22097403

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MATHEMATICAL STUDIES
STANDARD LEVEL
PAPER 1

Thursday 7 May 2009 (afternoon)

1 hour 30 minutes

Candidate session number

0	0								

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Working may be continued below the box, if necessary. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. Eight houses in a street are inhabited by different numbers of people, as shown in the table below.

House	A	B	C	D	E	F	G	H
Number of inhabitants	5	4	7	6	4	3	6	4

(a) The following statements refer to the number of inhabitants per house. Write down true (T) or false (F) for each.

　(i)　The mean is 5.

　(ii)　The range is 4.

　(iii)　The mode is 6.

　(iv)　The standard deviation is 1.4 correct to 2 significant figures.　　*[4 marks]*

(b) Calculate the interquartile range for the number of inhabitants per house.　　*[2 marks]*

Working:

Answers:

(a)　(i)　_____

　　(ii)　_____

　　(iii)　_____

　　(iv)　_____

(b)　_____

2.   Consider the two propositions p and q.

p: The sun is shining q: I will go swimming

Write in words the compound propositions

(a) $p \Rightarrow q$;   *[2 marks]*

(b) $\neg p \vee q$.   *[2 marks]*

The truth table for these compound propositions is given below.

p	q	$p \Rightarrow q$	$\neg p$	$\neg p \vee q$
T	T	T		T
T	F	F		F
F	T	T		T
F	F	T		T

(c)   Complete the column for $\neg p$.   *[1 mark]*

(d)   State the relationship between the compound propositions $p \Rightarrow q$ and $\neg p \vee q$.   *[1 mark]*

Working:

Answers:

(a) _____

_____

(b) _____

_____

(d) _____

0316

3.   A tree begins losing its leaves in October. The number of leaves that the tree loses each day increases by the same number on each successive day.

Date in October	1	2	3	4	....
Number of leaves lost	24	40	56	72	....

(a)   Calculate the number of leaves that the tree loses on the 21st October.   *[3 marks]*

(b)   Find the total number of leaves that the tree loses in the 31 days of the month of October.   *[3 marks]*

Working:

Answers:

(a) _____

(b) _____

4. A and B are points on a straight line as shown on the graph below.
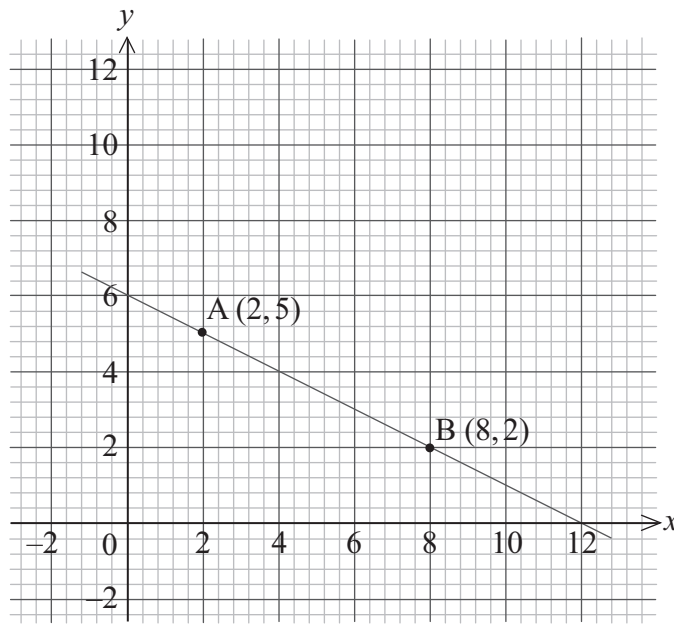
(a) Write down the *y*-intercept of the line AB. *[1 mark]*

(b) Calculate the gradient of the line AB. *[2 marks]*

The acute angle between the line AB and the *x*-axis is θ.

