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MARKSCHEME

May 2008

MATHEMATICS

Standard Level

Paper 1

16 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**: often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Write the marks in red on candidates' scripts, in the right hand margin.

- Show the **breakdown** of individual marks awarded using the abbreviations ***M1***, ***A1***, *etc*.
- Write down the total for each **question** (at the end of the question) and **circle** it.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks.
- Once a correct answer to a question or part-question is seen, ignore further working.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers. In this case, ignore mark breakdown (**M**, **A**, **R**).*

- Do **not** award a mixture of *N* and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.
- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets e.g. (M1)**.*

- Implied marks can only be awarded if **correct** work is seen or if implied in subsequent working (a correct answer does not necessarily mean that the implied marks are all awarded).
- Normally the correct work is seen or implied in the next line.

*Must be seen marks appear without **brackets e.g. M1**.*

- Must be seen marks can only be awarded if **correct** work is seen.
- If a must be seen ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). To award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded, but ***M*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). Apply a **MR** penalty of 1 mark to that question. Award the marks as usual and then write –1(**MR**) next to the total. Subtract 1 mark from the total for the question. A candidate should be penalized only once for a particular mis-read.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. The mark should be labelled **(d)** and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives:

$$f'(x) = \left(2\cos(5x-3)\right)5 \quad \left(=10\cos(5x-3)\right) \qquad \textbf{\textit{A1}}$$

Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy.

- **Rounding errors**: only applies to final answers not to intermediate steps.
- **Level of accuracy**: when this is not specified in the question the general rule applies: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

Candidates should be penalized ***once only IN THE PAPER*** *for an accuracy error* ***(AP)****. Award the marks as usual then write* ***(AP)*** *against the answer. On the* ***front*** *cover write –1****(AP)****. Deduct 1 mark from the total for the paper, not the question.*

- If a final correct answer is incorrectly rounded, apply the ***AP***.
- If the level of accuracy is not specified in the question, apply the ***AP*** for correct answers not given to three significant figures.
- Intermediate values are sometimes written as 3.24(741). This indicates that using 3.24 (or 3.25) is acceptable, but the more accurate value is 3.24741. The digits in brackets are not required for the marks. If candidates work with fewer than three significant figures, this could lead to an ***AP***.

If there is no working shown, and answers are given to the correct two significant figures, apply the ***AP***. However, do **not** accept answers to one significant figure without working.

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

13 Style

The markscheme often uses words to describe what the marks are for, followed by examples, using the e.g. notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are ***M*** *marks, the examples may include ones using poor notation, to indicate what is acceptable.*

SECTION A

QUESTION 1

(a) evidence of using $\sum f_i = 100$ ***(M1)***
$k = 4$ ***A1*** ***N2***

(b) (i) evidence of median position ***(M1)***
e.g. 50^{th} item, $26 + 10 + 20 = 56$
median $= 3$ ***A1*** ***N2***

(ii) $Q_1 = 1$ and $Q_3 = 5$ ***(A1)(A1)***
interquartile range $= 4$ (accept 1 to 5 or $5 - 1$, *etc.*) ***A1*** ***N3***

[7 marks]

QUESTION 2

(a) evidence of attempting to solve $f(x) = 0$ ***(M1)***
evidence of correct working ***A1***
e.g. $(x+1)(x-2)$, $\dfrac{1 \pm \sqrt{9}}{2}$
intercepts are $(-1, 0)$ and $(2, 0)$ (accept $x = -1$, $x = 2$) ***A1A1*** ***N1N1***

(b) evidence of appropriate method ***(M1)***
e.g. $x_v = \dfrac{x_1 + x_2}{2}$, $x_v = -\dfrac{b}{2a}$, reference to symmetry
$x_v = 0.5$ ***A1*** ***N2***

[6 marks]

QUESTION 3

(a) $\det \boldsymbol{M} = -4$ ***A1*** ***N1***

(b) $\boldsymbol{M}^{-1} = -\dfrac{1}{4}\begin{pmatrix} -1 & -1 \\ -2 & 2 \end{pmatrix}$ $\left(= \begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \right)$ ***A1A1*** ***N2***

> **Note:** Award ***A1*** for $-\dfrac{1}{4}$ and ***A1*** for the correct matrix.

