## MATHEMATICAL STUDIES <br> STANDARD LEVEL <br> PAPER 1

Wednesday 7 May 2008 (afternoon)
Candidate session number
1 hour 30 minutes

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## INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Working may be continued below the box, if necessary. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. (a) Calculate exactly $\frac{(3 \times 2.1)^{3}}{7 \times 1.2}$.
(b) Write the answer to part (a) correct to 2 significant figures.
(c) Calculate the percentage error when the answer to part (a) is written correct to 2 significant figures.
(d) Write your answer to part (c) in the form $a \times 10^{k}$ where $1 \leq a<10$ and $k \in \mathbb{Z}$.

## Working:

Answers:
(a)
(b)
(c)
(d)
$\qquad$
$\qquad$
(c)
d) $\qquad$
2. There are 120 teachers in a school. Their ages are represented by the cumulative frequency graph below.

(a) Write down the median age.
(b) Find the interquartile range for the ages.
(c) Given that the youngest teacher is 21 years old and the oldest is 72 years old, represent the information on a box and whisker plot using the scale below.


Working:

Answers:
(a)
(b) $\qquad$
3. Consider the function $f(x)=2 x^{3}-5 x^{2}+3 x+1$.
(a) Find $f^{\prime}(x)$.
(b) Write down the value of $f^{\prime}(2)$.
(c) Find the equation of the tangent to the curve of $y=f(x)$ at the point $(2,3)$.

Working:

Answers:
(a)
(b)
(c)
4. (a) Complete the following table of values for the height and weight of seven students. [4 marks]

|  | Values | Mode | Median | Mean | Standard <br> deviation |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Height (cm) | $151,158,171,163,184,148,171$ |  |  | 164 | 11.7 |
| Weight (kg) | $53,61,58,82,45,72,82$ | 82 | 61 |  |  |

The ages (in months) of seven students are 194, 205, 208, 210, 200, 226, 223.
(b) Represent these values in an ordered stem and leaf diagram.

## Working:

5. 200 people of different ages were asked to choose their favourite type of music from the choices Popular, Country and Western and Heavy Metal. The results are shown in the table below.

| Age/Music <br> choice | Popular | Country <br> and <br> Western | Heavy <br> Metal | Totals |
| :---: | :---: | :---: | :---: | :---: |
| $11-25$ | 35 | 5 | 50 | 90 |
| $26-40$ | 30 | 10 | 20 | 60 |
| $41-60$ | 20 | 25 | 5 | 50 |
| Totals | 85 | 40 | 75 | 200 |

It was decided to perform a chi-squared test for independence at the $5 \%$ level on the data.
(a) Write down the null hypothesis.
(b) Write down the number of degrees of freedom.
(c) Write down the chi-squared value.
(d) State whether or not you will reject the null hypothesis, giving a clear reason for your answer.

## Working:

6. Consider the following logic propositions:
$p:$ Sean is at school
$q:$ Sean is playing a game on his computer.

(b) Write in words, the converse of $p \Rightarrow \neg q$.
(c) Complete the following truth table for $p \Rightarrow \neg q$.

| $p$ | $q$ | $\neg q$ | $p \Rightarrow \neg q$ |
| :---: | :---: | :---: | :---: |
| T | T |  |  |
| T | F |  |  |
| F | T |  |  |
| F | F |  |  |

## Working:

Answers:
(a) $\qquad$
$\qquad$
$\qquad$
(b) $\qquad$
$\qquad$
$\qquad$
$\qquad$
7. Triangle ABC is such that AC is 7 cm , angle ABC is $65^{\circ}$ and angle ACB is $30^{\circ}$.
(a) Sketch the triangle writing in the side length and angles. [1 mark]
(b) Calculate the length of AB .
(c) Find the area of triangle ABC .

## Working:

Answers:
(b)
(c)
8. The first term of an arithmetic sequence is 0 and the common difference is 12 .
(a) Find the value of the $96^{\text {th }}$ term of the sequence.

The first term of a geometric sequence is 6 . The $6^{\text {th }}$ term of the geometric sequence is equal to the $17^{\text {th }}$ term of the arithmetic sequence given above.
(b) Write down an equation using this information.
(c) Calculate the common ratio of the geometric sequence.

## Working:

Answers:
(a)
(b)
(c)
9. The table below shows the number of words in the extended essays of an IB class.

| Number of <br> words | $3200 \leq w<3400$ | $3400 \leq w<3600$ | $3600 \leq w<3800$ | $3800 \leq w<4000$ | $4000 \leq w<4200$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Frequency | 2 | 5 | 8 | 17 | 3 |

(a) Draw a histogram on the grid below for the data in this table.