(c) Show θ on the diagram. *[1 mark]*

(d) Calculate the size of θ. *[2 marks]*

Working:

Answers:

(a) _____

(b) _____

(d) _____

0516

Turn over

5. Let $f(x) = x^2 - 6x + 8$.

(a) Factorise $x^2 - 6x + 8$. *[2 marks]*

(b) Hence, or otherwise, solve the equation $x^2 - 6x + 8 = 0$. *[2 marks]*

Let $g(x) = x + 3$.

(c) Write down the solutions to the equation $f(x) = g(x)$. *[2 marks]*

Working:

Answers:

(a) _____

(b) _____

(c) _____

0616

6. A questionnaire was given to all members of a school community to find out which drink was the most popular to have with breakfast. The results are given in the table below, classified by age.

	Hot Chocolate	Tea	Coffee	Milk
Children aged 12 years and less	55	10	1	34
Teenagers aged from 13 to 19 years	25	35	20	10
Adults aged 20 years and over	20	40	79	6

A χ^2 test was conducted to decide whether the type of drink was independent of age.

(a) Find the number of degrees of freedom for the χ^2 test. *[2 marks]*

(b) Write down the null hypothesis for the χ^2 test. *[1 mark]*

(c) Write down the critical value for the χ^2 test at the 5 % significance level. *[1 mark]*

The χ^2 test statistic is calculated to be 146 with a *p*-value of 6.62×10^{-29} (both numbers given correct to 3 significant figures).

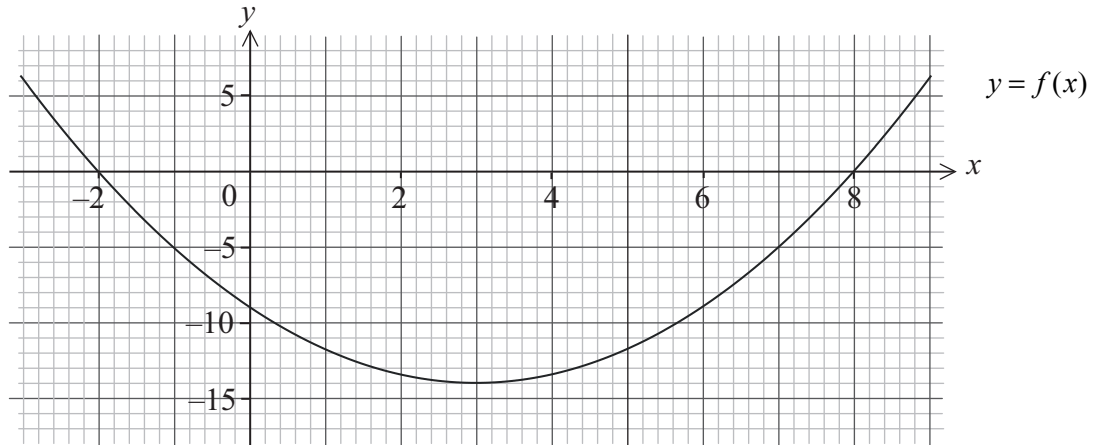(d) Write down the conclusion reached at the 5 % significance level. Give a clear reason for your answer. *[2 marks]*

Working:

Answers:

(a) _____

(b) _____

_____

(c) _____

(d) _____

_____

0716

Turn over

7.   The graph of a quadratic function $y = f(x)$ is given below.

(a)   Write down the equation of the axis of symmetry.   *[2 marks]*

(b)   Write down the coordinates of the minimum point.   *[2 marks]*

(c)   Write down the range of $f(x)$.   *[2 marks]*

Working:

Answers:

(a)   _____

(b)   _____

(c)   _____

8.   The annual fees paid to a school for the school years 2000, 2001 and 2002 increase as a geometric progression. The table below shows the fee structure.

Year	Fees (USD)
2000	8000.00
2001	8320.00
2002	8652.80

(a)   Calculate the common ratio for the increasing sequence of fees.   *[2 marks]*

In parts (b) and (c) give your answer correct to 2 decimal places.