(c) $\boldsymbol{X} = \boldsymbol{M}^{-1}\begin{pmatrix} 4 \\ 8 \end{pmatrix}$ $\left(\boldsymbol{X} = -\dfrac{1}{4}\begin{pmatrix} -1 & -1 \\ -2 & 2 \end{pmatrix}\begin{pmatrix} 4 \\ 8 \end{pmatrix} \right)$ ***M1***

$\boldsymbol{X} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ $(x = 3,\ y = -2)$ ***A1A1*** ***N0***

> **Note:** Award no marks for an **algebraic** solution of the system $2x + y = 4$, $2x - y = 8$.

[6 marks]

QUESTION 4

(a) evidence of choosing the formula $\cos 2A = 2\cos^2 A - 1$ ***(M1)***

Note: If they choose another correct formula, do not award the ***M1*** unless there is evidence of finding $\sin^2 A = 1 - \frac{1}{9}$.

correct substitution ***A1***

e.g. $\cos 2A = \left(\frac{1}{3}\right)^2 - \frac{8}{9}$, $\cos 2A = 2 \times \left(\frac{1}{3}\right)^2 - 1$

$\cos 2A = -\frac{7}{9}$ ***A1*** ***N2***

(b) **METHOD 1**

evidence of using $\sin^2 B + \cos^2 B = 1$ ***(M1)***

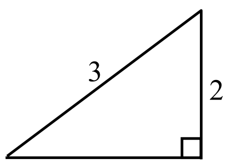
e.g. $\left(\frac{2}{3}\right)^2 + \cos^2 B = 1$, $\sqrt{\frac{5}{9}}$ (seen anywhere),

$\cos B = \pm\sqrt{\frac{5}{9}}$ $\left(= \pm\frac{\sqrt{5}}{3} \right)$ ***(A1)***

$\cos B = -\sqrt{\frac{5}{9}}$ $\left(= -\frac{\sqrt{5}}{3} \right)$ ***A1*** ***N2***

METHOD 2

diagram ***M1***
e.g.



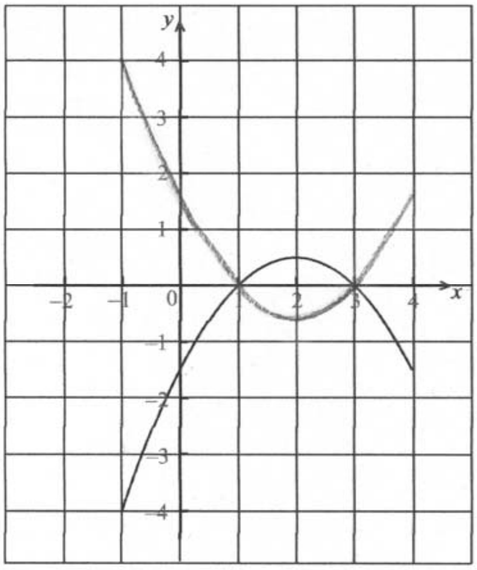
for finding third side equals $\sqrt{5}$ ***(A1)***

$\cos B = -\frac{\sqrt{5}}{3}$ ***A1*** ***N2***

[6 marks]

QUESTION 5

(a)



M1A1 ***N2***

> **Note:** Award ***M1*** for evidence of reflection in x-axis,
> ***A1*** for correct vertex **and** all intercepts approximately correct.

(b) (i) $g(-3) = f(0)$ ***(A1)***

$f(0) = -1.5$ ***A1*** ***N2***

(ii) translation (accept shift, slide, *etc.*) of $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$ ***A1A1*** ***N2***

[6 marks]

QUESTION 6

(a) (i) correct calculation *(A1)*

e.g. $\frac{9}{20}+\frac{5}{20}-\frac{2}{20}$, $\frac{4+2+3+3}{20}$

$\mathrm{P}(\text{male or tennis})=\frac{12}{20}\ \left(=\frac{3}{5}\right)$ ***A1*** ***N2***

(ii) correct calculation *(A1)*

e.g. $\frac{6}{20}\div\frac{11}{20}$, $\frac{3+3}{11}$

$\mathrm{P}(\text{not football}\mid\text{female})=\frac{6}{11}$ ***A1*** ***N2***

