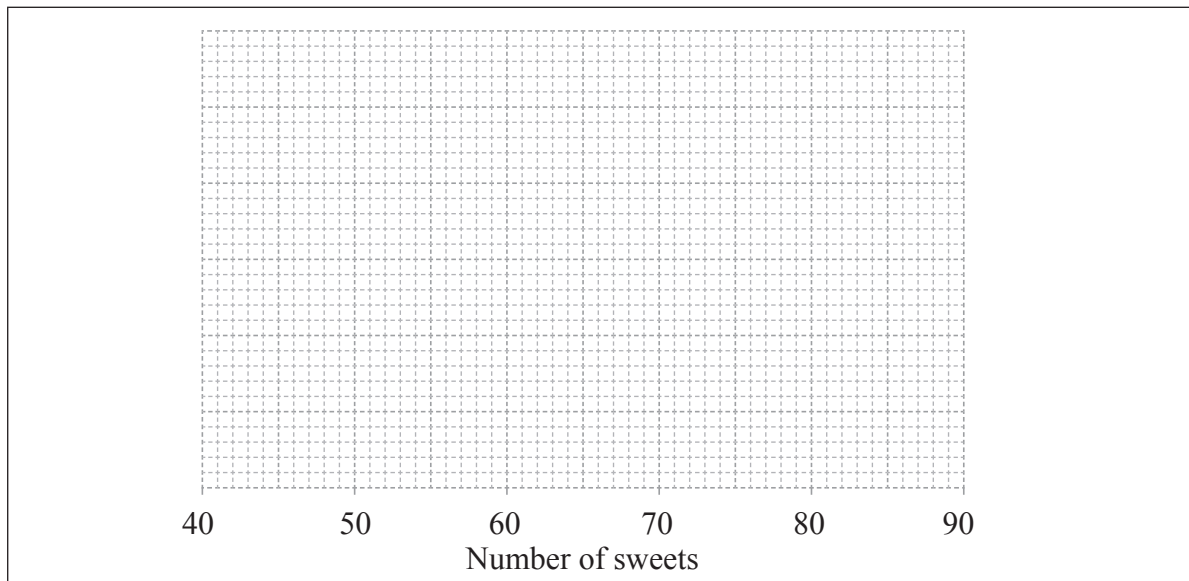
(ii) the lower quartile;

(iii) the upper quartile.

[3 marks]

(b) Draw a box and whisker plot of the pupils' estimates using the grid below.

[3 marks]



Working:

Answers:

- (a) (i)
- (ii)
- (iii)



3. In a particular school, students must choose at least one of three optional subjects: art, psychology or history.

Consider the following propositions

a: I choose art,
p: I choose psychology,
h: I choose history.

- (a) Write, in words, the compound proposition

$$\neg h \wedge (p \vee a). \quad [3 \text{ marks}]$$

- (b) Complete the truth table for $\neg a \Rightarrow p$. [1 mark]

<i>a</i>	<i>p</i>	$\neg a$	$\neg a \Rightarrow p$
T	T	F	
T	F	F	
F	T	T	
F	F	T	

- (c) State whether $\neg a \Rightarrow p$ is a tautology, a contradiction **or** neither. Justify your answer. [2 marks]

Working:

Answers:

- (a)
-
- (c)
-



4. The planet Earth takes one year to revolve around the Sun. Assume that a year is 365 days and the path of the Earth around the Sun is the circumference of a circle of radius 150 000 000 km.

