22097306

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MATHEMATICS
STANDARD LEVEL
PAPER 2

Friday 8 May 2009 (morning)

1 hour 30 minutes

Candidate session number

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

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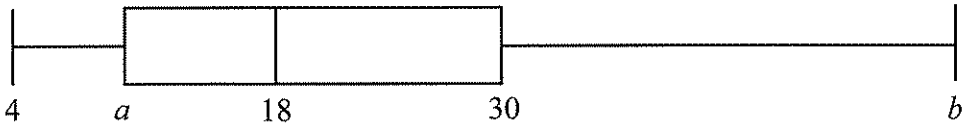
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 5]*

The following diagram is a box and whisker plot for a set of data.



The interquartile range is 20 and the range is 40.

(a) Write down the median value. *[1 mark]*

(b) Find the value of

(i) a;

(ii) b. *[4 marks]*

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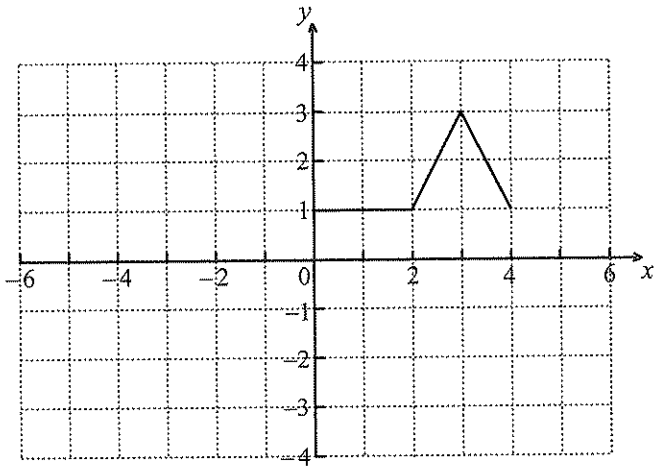
2. *[Maximum mark: 6]*

Consider the graph of f shown below.



(a) On the **same** grid sketch the graph of $y = f(-x)$. *[2 marks]*

(This question continues on the following page)

(Question 2 continued)

The following four diagrams show **images** of f under different transformations.

