## MATHEMATICAL STUDIES <br> STANDARD LEVEL <br> PAPER 1

Friday 7 November 2008 (afternoon)
Candidate session number
1 hour 30 minutes

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## INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Working may be continued below the box, if necessary. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. Given that $h=\sqrt{\ell^{2}-\frac{d^{2}}{4}}$,
(a) Calculate the exact value of $h$ when $\ell=0.03625$ and $d=0.05$.
(b) Write down the answer to part (a) correct to three decimal places.
(c) Write down the answer to part (a) correct to three significant figures.
(d) Write down the answer to part (a) in the form $a \times 10^{k}$, where $1 \leq a<10, k \in \mathbb{Z}$.

## Working:

2. The grades obtained by a group of 20 IB students are listed below:

| 6 | 2 | 5 | 3 | 5 | 5 | 6 | 2 | 6 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 7 | 6 | 2 | 4 | 2 | 4 | 3 | 4 | 5 | 6 |

(a) Complete the following table for the grades obtained by the students.

| Grade | Frequency |
| :---: | :---: |
| 1 |  |
| 2 |  |
| 3 | 2 |
| 4 |  |
| 5 | 4 |
| 6 |  |
| 7 | 1 |

(b) Write down the modal grade obtained by the students.
(c) Calculate the median grade obtained by the students.

One student is chosen at random from the group.
(d) Find the probability that this student obtained either grade 4 or grade 5 .

## Working:

3. The diagram below shows the cumulative frequency distribution of the heights in metres of 600 trees in a wood.

(a) Write down the median height of the trees.
(b) Calculate the interquartile range of the heights of the trees.

## (Question 3 continued)

(c) Given that the smallest tree in the wood is 3 m high and the tallest tree is 28 m high, draw the box and whisker plot on the grid below that shows the distribution of trees in the wood.


Working:
4. Let $p$ and $q$ represent the propositions

$$
\begin{aligned}
& p: \text { food may be taken into the cinema } \\
& q \text { : drinks may be taken into the cinema }
\end{aligned}
$$

(a) Complete the truth table below for the symbolic statement $\neg(p \vee q)$.

| $p$ | $q$ | $p \vee q$ | $\neg(p \vee q)$ |
| :---: | :---: | :---: | :---: |
| T | T |  |  |
| T | F |  |  |
| F | T |  |  |
| F | F |  |  |

(b) Write down in words the meaning of the symbolic statement $\neg(p \vee q)$.
(c) Write in symbolic form the compound statement:
"no food and no drinks may be taken into the cinema".

## Working:

Answers:
(b) $\qquad$
$\qquad$
(c)
$\qquad$
$\qquad$
5. The exchange rate between Indian rupees (INR) and Singapore dollars (S\$) is $100 \mathrm{INR}=\mathrm{S} \$ 3.684$

Kwai Fan changes S\$500 to Indian rupees.
(a) Calculate the number of Indian rupees she will receive using this exchange rate.

Give your answer correct to the nearest rupee.
On her return to Singapore, Kwai Fan has 2500 Indian rupees left from her trip. She wishes to exchange these rupees back to Singapore dollars. There is a $3 \%$ commission charge for this transaction and the exchange rate is $100 \mathrm{INR}=\mathrm{S} \$ 3.672$.
(b) Calculate the commission in Indian rupees that she is charged for this exchange.
(c) Calculate the amount of money she receives in Singapore dollars, correct to two decimal places.

## Working:

6. The distribution of the weights, correct to the nearest kilogram, of the members of a football club is shown in the following table.

| Weight (kg) | $40-49$ | $50-59$ | $60-69$ | $70-79$ |
| :--- | :---: | :---: | :---: | :---: |
| Frequency | 6 | 18 | 14 | 4 |

(a) On the grid below draw a histogram to show the above weight distribution.

(b) Write down the mid-interval value for the $40-49$ interval.
(c) Find an estimate of the mean weight of the members of the club.
(d) Write down an estimate of the standard deviation of their weights.