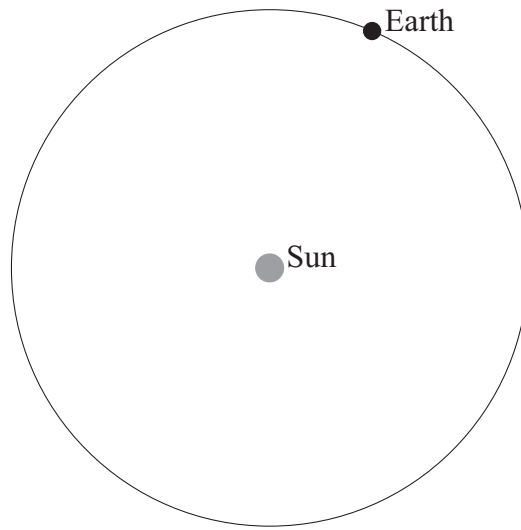


diagram not to scale

- (a) Calculate the distance travelled by the Earth in **one day**. *[4 marks]*
- (b) Give your answer to part (a) in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$. *[2 marks]*

Working:

Answers:

- (a)
- (b)



5. The straight line L passes through the points $A(-1, 4)$ and $B(5, 8)$.

(a) Calculate the gradient of L . [2 marks]

(b) Find the equation of L . [2 marks]

The line L also passes through the point $P(8, y)$.

(c) Find the value of y . [2 marks]

Working:

Answers:

(a)

(b)

(c)



6. 80 matches were played in a football tournament. The following table shows the number of goals scored in all matches.

Number of goals	0	1	2	3	4	5
Number of matches	16	22	19	17	1	5

(a) Find the mean number of goals scored per match. *[2 marks]*

(b) Find the median number of goals scored per match. *[2 marks]*

A local newspaper claims that the mean number of goals scored per match is two.

(c) Calculate the percentage error in the local newspaper's claim. *[2 marks]*

Working:

Answers:

- (a)
- (b)
- (c)



7. 10 000 people attended a sports match. Let x be the number of adults attending the sports match and y be the number of children attending the sports match.

(a) Write down an equation in x and y . *[1 mark]*

The cost of an adult ticket was 12 Australian dollars (AUD). The cost of a child ticket was 5 Australian dollars (AUD).

(b) Find the total cost for a family of 2 adults and 3 children. *[2 marks]*

The total cost of tickets sold for the sports match was 108 800 AUD.

(c) Write down a second equation in x and y . *[1 mark]*

(d) Write down the value of x and the value of y . *[2 marks]*

Working:

Answers:

- (a)
- (b)
- (c)
- (d)



8. 75 metal spherical cannon balls, each of diameter 10 cm, were excavated from a Napoleonic War battlefield.

(a) Calculate the total volume of all 75 metal cannon balls excavated. *[3 marks]*

The cannon balls are to be melted down to form a sculpture in the shape of a cone. The base radius of the cone is 20 cm.

(b) Calculate the height of the cone, assuming that no metal is wasted. *[3 marks]*

Working:

Answers:

(a)

(b)



9. A survey was carried out at an international airport. A number of travellers were interviewed and asked for their flight destinations. The results are shown in the table below.

Destination	America	Africa	Asia
Number of males	45	62	37
Number of females	35	46	25

One traveller is to be chosen at random from all those interviewed.

- (a) Find the probability that this traveller was going to Africa. [2 marks]

One female traveller is to be chosen at random from all those interviewed.

- (b) Find the probability that this female traveller was going to Asia. [2 marks]

One traveller is to be chosen at random from those **not** going to America.

- (c) Find the probability that the chosen traveller is female. [2 marks]