Diagram A

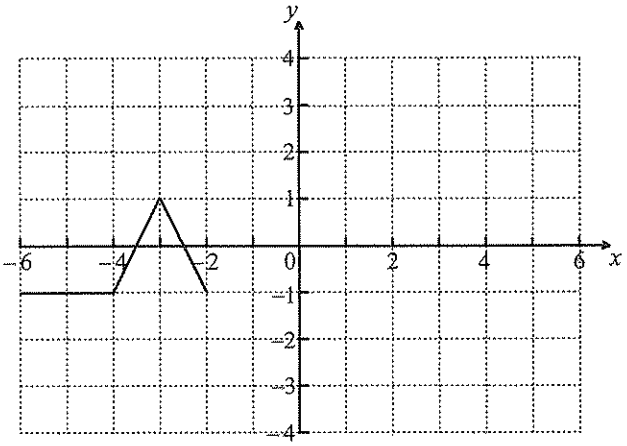


Diagram B

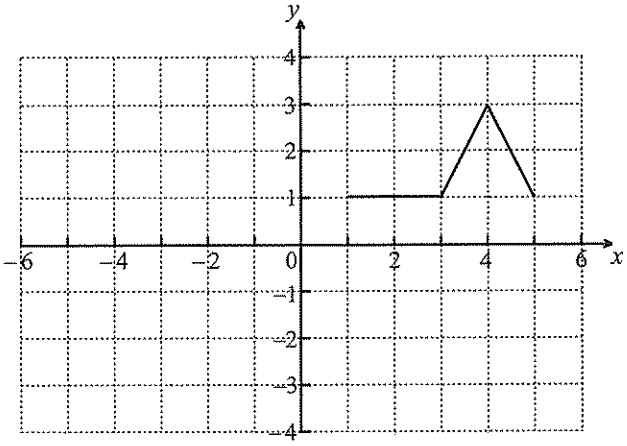


Diagram C

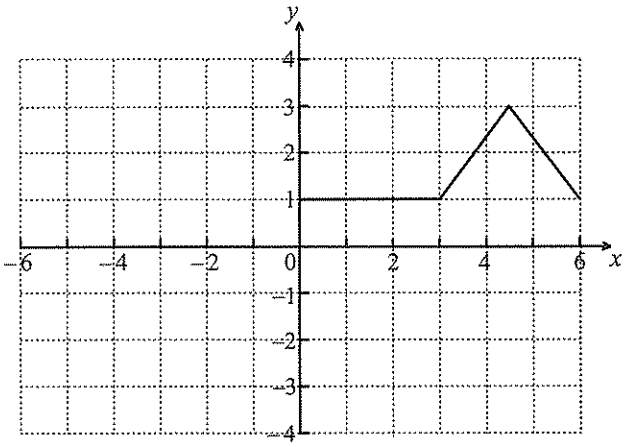
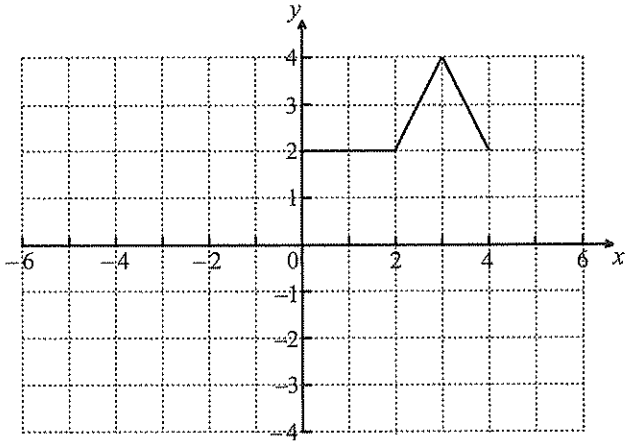


Diagram D



(b) Complete the following table. *[2 marks]*

Description of transformation	**Diagram letter**
Horizontal stretch with scale factor 1.5	
Maps f to $f(x)+1$	

(c) Give a full geometric description of the transformation that gives the image in Diagram A. *[2 marks]*

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3. *[Maximum mark: 5]*

Solve the equation $e^x = 4\sin x$, for $0 \leq x \leq 2\pi$.

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4. *[Maximum mark: 8]*

The diagram below shows a triangle ABD with $AB = 13$ cm and $AD = 6.5$ cm.
Let C be a point on the line BD such that $BC = AC = 7$ cm.

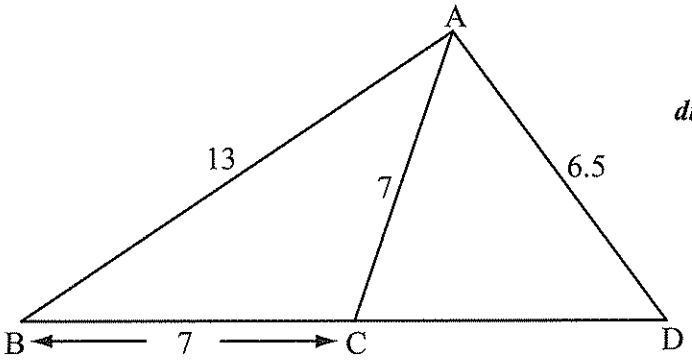

diagram not to scale

(a) Find the size of angle ACB. *[3 marks]*

(b) Find the size of angle CAD. *[5 marks]*

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5. *[Maximum mark: 7]*

(a) Expand $\sum_{r=4}^{7} 2^r$ as the sum of four terms. *[1 mark]*

(b) (i) Find the value of $\sum_{r=4}^{30} 2^r$.

(ii) Explain why $\sum_{r=4}^{\infty} 2^r$ cannot be evaluated. *[6 marks]*

6. *[Maximum mark: 7]*

Consider the curve $y = \ln(3x-1)$. Let P be the point on the curve where $x = 2$.

(a) Write down the gradient of the curve at P. *[2 marks]*

(b) The normal to the curve at P cuts the x-axis at R. Find the coordinates of R. *[5 marks]*

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7. *[Maximum mark: 7]*

The quadratic equation $kx^2 + (k-3)x + 1 = 0$ has two equal real roots.