The fees continue to increase in the same ratio.

(b)   Find the fees paid for 2006.   *[2 marks]*

A student attends the school for eight years, starting in 2000.

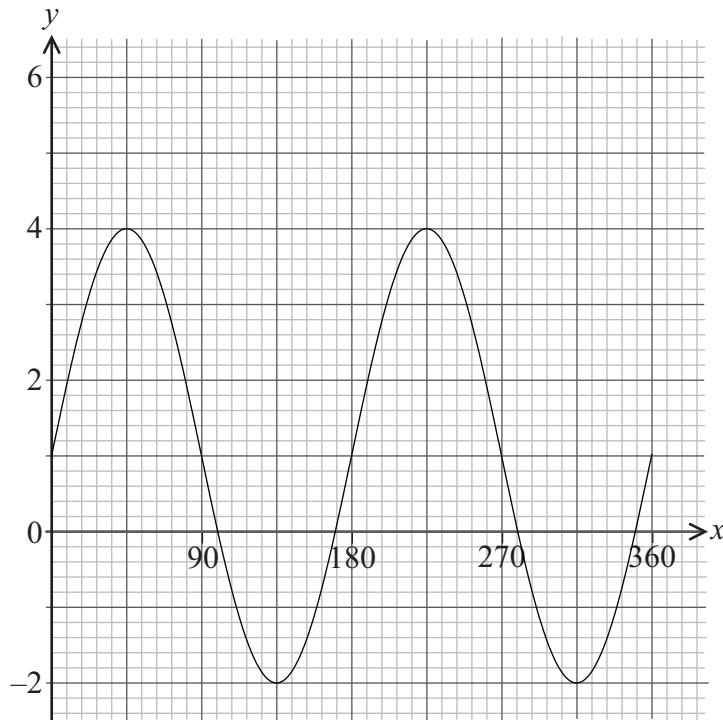(c)   Find the **total** fees paid for these eight years.   *[2 marks]*

Working:

Answers:

(a) _____

(b) _____

(c) _____

9. The graph of the function $f(x) = a\sin(bx) + c$ is shown below for $0° \le x \le 360°$.

(a) Write down the period of $f(x)$. *[1 mark]*

(b) Write down the value of

(i) a;

(ii) b;

(iii) c. *[3 marks]*

(c) Find the number of solutions to the equation $f(x) = 3$ in the interval $90° \le x \le 270°$. *[2 marks]*

Working:

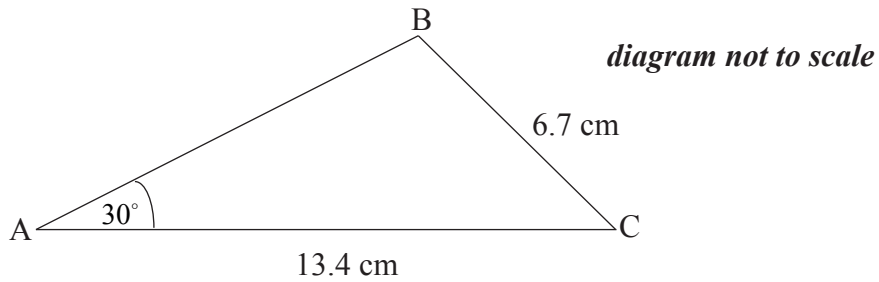*Answers:*

(a) _____

(b) (i) _____

(ii) _____

(iii) _____

(c) _____

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10. The diagram shows triangle ABC in which angle $BAC = 30°$, $BC = 6.7$ cm and $AC = 13.4$ cm.

diagram not to scale

(a) Calculate the size of angle ACB. *[4 marks]*

Nadia makes an accurate drawing of triangle ABC. She measures angle BAC and finds it to be $29°$.

