N08/5/MATSD/SP2/ENG/TZ0/XX

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MATHEMATICAL STUDIES
STANDARD LEVEL
PAPER 2

Monday 10 November 2008 (morning)

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Please start each question on a new page. You are advised to show all working, where possible. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. *[Maximum mark: 17]*

Throughout this question *all* the numerical answers must be given correct to the nearest whole number.

Park School started in January 2000 with 100 students. Every full year, there is an increase of 6 % in the number of students.

(a) Find the number of students attending Park School in

(i) January 2001;

(ii) January 2003. *[4 marks]*

(b) Show that the number of students attending Park School in January 2007 is 150. *[2 marks]*

Grove School had 110 students in January 2000. Every full year, the number of students is 10 more than in the previous year.

(c) Find the number of students attending Grove School in January 2003. *[2 marks]*

(d) Find the year in which the number of students attending Grove School will be first 60 % **more than** in January 2000. *[4 marks]*

Each January, one of these two schools, the one that has more students, is given extra money to spend on sports equipment.

(e) (i) Decide which school gets the money in 2007. Justify your answer.

(ii) Find the first year in which Park School will be given this extra money. *[5 marks]*

2. *[Maximum mark: 14]*

The quadrilateral ABCD shown below represents a sandbox. AB and BC have the same length. AD is 9 m long and CD is 4.2 m long. Angles ADC and ABC are 95° and 130° respectively.

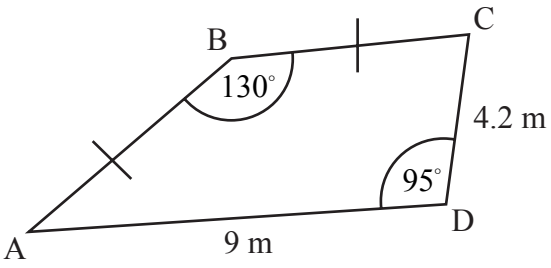

diagram not to scale

(a) Find the length of AC. *[3 marks]*

(b) (i) Write down the size of angle BCA.

(ii) Calculate the length of AB. *[4 marks]*

(c) Show that the area of the sandbox is 31.1 m^2 correct to 3 s.f. *[4 marks]*

The sandbox is a prism. Its edges are 40 cm high. The sand occupies one third of the volume of the sandbox.

(d) Calculate the volume of sand in the sandbox. *[3 marks]*

3. *[Maximum mark: 18]*

Jorge conducted a survey of 200 drivers. He asked two questions:

How long have you had your driving licence?
Do you wear a seat belt when driving?

The replies are summarized in the table below.

	Wear a seat belt	**Do not wear a seat belt**
Licence less than 2 years	38	42
Licence between 2 and 15 years	30	45
Licence more than 15 years	30	15

(a) Jorge applies a χ^2 test at the 5 % level to investigate whether wearing a seat belt is associated with the time a driver has had their licence.

(i) Write down the null hypothesis, H_0.

(ii) Write down the number of degrees of freedom.

(iii) Show that the expected number of drivers that wear a seat belt and have had their driving licence for more than 15 years is 22, correct to the nearest whole number.

(iv) Write down the χ^2 test statistic for this data.

(v) Does Jorge accept H_0? Give a reason for your answer. *[8 marks]*

(b) Consider the 200 drivers surveyed. One driver is chosen at random. Calculate the probability that

(i) this driver wears a seat belt;

(ii) the driver does not wear a seat belt, **given that** the driver has held a licence for more than 15 years. *[4 marks]*

(c) Two drivers are chosen at random. Calculate the probability that

(i) both wear a seat belt.

(ii) at least one wears a seat belt. *[6 marks]*

4. *[Maximum mark: 18]*

The temperature in °C of a pot of water removed from the cooker is given by $T(m) = 20 + 70 \times 2.72^{-0.4m}$, where m is the number of minutes after the pot is removed from the cooker.

(a) Show that the temperature of the water when it is removed from the cooker is 90 °C. *[2 marks]*

The following table shows values for m and $T(m)$.

m	1	2	4	6	8	10
$T(m)$	66.9	51.4	34.1	26.3	22.8	s

(b) (i) Write down the value of s.

(ii) Draw the graph of $T(m)$ for $0 \le m \le 10$. Use a scale of 1 cm to represent 1 minute on the horizontal axis and a scale of 1 cm to represent 10 °C on the vertical axis.

(iii) **Use your graph** to find how long it takes for the temperature to reach 56 °C. Show your method clearly.

(iv) Write down the temperature approached by the water after a long time. Justify your answer. *[9 marks]*

Consider the function $S(m) = 20m - 40$ for $2 \le m \le 6$.
The function $S(m)$ represents the temperature of soup in a pot placed on the cooker two minutes after the water has been removed. The soup is then heated.

(c) Draw the graph of $S(m)$ on the same set of axes used for part (b). *[2 marks]*

(d) Comment on the meaning of the constant **20** in the formula for $S(m)$ in relation to the temperature of the soup. *[1 mark]*

(e) (i) **Use your graph** to solve the equation $S(m) = T(m)$. Show your method clearly.

(ii) Hence describe by using inequalities the set of values of m for which $S(m) > T(m)$. *[4 marks]*

5. *[Maximum mark: 23]*

Consider the curve $y = x^3 + \frac{3}{2}x^2 - 6x - 2$

(a) (i) Write down the value of y when x is 2.

(ii) Write down the coordinates of the point where the curve intercepts the y-axis. *[3 marks]*

(b) Sketch the curve for $-4 \le x \le 3$ and $-10 \le y \le 10$. Indicate clearly the information found in (a). *[4 marks]*

(c) Find $\frac{dy}{dx}$. *[3 marks]*

(d) Let L_1 be the tangent to the curve at $x = 2$.
Let L_2 be a tangent to the curve, parallel to L_1.

(i) Show that the gradient of L_1 is 12.

(ii) Find the x-coordinate of the point at which L_2 and the curve meet.

(iii) Sketch and label L_1 and L_2 on the diagram drawn in (b). *[8 marks]*

(e) It is known that $\frac{dy}{dx} > 0$ for $x < -2$ and $x > b$ where b is positive.

(i) Using your graphic display calculator, or otherwise, find the value of b.

(ii) Describe the behaviour of the curve in the interval $-2 < x < b$.

(iii) Write down the equation of the tangent to the curve at $x = -2$. *[5 marks]*