(a) Find the possible values of k. *[5 marks]*

(b) **Write down** the values of k for which $x^2 + (k-3)x + k = 0$ has two equal real roots. *[2 marks]*

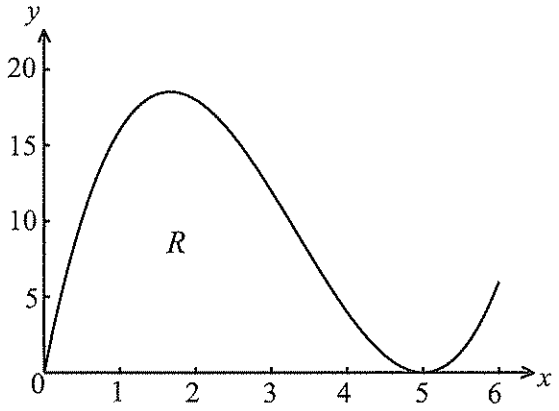
*Do **NOT** write on this page.*

SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 14]*

Let $f(x) = x(x-5)^2$, for $0 \le x \le 6$. The following diagram shows the graph of f.

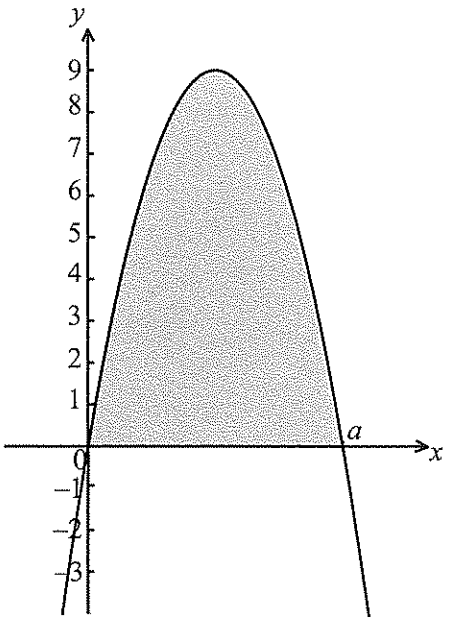


Let R be the region enclosed by the x-axis and the curve of f.

(a) Find the area of R. *[3 marks]*

(b) Find the volume of the solid formed when R is rotated through 360° about the x-axis. *[4 marks]*

(c) The diagram below shows a part of the graph of a quadratic function $g(x) = x(a-x)$. The graph of g crosses the x-axis when $x = a$.



The area of the shaded region is equal to the area of R. Find the value of a. *[7 marks]*

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9. *[Maximum mark: 13]*

A van can take either Route A or Route B for a particular journey.

If Route A is taken, the journey time may be assumed to be normally distributed with mean 46 minutes and a standard deviation 10 minutes.

If Route B is taken, the journey time may be assumed to be normally distributed with mean μ minutes and standard deviation 12 minutes.

(a) For Route A, find the probability that the journey takes **more** than 60 minutes. *[2 marks]*

(b) For Route B, the probability that the journey takes **less** than 60 minutes is 0.85. Find the value of μ. *[3 marks]*

(c) The van sets out at 06:00 and needs to arrive before 07:00.

(i) Which route should it take?

(ii) Justify your answer. *[3 marks]*

(d) On five consecutive days the van sets out at 06:00 and takes Route B. Find the probability that

(i) it arrives before 07:00 on all five days;

(ii) it arrives before 07:00 on at least three days. *[5 marks]*

*Do **NOT** write on this page.*

10. *[Maximum mark: 18]*

Let $f(x) = 3\sin x + 4\cos x$, for $-2\pi \le x \le 2\pi$.

(a) Sketch the graph of f. *[3 marks]*

(b) Write down

(i) the amplitude;

(ii) the period;

(iii) the x-intercept that lies between $-\frac{\pi}{2}$ and 0. *[3 marks]*

(c) Hence write $f(x)$ in the form $p\sin(qx+r)$. *[3 marks]*

(d) Write down one value of x such that $f'(x) = 0$. *[2 marks]*

(e) Write down the two values of k for which the equation $f(x) = k$ has exactly two solutions. *[2 marks]*

(f) Let $g(x) = \ln(x+1)$, for $0 \le x \le \pi$. There is a value of x, between 0 and 1, for which the gradient of f is equal to the gradient of g. Find this value of x. *[5 marks]*