22087405

International Baccalaureate®
Baccalauréat International
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MATHEMATICAL STUDIES
STANDARD LEVEL
PAPER 1

Wednesday 7 May 2008 (afternoon)

1 hour 30 minutes

Candidate session number

0	0							

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

0118

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Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Working may be continued below the box, if necessary. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. (a) (i) Complete the truth table below.

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T			F	F	
T	F			F	T	
F	T			T	F	
F	F			T	T	

(ii) State whether the compound propositions $\neg(p \wedge q)$ and $\neg p \vee \neg q$ are equivalent. *[4 marks]*

Consider the following propositions.

p: Amy eats sweets
q: Amy goes swimming.

(b) Write, in symbolic form, the following proposition. *[2 marks]*

Amy either eats sweets or goes swimming, but not both.

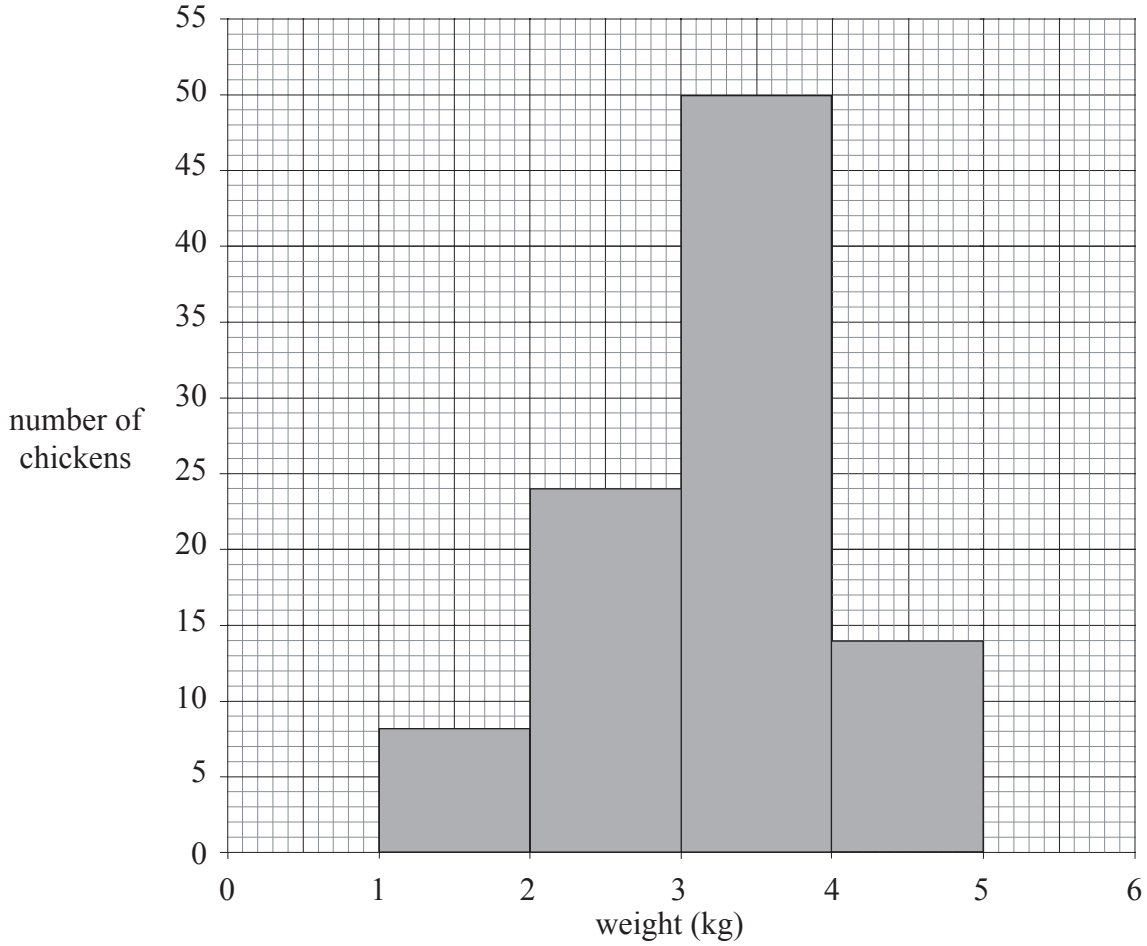
Working:

Answers:

(a) (ii) _____

(b) _____

2. The following histogram shows the weights of a number of frozen chickens in a supermarket. The weights are grouped such that $1 \leq \text{weight} < 2, 2 \leq \text{weight} < 3$ and so on.



