## MATHEMATICAL STUDIES <br> STANDARD LEVEL <br> PAPER 1

Thursday 7 May 2009 (afternoon)
1 hour 30 minutes
Candidate session number


## INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Working may be continued below the box, if necessary. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. A rectangle is 2680 cm long and 1970 cm wide.
(a) Find the perimeter of the rectangle, giving your answer in the form $a \times 10^{k}$, where $1 \leq a<10$ and $k \in \mathbb{Z}$.
(b) Find the area of the rectangle, giving your answer correct to the nearest thousand square centimetres.

## Working:

2. A plumber in Australia charges 90 AUD per hour for work, plus a fixed cost. His total charge is represented by the cost function $C=60+90 t$, where $t$ is in hours.
(a) Write down the fixed cost. [1 mark]
(b) It takes $3 \frac{1}{2}$ hours to complete a job for Paula. Find the total cost. [2 marks]
(c) Steve received a bill for 510 AUD. Calculate the time it took the plumber to complete the job.

Working:
3. The following table shows the number of errors per page in a 100 page document.

| Number of errors | 0 | 1 | 2 | 3 | 4 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Number of pages | 28 | 24 | 20 | 17 | 11 |

(a) State whether the data is discrete, continuous or neither.
[1 mark]
(b) Find the mean number of errors per page.
(c) Find the median number of errors per page.
(d) Write down the mode.

## Working:

Answers:
(a)
(b)
(c)
(d) $\qquad$
4. Consider the following sequence:

$$
57,55,53 \ldots, 5,3
$$

(a) Find the number of terms of the sequence.
(b) Find the sum of the sequence.

Working:

Answers:
(a)
(b)
5. The following diagram shows the graph of $y=a \cos b x$ for $0^{\circ} \leq x \leq 240^{\circ}$.

(a) Write down
(i) the period;
(ii) the amplitude.
(b) Find the value of
(i) $a$;
(ii) $b$.

Working:

Answers:
(a) (i) $\qquad$
(ii)
(b) (i) $\qquad$
(ii) $\qquad$
6. A shop keeper recorded daily sales $s$ of ice cream along with the daily maximum temperature $t^{\circ} \mathrm{C}$. The results for one week are shown below.

| $t$ | 29 | 31 | 34 | 23 | 19 | 20 | 27 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $s$ | 104 | 92 | 112 | 48 | 56 | 72 | 66 |

(a) Write down the equation of the regression line for $s$ on $t$.
(b) Use your equation to predict the ice cream sales on a day when the maximum temperature is $24^{\circ} \mathrm{C}$. Give your answer correct to the nearest whole number.

## Working:

7. The diagram below shows the line PQ , whose equation is $x+2 y=12$. The line intercepts the axes at P and Q respectively.

(a) Find the coordinates of P and of Q .
(b) A second line with equation $x-y=3$ intersects the line PQ at the point A . Find the coordinates of A.

## Working:

8. Inge borrows $€ 4500$ for 2 years.
(a) Bank 1 charges compound interest at a rate of $15 \%$ per annum, compounded quarterly.

Calculate the total amount to be repaid at the end of the 2 years. Give your answer correct to two decimal places.
(b) Bank 2 charges simple interest, and Inge would repay the same total amount as for Bank 1. Find the rate of interest charged by Bank 2.

Working:
9. The coordinates of the vertices of a triangle ABC are $\mathrm{A}(4,3), \mathrm{B}(7,-3)$ and $\mathrm{C}(0.5, p)$.
(a) Calculate the gradient of the line AB .
(b) Given that the line AC is perpendicular to the line AB
(i) write down the gradient of the line AC ;
(ii) find the value of $p$.

## Working:

Answers:
(a)
(b) (i)
(ii) $\qquad$
10. For events $A$ and $B$, the probabilities are $\mathrm{P}(A)=\frac{4}{13}$ and $\mathrm{P}(B)=\frac{5}{13}$.
(a) If events $A$ and $B$ are mutually exclusive, write down the value of $\mathrm{P}(A \cap B)$.
(b) If events $A$ and $B$ are independent, find the value of $\mathrm{P}(A \cap B)$.
(c) If $\mathrm{P}(A \cup B)=\frac{7}{13}$, find the value of $\mathrm{P}(A \cap B)$.

## Working:

Answers:
(a)
(b)
(c) $\qquad$
11. Consider $f: x \mapsto x^{2}-4$.
(a) Find $f^{\prime}(x)$.
[1 mark]

Let $L$ be the line with equation $y=3 x+2$.
(b) Write down the gradient of a line parallel to $L$.
(c) Let P be a point on the curve of $f$. At P , the tangent to the curve is parallel to $L$. Find the coordinates of P .

## Working:

Answers:
(a)
(b)
(c)
12. The following table gives the exchange rate from US dollars to euros and from US dollars to Japanese Yen. Give all answers in this question correct to two decimal places.

| 1 USD | 0.6337 EUROS |
| :--- | :--- |
| 1 USD | 99.7469 YEN |

(a) Enrico has 475 USD.
(i) How many euros is this worth?

Enrico goes to a bank to exchange his dollars. The bank charges $3 \%$ commission.
(ii) How many euros does Enrico receive?
(b) Find the exchange rate from euros to yen.

## Working:

Answers:
(a) (i)
(ii)
(b) $\qquad$
13. A weighted die has 2 red faces, 3 green faces and 1 black face. When the die is thrown, the black face is three times as likely to appear on top as one of the other five faces. The other five faces have equal probability of appearing on top.

The following table gives the probabilities.

| Red 1 | Red 2 | Green 1 | Green 2 | Green 3 | Black |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{m}{8}$ | $\frac{1}{8}$ | $\frac{n}{8}$ |

(a) Find the value of
(i) $m$;
(ii) $n$.

The die is thrown once.
(b) Given that the face on top is not red, find the probability that it is black.

The die is now thrown twice.
(c) Calculate the probability that black appears on top both times.

## Working:

Answers:
(a) (i)
(ii)
(b)
(c) $\qquad$
14. A quadratic curve with equation $y=a x(x-b)$ is shown in the following diagram.


The $x$-intercepts are at $(0,0)$ and $(6,0)$, and the vertex $V$ is at $(h, 8)$.
(a) Find the value of $h$.
(b) Find the equation of the curve.

## Working:

Answers:
(a)
(b)
15. The right pyramid shown in the diagram has a square base with sides of length 40 cm . The height of the pyramid is also 40 cm .

diagram not to scale
(a) Find the length of OB .
(b) Find the size of angle OBP.

## Working:

Answers:
(a)
(b)