(b) Calculate the percentage error in Nadia's measurement of angle BAC. *[2 marks]*

Working:

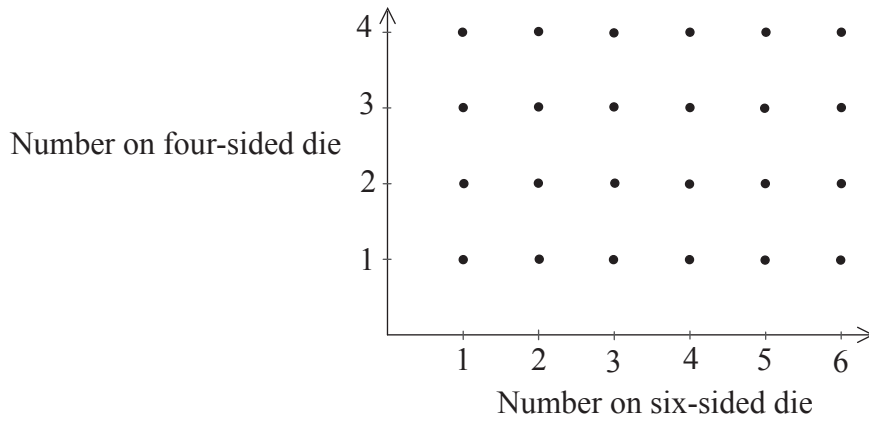*Answers:*

(a) _____

(b) _____

11. A fair six-sided die has the numbers 1, 2, 3, 4, 5, 6 written on its faces. A fair four-sided die has the numbers 1, 2, 3, and 4 written on its faces. The two dice are rolled.

The following diagram shows the possible outcomes.

(a) Find the probability that the two dice show the same number. *[2 marks]*

(b) Find the probability that the difference between the two numbers shown on the dice is 1. *[2 marks]*

(c) Find the probability that the number shown on the four-sided die is greater than the number shown on the six-sided die, given that the difference between the two numbers is 1. *[2 marks]*

Working:

Answers:

(a) _____

(b) _____

(c) _____

12. An amount, *C*, of Australian Dollars (AUD) is invested for 5 years at 2.5 % yearly simple interest. The interest earned on this investment is 446.25 AUD.

(a) Calculate the value of *C*.   *[2 marks]*

5000 AUD is invested at a nominal annual interest rate of 2.5 % **compounded half yearly**.

(b) Calculate the length of time in years for the interest on this investment to exceed 446.25 AUD.   *[4 marks]*

Working:

Answers:

(a) _____

(b) _____

13. Tennis balls are sold in cylindrical tubes that contain four balls. The radius of each tennis ball is 3.15 cm and the radius of the tube is 3.2 cm. The length of the tube is 26 cm.

   (a)   Find the volume of one tennis ball.   *[2 marks]*

   (b)   Calculate the volume of the empty space in the tube when four tennis balls have been placed in it.   *[4 marks]*

Working:

Answers:

(a) _____

(b) _____

14. The number of cells, C, in a culture is given by the equation $C = p \times 2^{0.5t} + q$, where t is the time in hours measured from 12:00 on Monday and p and q are constants.

The number of cells in the culture at 12:00 on Monday is 47.
The number of cells in the culture at 16:00 on Monday is 53.

Use the above information to

(a)   write down two equations in p and q ;   *[2 marks]*

(b)   calculate the value of p and of q ;   *[2 marks]*

(c)   find the number of cells in the culture at 22:00 on Monday.   *[2 marks]*

Working:

Answers:

(a)   _____

      _____

(b)   _____

(c)   _____

15. The straight line, *L*, has equation $2y - 27x - 9 = 0$.

    (a)   Find the gradient of *L*.   *[2 marks]*

Sarah wishes to draw the tangent to $f(x) = x^4$ parallel to *L*.

    (b)   Write down $f'(x)$.   *[1 mark]*

    (c)   (i)   Find the *x* coordinate of the point at which the tangent must be drawn.

            (ii)   Write down the value of $f(x)$ at this point.   *[3 marks]*

Working:

Answers:

(a) _____

(b) _____

(c) (i) _____

      (ii) _____

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