(b) $\mathrm{P}(\text{first not football})=\frac{11}{20}$, $\mathrm{P}(\text{second not football})=\frac{10}{19}$ ***A1***

$\mathrm{P}(\text{neither football})=\frac{11}{20}\times\frac{10}{19}$ ***A1***

$\mathrm{P}(\text{neither football})=\frac{110}{380}\ \left(=\frac{11}{38}\right)$ ***A1*** ***N1***

[7 marks]

QUESTION 7

(a) evidence of factorising 3/division by 3 ***A1***

e.g. $\int_1^5 3f(x)\,dx = 3\int_1^5 f(x)\,dx$, $\frac{12}{3}$, $\int_1^5 \frac{3f(x)\,dx}{3}$ (do not accept 4 as this is show that)

evidence of stating that reversing the limits changes the sign ***A1***

e.g. $\int_5^1 f(x)\,dx = -\int_1^5 f(x)\,dx$

$\int_5^1 f(x)\,dx = -4$ ***AG*** ***N0***

(b) evidence of correctly combining the integrals (seen anywhere) ***(A1)***

e.g. $I = \int_1^2 (x + f(x))\,dx + \int_2^5 (x + f(x))\,dx = \int_1^5 (x + f(x))\,dx$

evidence of correctly splitting the integrals (seen anywhere) ***(A1)***

e.g. $I = \int_1^5 x\,dx + \int_1^5 f(x)\,dx$

$\int x\,dx = \frac{x^2}{2}$ (seen anywhere) ***A1***

$\int_1^5 x\,dx = \left[\frac{x^2}{2}\right]_1^5 = \frac{25}{2} - \frac{1}{2}$ $\left(= \frac{24}{2}, 12\right)$ ***A1***

$I = 16$ ***A1*** ***N3***

[7 marks]

SECTION B

QUESTION 8

(a) (i) evidence of combining vectors ***(M1)***

e.g. $\vec{AB} = \vec{OB} - \vec{OA}$ (or $\vec{AD} = \vec{AO} + \vec{OD}$ in part (ii))

$\vec{AB} = \begin{pmatrix} 2 \\ -4 \\ -2 \end{pmatrix}$ ***A1*** ***N2***

(ii) $\vec{AD} = \begin{pmatrix} 2 \\ k-5 \\ -2 \end{pmatrix}$ ***A1*** ***N1***

[3 marks]

(b) evidence of using perpendicularity $\Rightarrow$ scalar product $= 0$ ***(M1)***

e.g. $\begin{pmatrix} 2 \\ -4 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ k-5 \\ -2 \end{pmatrix} = 0$

$4 - 4(k-5) + 4 = 0$ ***A1***

$-4k + 28 = 0$ (accept any correct equation clearly leading to $k = 7$) ***A1***

$k = 7$ ***AG*** ***N0***

[3 marks]

(c) $\vec{AD} = \begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix}$ ***(A1)***

$\vec{BC} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ ***A1***

evidence of correct approach ***(M1)***

e.g. $\vec{OC} = \vec{OB} + \vec{BC}$, $\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$, $\begin{pmatrix} x-3 \\ y-1 \\ z-2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$

$\vec{OC} = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$ ***A1*** ***N3***

[4 marks]

continued…

Question 8 continued

(d) **METHOD 1**

choosing appropriate vectors, $\overrightarrow{BA}$, $\overrightarrow{BC}$ ***(A1)***
finding the scalar product ***M1***
e.g. $-2(1)+4(1)+2(-1)$, $2(1)+(-4)(1)+(-2)(-1)$
$\cos A\hat{B}C = 0$ ***A1*** ***N1***

METHOD 2

$\overrightarrow{BC}$ parallel to $\overrightarrow{AD}$ (may show this on a diagram with points labelled) ***R1***

$\overrightarrow{BC} \perp \overrightarrow{AB}$ (may show this on a diagram with points labelled) ***R1***

$A\hat{B}C = 90^\circ$
$\cos A\hat{B}C = 0$ ***A1*** ***N1***
[3 marks]

Total [13 marks]