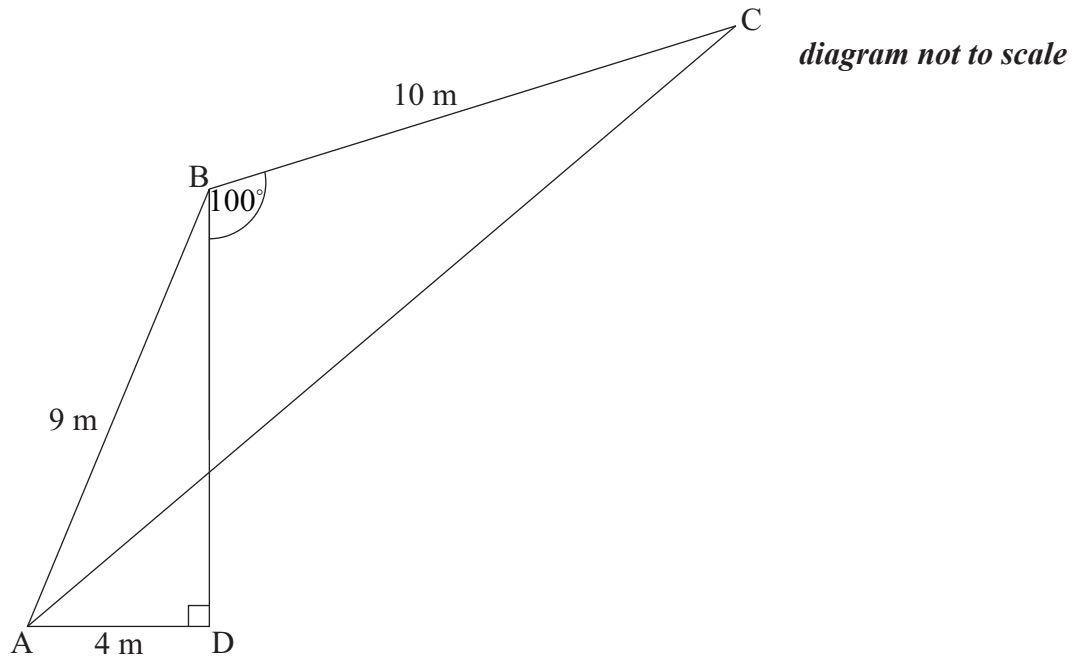
Working:

Answers:

- (a)
- (b)
- (c)



10. In the diagram, $AD = 4\text{ m}$, $AB = 9\text{ m}$, $BC = 10\text{ m}$, $\hat{BDA} = 90^\circ$ and $\hat{DBC} = 100^\circ$.



- (a) Calculate the size of \hat{ABC} . [3 marks]
- (b) Calculate the length of AC. [3 marks]

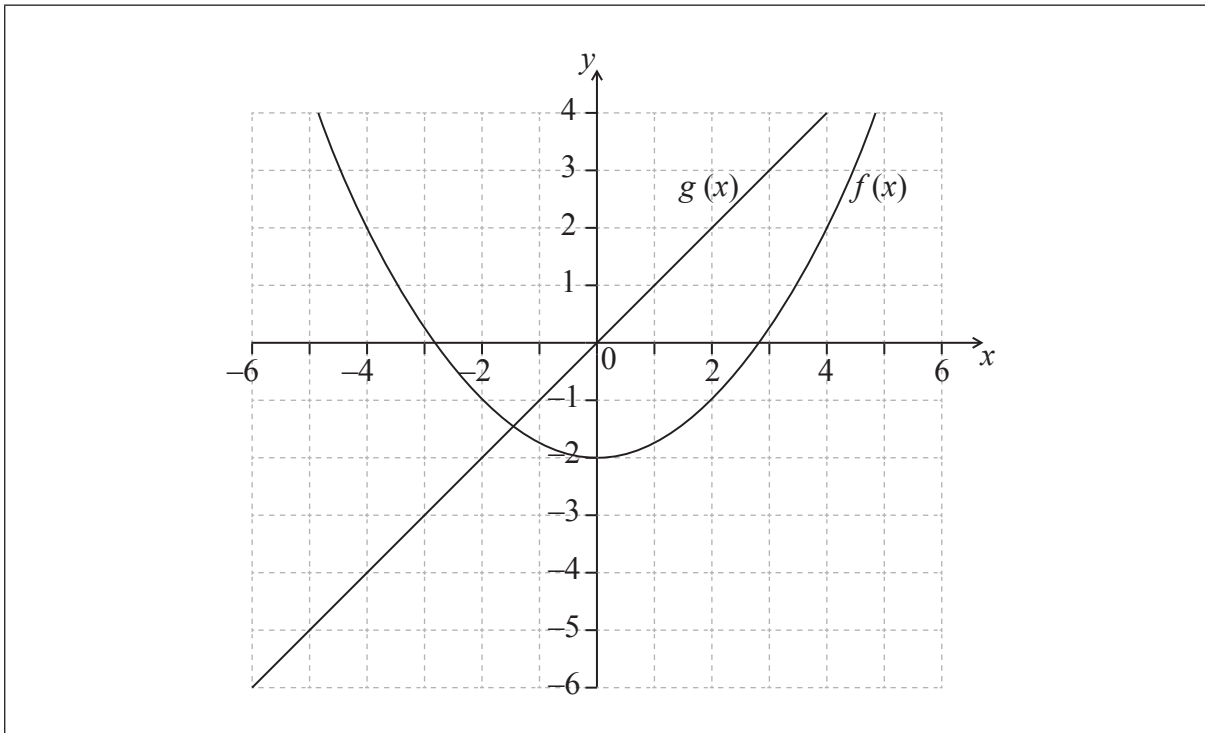
Working:

Answers:

- (a)
- (b)



11. The figure shows the graphs of the functions $f(x) = \frac{1}{4}x^2 - 2$ and $g(x) = x$.



- (a) Differentiate $f(x)$ with respect to x . [1 mark]
- (b) Differentiate $g(x)$ with respect to x . [1 mark]
- (c) Calculate the value of x for which the gradients of the two graphs are the same. [2 marks]
- (d) Draw the tangent to the parabola at the point with the value of x found in part (c). [2 marks]

Working:

Answers:

- (a)
- (b)
- (c)



12. A manufacturer in England makes 16 000 garden statues. 12 % are defective and cannot be sold.

(a) Find the number of statues that are non-defective.

[2 marks]

The manufacturer sells each non-defective statue for 5.25 British pounds (GBP) to an American company. The exchange rate from GBP to US dollars (USD) is 1 GBP = 1.6407 USD.

(b) Calculate the amount in USD paid by the American company for all the non-defective statues. Give your answer correct to **two decimal places**.

[2 marks]

The American company sells one of the statues to an Australian customer for 12 USD. The exchange rate from Australian dollars (AUD) to USD is 1 AUD = 0.8739 USD.

(c) Calculate the amount that the Australian customer pays, in AUD, for this statue. Give your answer correct to **two decimal places**.

[2 marks]

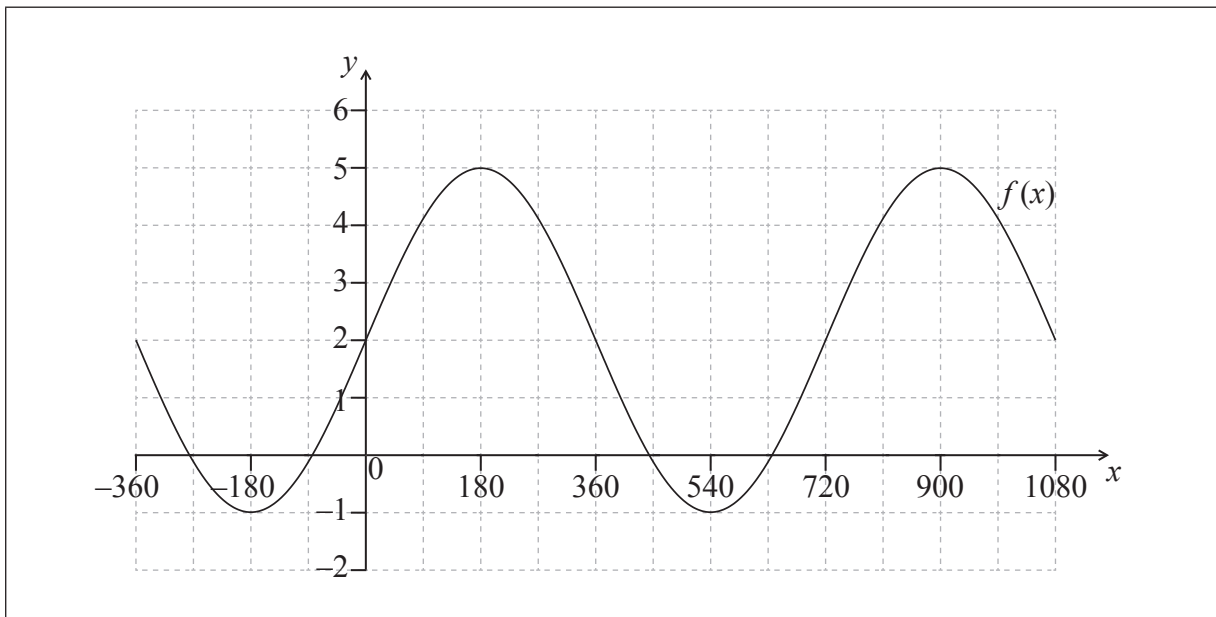
Working:

Answers:

- (a)
- (b)
- (c)



13. The graph of the function $f(x) = a \sin(bx) + c$ is shown below for $-360^\circ \leq x \leq 1080^\circ$.



- (a) Write down the period of $f(x)$. [1 mark]
- (b) Write down the value of
 - (i) a ;
 - (ii) b ;
 - (iii) c . [3 marks]

P is one of the points where the graph $y = f(x)$ intersects the x -axis. The x -coordinate of P lies between -180° and 180° .

- (c) (i) Mark and label the point P on the graph above.
- (ii) **Estimate** the x -coordinate of P . [2 marks]

Working:

Answers:

- (a)
- (b) (i)
- (ii)
- (iii)
- (c) (ii)



14. Shiyun bought a car in 1999. The value of the car V , in USD, is depreciating according to the exponential model

$$V = 25\,000 \times 1.5^{-0.2t}, \quad t \geq 0,$$

where t is the time, in years, that Shiyun has owned the car.

- (a) Write down the value of the car when Shiyun bought it. *[1 mark]*

- (b) Calculate the value of the car three years after Shiyun bought it. Give your answer correct to **two decimal places**. *[2 marks]*

- (c) Calculate the time for the car to depreciate to half of its value since Shiyun bought it. *[3 marks]*

Working:

Answers:

- (a)
- (b)
- (c)



15. In the diagram, $\hat{BAC} = 90^\circ$. The length of the three sides are x cm, $(x+7)$ cm and $(x+8)$ cm.

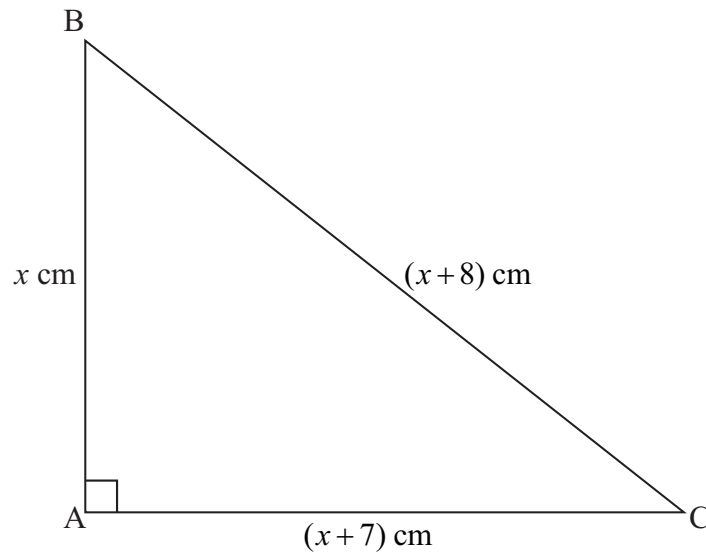


diagram not to scale

- (a) Write down and **simplify** a quadratic equation in x which links the three sides of the triangle. [3 marks]
- (b) Solve the quadratic equation found in part (a). [2 marks]
- (c) Write down the value of the perimeter of the triangle. [1 mark]

Working:

Answers:

- (a)
- (b)
- (c)