## Working:

7. A straight line, $L_{1}$, has equation $x+4 y+34=0$.
(a) Find the gradient of $L_{1}$.

The equation of line $L_{2}$ is $y=m x . L_{2}$ is perpendicular to $L_{1}$.
(b) Find the value of $m$.
(c) Find the coordinates of the point of intersection of the lines $L_{1}$ and $L_{2}$.

## Working:

Answers:
(a)
(b)
(c)
$\qquad$
c) $\qquad$
8. Given the arithmetic sequence: $u_{1}=124, u_{2}=117, u_{3}=110, u_{4}=103, \ldots$
(a) Write down the common difference of the sequence.
(b) Calculate the sum of the first 50 terms of the sequence.
$u_{k}$ is the first term in the sequence which is negative.
(c) Find the value of $k$.

## Working:

9. The Venn diagram shows the numbers of pupils in a school according to whether they study the sciences Physics $(P)$, Chemistry ( $C$ ), Biology $(B)$.

(a) Write down the number of pupils that study Chemistry only.
(b) Write down the number of pupils that study exactly two sciences.
(c) Write down the number of pupils that do not study Physics.
(d) Find $n[(P \cup B) \cap C]$.

## Working:

Answers:
(a)
(b)
(c)
(d) $\qquad$
10. Eva invests USD2000 at a nominal annual interest rate of $8 \%$ compounded half-yearly.
(a) Calculate the value of her investment after 5 years, correct to the nearest dollar. [3 marks]

Toni invests USD1500 at an annual interest rate of 7.8 \% compounded yearly.
(b) Find the number of complete years it will take for his investment to double in value.

Working:

Answers:
(a)
(b)
11. The marks obtained by 8 candidates in Physics and Chemistry tests are given below.

| Physics (x) | 6 | 8 | 10 | 11 | 10 | 5 | 4 | 12 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Chemistry (y) | 8 | 11 | 14 | 13 | 11 | 7 | 5 | 15 |

(a) Write down the product moment correlation coefficient, $r$.
(b) Write down, in the form $y=m x+c$, the equation of the regression line $y$ on $x$ for the 8 candidates.

A ninth candidate obtained a score of 7 in the Physics test but was absent for the Chemistry test.
(c) Use your answer to (b) to estimate the score he would have obtained on the Chemistry test.
(d) Give a reason why it is valid to use this regression line to estimate the score on the Chemistry test.

12. The diagram shows a pyramid VABCD which has a square base of length 10 cm and edges of length $13 \mathrm{~cm} . \mathrm{M}$ is the midpoint of the side BC .

diagram not to scale
(a) Calculate the length of VM.
(b) Calculate the vertical height of the pyramid.
(c) Calculate the angle between a sloping face of the pyramid and its base.

Working:
13. (a) Factorise the expression $x^{2}-k x$.
(b) Hence solve the equation $x^{2}-k x=0$.

The diagram below shows the graph of the function $f(x)=x^{2}-k x$ for a particular value of $k$.

(c) Write down the value of $k$ for this function.
(d) Find the minimum value of the function $y=f(x)$.

## Working:

14. The function $f(x)=a^{x}+b$ is defined by the mapping diagram below.

$\begin{array}{lll}\text { (a) } & \text { Find the values of } a \text { and } b . & {[3 \mathrm{marks}]} \\ \text { (b) Write down the image of } 2 \text { under the function } f . & \text { [1 mark] } \\ \text { (c) } & \text { Find the value of } c . & {[2 \mathrm{marks}]}\end{array}$

Working:

Answers:
(a)
(b)
(c) $\qquad$
15. The diagram below shows the graphs of two sine functions, $f(x)$ and $g(x)$, for $-180^{\circ} \leq x \leq 180^{\circ}$.

(a) Write down
(i) the equation of $f(x)$;
(ii) the equation of $g(x)$.
(b) Use your graphic display calculator to solve the equation $f(x)=g(x)$ in the interval $-90^{\circ} \leq x \leq 90^{\circ}$.

## Working:

Answers:
(a) (i)
(ii)
(b) $\qquad$