(a) On the graph above, draw in the frequency polygon. *[2 marks]*

(b) Find the total number of chickens. *[1 mark]*

(c) Write down the modal group. *[1 mark]*

(This question continues on the following page)

(Question 2 continued)

Gabriel chooses a chicken at random.

(d) Find the probability that this chicken weighs less than 4 kg. *[2 marks]*

Working:

Answers:

(b) _____

(c) _____

(d) _____

3. Triangle ABC is drawn such that angle ABC is 90º, angle ACB is 60º and AB is 7.3 cm.

(a) (i) Sketch a diagram to illustrate this information. Label the points A, B, C. Show the angles 90º, 60º and the length 7.3 cm on your diagram.

(ii) Find the length of BC. *[3 marks]*

Point D is on the straight line AC extended and is such that angle CDB is 20º.

(b) (i) Show the point D and the angle 20º on your diagram.

(ii) Find the size of angle CBD. *[3 marks]*

Working:

Answers:

(a) (ii) _____

(b) (ii) _____

4. Consider the numbers $\sqrt{3}$, 6, $2\frac{1}{2}$, π, –5, and the sets $\mathbb{N}$, $\mathbb{Z}$, and $\mathbb{Q}$. Complete the following table by placing a tick in the appropriate box if the number is an element of the set.

	$\sqrt{3}$	6	$2\frac{1}{2}$	π	–5
$\mathbb{N}$					
$\mathbb{Z}$					
$\mathbb{Q}$					

[6 marks]

Working:

5. A survey was conducted of the number of bedrooms in 208 randomly chosen houses. The results are shown in the following table.

Number of bedrooms	1	2	3	4	5	6
Number of houses	41	60	52	32	15	8

(a) State whether the data is discrete or continuous. *[1 mark]*

(b) Write down the mean number of bedrooms per house. *[2 marks]*

(c) Write down the standard deviation of the number of bedrooms per house. *[1 mark]*

(d) Find how many houses have a number of bedrooms greater than one standard deviation above the mean. *[2 marks]*

Working:

Answers:

(a) _____

(b) _____

(c) _____

(d) _____

6. (a) Write down the gradient of the line $y = 3x + 4$. *[1 mark]*

(b) Find the gradient of the line which is perpendicular to the line $y = 3x + 4$. *[1 mark]*

(c) Find the equation of the line which is perpendicular to $y = 3x + 4$ and which passes through the point $(6, 7)$. *[2 marks]*

(d) Find the coordinates of the point of intersection of these two lines. *[2 marks]*

Working:

Answers:

(a) _____

(b) _____

(c) _____

(d) _____

7. The depth, in metres, of water in a harbour is given by the function $d = 4\sin(0.5t^\circ) + 7$, where t is in minutes, $0 \le t \le 1440$.

(a) Write down the amplitude of d. *[1 mark]*

(b) Find the maximum value of d. *[1 mark]*

(c) Find the period of d. Give your answer in **hours**. *[2 marks]*

On Tuesday, the minimum value of d occurs at 14:00.

(d) Find when the next **maximum** value of d occurs. *[2 marks]*

Working:

Answers:

(a) _____

(b) _____

(c) _____

(d) _____

8. Emma places € 8000 in a bank account that pays a nominal interest rate of 5 % per annum, compounded quarterly.

(a) Calculate the amount of money that Emma would have in her account after 15 years. Give your answer correct to the nearest Euro. *[3 marks]*

(b) After a period of time she decides to withdraw the money from this bank. There is € 9058.17 in her account. Find the number of months that Emma had left her money in the account. *[3 marks]*

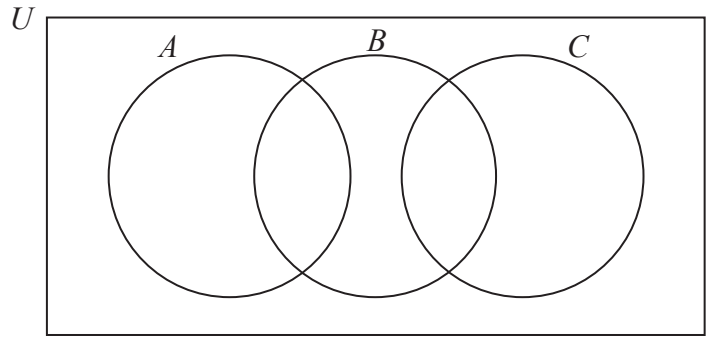
Working:

Answers:

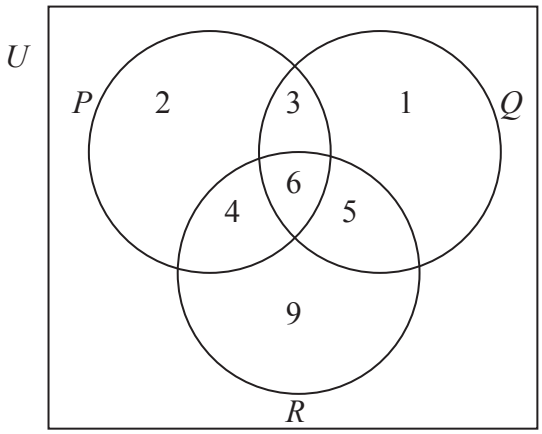
(a) _____

(b) _____

9. (a) Shade $(A \cup B) \cap C'$ on the diagram below. *[2 marks]*



(b) In the Venn diagram below, the number of elements in each region is given.
Find $n((P \cap Q) \cup R)$. *[2 marks]*



(c) U is the set of positive integers, $\mathbb{Z}^+$.
E is the set of even numbers.
M is the set of multiples of 3.

(i) List the first six elements of the set M.

(ii) List the first six elements of the set $E' \cap M$. *[2 marks]*

Working:

Answers:

(b) _____

(c) (i) _____

(ii) _____

10. (a) Factorise the expression $x^2 - 3x - 10$. *[2 marks]*

(b) A function is defined as $f(x) = 1 + x^3$ for $x \in \mathbb{Z}$, $-3 \leq x \leq 3$.

(i) List the elements of the domain of $f(x)$.

(ii) Write down the range of $f(x)$. *[4 marks]*

Working:

Answers:

(a) _____

(b) (i) _____

(ii) _____

11. The table below shows some exchange rates for the Japanese Yen (JPY).

Currency	**1 JPY**
Canadian Dollar	0.010406
Chinese Yuan	0.07127
Euro	0.0072591
Norwegian Kroner	0.057319

Minbin has 1250 Japanese Yen which she wishes to exchange for Chinese Yuan.

(a) Calculate how many Yuan she will receive. Give your answer to the nearest Yuan. *[2 marks]*

Rupert has 855 Canadian Dollars which he wishes to exchange for Japanese Yen.

(b) Calculate how many Yen he will receive. Give your answer to the nearest Yen. *[2 marks]*

(c) Find how many Norwegian Kroner there are to the Euro. Give your answer correct to 2 decimal places. *[2 marks]*

Working:

Answers:

(a) _____

(b) _____

(c) _____

12. Consider the function $f(x) = \frac{1}{2}x^3 - 2x^2 + 3$.

(a) Find $f'(x)$. *[2 marks]*

(b) Find $f''(x)$. *[2 marks]*

(c) Find the equation of the tangent to the curve of f at the point $(1, 1.5)$. *[2 marks]*

Working:

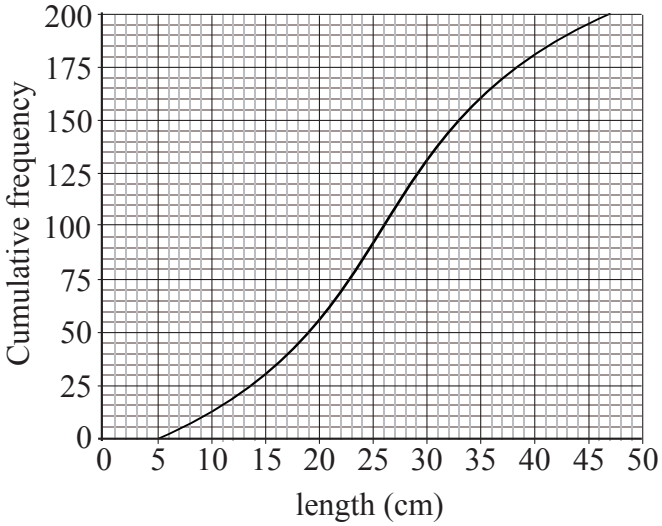
Answers:

(a) _____

(b) _____

(c) _____

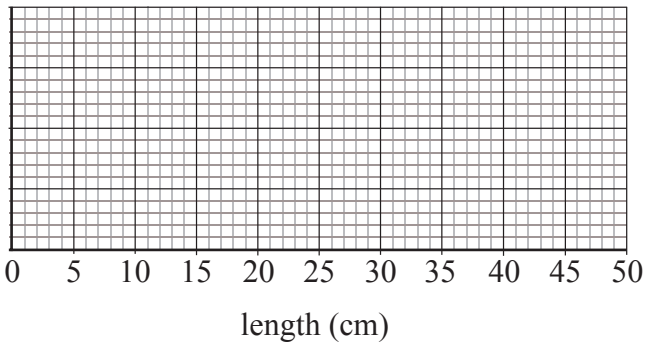
13. A random sample of 200 females measured the length of their hair in cm. The results are displayed in the cumulative frequency curve below.



(a) Write down the median length of hair in the sample. *[1 mark]*

(b) Find the interquartile range for the length of hair in the sample. *[2 marks]*

(c) Given that the shortest length was 6 cm and the longest 47 cm, draw and label a box and whisker plot for the data on the grid provided below. *[3 marks]*



Working:

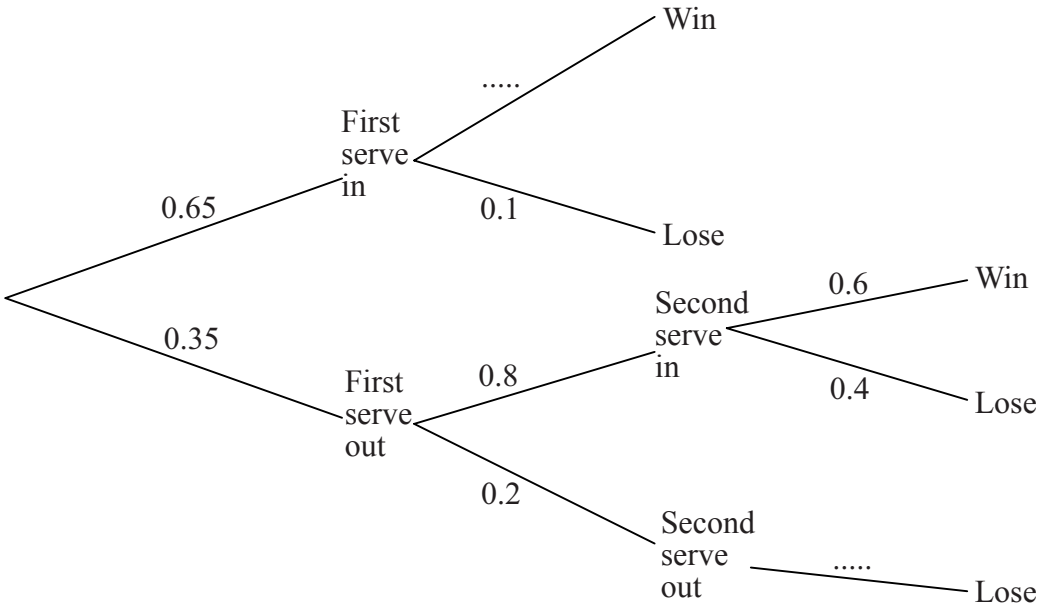
Answers:

(a) _____

(b) _____

14. When Andy plays tennis, 65 % of his first serves go into the correct area of the court.
If the first serve goes into the correct area, his chance of winning the point is 90 %.
If his first serve does not go into the correct area, Andy is allowed a second serve and, of these, 80 % go into the correct area.
If the second serve goes into the correct area, his chance of winning the point is 60 %.
If neither serve goes into the correct area, Andy loses the point.

(a) Complete the tree diagram below. *[2 marks]*



(b) Find the probability that Andy loses the point. *[4 marks]*

Working:

Answers:

(b)

15. The function $f(x)$ is such that $f'(x) < 0$ for $1 < x < 4$. At the point P$(4, 2)$ on the graph of $f(x)$ the gradient is zero.

(a) Write down the equation of the tangent to the graph of $f(x)$ at P. *[2 marks]*

(b) State whether $f(4)$ is greater than, equal to or less than $f(2)$. *[2 marks]*

(c) Given that $f(x)$ is increasing for $4 \leq x < 7$, what can you say about the point P? *[2 marks]*

Working:

Answers:

(a) _____

(b) _____

(c) _____