QUESTION 9

(a) (i) range of f is $[-1, 1]$, $(-1 \le f(x) \le 1)$ ***A2*** ***N2***

(ii) $\sin^3 x = 1 \Rightarrow \sin x = 1$ ***A1***

justification for one solution on $[0, 2\pi]$ ***R1***

e.g. $x = \frac{\pi}{2}$, unit circle, sketch of $\sin x$

1 solution (seen anywhere) ***A1*** ***N1***

[5 marks]

(b) $f'(x) = 3\sin^2 x \cos x$ ***A2*** ***N2***

[2 marks]

(c) using $V = \int_a^b \pi y^2 \mathrm{d}x$ ***(M1)***

$V = \int_0^{\frac{\pi}{2}} \pi \left(\sqrt{3} \sin x \cos^{\frac{1}{2}} x \right)^2 \mathrm{d}x$ ***(A1)***

$= \pi \int_0^{\frac{\pi}{2}} 3\sin^2 x \cos x \, \mathrm{d}x$ ***A1***

$V = \pi \left[\sin^3 x \right]_0^{\frac{\pi}{2}} \quad \left(= \pi \left(\sin^3 \left(\frac{\pi}{2} \right) - \sin^3 0 \right) \right)$ ***A2***

evidence of using $\sin \frac{\pi}{2} = 1$ and $\sin 0 = 0$ ***(A1)***

e.g. $\pi(1-0)$

$V = \pi$ ***A1*** ***N1***

[7 marks]

Total [14 marks]

QUESTION 10

(a) evidence of using area of a triangle *(M1)*

e.g. $A = \frac{1}{2} \times 2 \times 2 \times \sin\theta$

$A = 2\sin\theta$ ***A1*** ***N2***

[2 marks]

(b) **METHOD 1**

$\mathrm{P\hat{O}A} = \pi - \theta$ ***(A1)***

area $\Delta\mathrm{OPA} = \frac{1}{2} 2 \times 2 \times \sin(\pi - \theta) \quad \left(= 2\sin(\pi - \theta)\right)$ ***A1***

since $\sin(\pi - \theta) = \sin\theta$ ***R1***

then both triangles have the same area ***AG*** ***N0***

METHOD 2

triangle OPA has the same height and the same base as triangle OPB ***R3***

then both triangles have the same area ***AG*** ***N0***

[3 marks]

(c) area semicircle $= \frac{1}{2} \times \pi(2)^2 \quad (= 2\pi)$ ***A1***

area $\Delta\mathrm{APB} = 2\sin\theta + 2\sin\theta \quad (= 4\sin\theta)$ ***A1***

$S =$ area of semicircle $-$ area $\Delta\mathrm{APB} \quad (= 2\pi - 4\sin\theta)$ ***M1***

$S = 2(\pi - 2\sin\theta)$ ***AG*** ***N0***

[3 marks]

continued …

Question 10 continued

(d) **METHOD 1**

attempt to differentiate *(M1)*

e.g. $\frac{dS}{d\theta} = -4\cos\theta$

setting derivative equal to 0 *(M1)*

correct equation ***A1***

e.g. $-4\cos\theta = 0$, $\cos\theta = 0$, $4\cos\theta = 0$

$\theta = \frac{\pi}{2}$ ***A1*** ***N3***

EITHER

evidence of using second derivative *(M1)*

$S''(\theta) = 4\sin\theta$ ***A1***

$S''\left(\frac{\pi}{2}\right) = 4$ ***A1***

it is a minimum because $S''\left(\frac{\pi}{2}\right) > 0$ ***R1*** ***N0***

OR

evidence of using first derivative *(M1)*

for $\theta < \frac{\pi}{2}$, $S'(\theta) < 0$ (may use diagram) ***A1***

for $\theta > \frac{\pi}{2}$, $S'(\theta) > 0$ (may use diagram) ***A1***

it is a minimum since the derivative goes from negative to positive ***R1*** ***N0***

METHOD 2

$2\pi - 4\sin\theta$ is minimum when $4\sin\theta$ is a maximum ***R3***

$4\sin\theta$ is a maximum when $\sin\theta = 1$ *(A2)*

$\theta = \frac{\pi}{2}$ ***A3*** ***N3***

[8 marks]

(e) S is greatest when $4\sin\theta$ is smallest (or equivalent) *(R1)*

$\theta = 0$ (or π) ***A1*** ***N2***

[2 marks]

Total [18 marks]