(b) Write down the modal group.

The maximum word count is 4000 words.
(c) Write down the probability that a student chosen at random is on or over the word count.

## Working:

10. Jane plans to travel from Amsterdam to Chicago. She changes 1500 Euros (EUR) to US Dollars (USD) at an exchange rate of 1 EUR to 1.33 USD. Give all answers in this question correct to two decimal places.
(a) Calculate the number of USD Jane receives.

Jane spends 1350 USD and then decides to convert the remainder back to EUR at a rate of 1 EUR to 1.38 USD.
(b) Calculate the amount of EUR Jane receives.

If Jane had waited until she returned to Amsterdam she could have changed her USD at a rate of 1 EUR to 1.36 USD but the bank would have charged $0.8 \%$ commission.
(c) Calculate the amount of EUR Jane gained or lost by changing her money in Chicago.

Working:
11. (a) Consider the numbers $2, \sqrt{3},-\frac{2}{3}$ and the sets $\mathbb{N}, \mathbb{Z}, \mathbb{Q}$ and $\mathbb{R}$.

Complete the table below by placing a tick in the appropriate box if the number is an element of the set, and a cross if it is not.
(i)

|  | $\mathbb{N}$ | $\mathbb{Z}$ | $\mathbb{Q}$ | $\mathbb{R}$ |
| :---: | :---: | :---: | :---: | :---: |
| 2 |  |  |  |  |
| $\sqrt{3}$ |  |  |  |  |
| $-\frac{2}{3}$ |  |  |  |  |

(b) A function $f$ is given by $f: x \mapsto 2 x^{2}-3 x, x \in\{-2,2,3\}$.
(i) Draw a mapping diagram to illustrate this function.
(ii) Write down the range of function $f$.

## Working:

(b) (ii)
12. The following table shows the monthly payments needed to repay a loan of $\$ 1000$ with various rates and time periods.

| Table of Monthly Repayments per $\$ 1000$ <br> Annual interest rate |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Loan Term <br> (months) | $\mathbf{5} \%$ | $\mathbf{5 . 5} \%$ | $\mathbf{6} \%$ | $\mathbf{6 . 5} \%$ |
| 12 | 87.50 | 87.92 | 88.34 | 88.75 |
| 18 | 59.74 | 60.21 | 60.62 | 61.12 |
| 24 | 45.94 | 46.38 | 46.84 | 47.25 |
| 30 | 37.66 | 38.11 | 38.57 | 39.04 |
| 36 | 32.16 | 32.62 | 33.09 | 33.56 |
| 42 | 28.25 | 28.72 | 29.20 | 29.70 |
| 48 | 25.33 | 25.81 | 26.30 | 26.80 |

Sarah takes out a personal loan for $\$ 24000$ to buy a car. She negotiates a loan for three years at $6 \%$ per annum interest.
(a) Calculate the exact monthly repayment she will make.
(b) Find the exact total of the repayments she will make.

Beryl took out a loan of $\$ 10000$ for 18 months. The total she paid for the loan was $\$ 10837.80$.
(c) Find the rate of interest charged on the loan.

## Working:

13. Bob invests 3000 USD in a bank that offers simple interest at a rate of $4 \%$ per annum.
(a) Calculate the number of years that it takes for Bob's money to double.

Charles invests 3000 USD in a bank that offers compound interest at a rate of $3.5 \%$ per annum, compounded half-yearly.
(b) Calculate the number of years that it takes for Charles's money to double.

## Working:

Answers:
(a)
(b)
14. A race track is made up of a rectangular shape 750 m by 500 m with semi-circles at each end as shown in the diagram.


Michael drives around the track once at an average speed of $140 \mathrm{kmh}^{-1}$.
(a) Calculate the distance that Michael travels.
(b) Calculate how long Michael takes in seconds.

Working:

Answers:
(a)
(b)
15. In an experiment it is found that a culture of bacteria triples in number every four hours.

There are 200 bacteria at the start of the experiment.

| Hours | 0 | 4 | 8 | 12 | 16 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| No. of bacteria | 200 | 600 | $a$ | 5400 | 16200 |

(a) Find the value of $a$.
(b) Calculate how many bacteria there will be after one day.
(c) Find how long it will take for there to be two million bacteria.

## Working:

Answers:
(a)
(b)
(c) $\qquad$

