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MATHEMATICAL STUDIES
STANDARD LEVEL
PAPER 2

Friday 8 May 2009 (morning)

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Please start each question on a new page. You are advised to show all working, where possible. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. *[Maximum mark: 12]*

A survey of 100 families was carried out, asking about the pets they own. The results are given below.

56 owned dogs *(S)*
38 owned cats *(Q)*
22 owned birds *(R)*
16 owned dogs and cats, but not birds
8 owned birds and cats, but not dogs
3 owned dogs and birds, but not cats
4 owned all three types of pets

(a) Draw a Venn diagram to represent this information. *[5 marks]*

(b) Find the number of families who own no pets. *[2 marks]*

(c) Find the percentage of families that own exactly one pet. *[3 marks]*

(d) A family is chosen at random. Find the probability that they own a cat, given that they own a bird. *[2 marks]*

2. *[Maximum mark: 22]*

(i) A manufacturer claims that fertilizer has an effect on the height of rice plants. He measures the height of fertilized and unfertilized plants. The results are given in the following table.

Plant height	Fertilized plants	Unfertilized plants
>75 cm	115	80
50 – 75 cm	45	65
<50 cm	20	35

A chi-squared test is performed to decide if the manufacturer's claim is justified at the **1 %** level of significance.

(a) Write down the null and alternate hypotheses for this test. *[2 marks]*

(b) For the number of fertilized plants with height greater than 75 cm, show that the expected value is 97.5. *[3 marks]*

(c) Write down the value of χ^2_{calc}. *[2 marks]*

(d) Write down the number of degrees of freedom. *[1 mark]*

(e) Write down the critical value of χ^2, at the **1 %** level of significance. *[1 mark]*

(f) Is the manufacturer's claim justified? Give a reason for your answer. *[2 marks]*

(This question continues on the following page)

(Question 2 continued)

(ii) The population of fleas on a dog after t days, is modelled by

$$N = 4 \times (2)^{\frac{t}{4}}, t \geq 0$$

Some values of N are shown in the table below.

t	0	4	8	12	16	20
N	p	8	16	32	q	128

(a) Write down the value of

(i) p;

(ii) q. *[3 marks]*

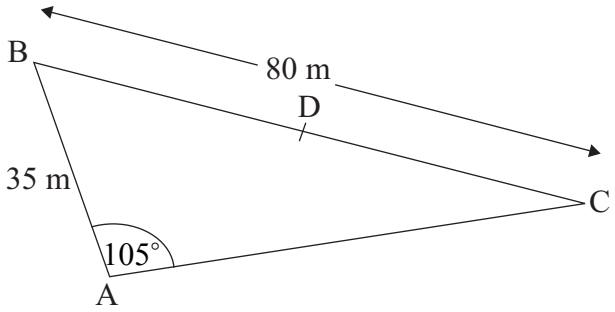
(b) Using the values in the table above, draw the graph of N for $0 \leq t \leq 20$. Use 1 cm to represent 2 days on the horizontal axis and 1 cm to represent 10 fleas on the vertical axis. *[6 marks]*

(c) **Use your graph** to estimate the number of days for the population of fleas to reach 55. *[2 marks]*

3. *[Maximum mark: 18]*

A farmer has a triangular field, ABC, as shown in the diagram.
AB = 35 m, BC = 80 m and $B\hat{A}C = 105°$, and D is the midpoint of BC.

diagram not to scale



(a) Find the size of $B\hat{C}A$. *[3 marks]*

(b) Calculate the length of AD. *[5 marks]*

The farmer wants to build a fence around ABD.

(c) Calculate the total length of the fence. *[2 marks]*

(d) The farmer pays 802.50 USD for the fence. Find the cost per metre. *[2 marks]*

(e) Calculate the area of the triangle ABD. *[3 marks]*

(f) A layer of earth 3 cm thick is removed from ABD. Find the volume removed in cubic metres. *[3 marks]*

4. *[Maximum mark: 19]*

(i) A geometric sequence has second term 12 and fifth term 324.

(a) Calculate the value of the common ratio. *[4 marks]*

(b) Calculate the 10^{th} term of this sequence. *[3 marks]*

(c) The k^{th} term is the first term which is greater than 2000. Find the value of k. *[3 marks]*

(ii) Consider the following propositions

p: The number is a multiple of five.
q: The number is even.
r: The number ends in zero.

(a) Write in words $(q \wedge \neg r) \Rightarrow \neg p$ *[3 marks]*

(b) Consider the statement "If the number is a multiple of five, and is not even then it will not end in zero".

(i) Write this statement in symbolic form.

(ii) Write the contrapositive of this statement in symbolic form. *[6 marks]*

5. *[Maximum mark: 19]*

A function is defined by $f(x) = \frac{5}{x^2} + 3x + c,\ x \neq 0,\ c \in \mathbb{Z}$.

(a) Write down an expression for $f'(x)$. *[4 marks]*

Consider the graph of f. The graph of f passes through the point P(1, 4).

(b) Find the value of c. *[2 marks]*

(c) There is a local minimum at the point Q.

(i) Find the coordinates of Q.

(ii) Find the set of values of x for which the function is decreasing. *[7 marks]*

Let T be the tangent to the graph of f at P.

(d) (i) Show that the gradient of T is –7.

(ii) Find the equation of T. *[4 marks]*

(e) T intersects the graph again at R. Use your graphic display calculator to find the coordinates of R. *[2 marks]*