M08/5/MATME/SP1/ENG/TZ1/XX+

22087303

International Baccalaureate®
Baccalauréat International
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MATHEMATICS
STANDARD LEVEL
PAPER 1

Wednesday 7 May 2008 (afternoon)

1 hour 30 minutes

Candidate session number

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 7]*

Let $\boldsymbol{A} = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 3 & 0 \\ -2 & 1 \end{pmatrix}$.

Find

(a) $\boldsymbol{A} + \boldsymbol{B}$; *[2 marks]*

(b) $-3\boldsymbol{A}$; *[2 marks]*

(c) $\boldsymbol{AB}$. *[3 marks]*

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2. *[Maximum mark: 6]*

Let $p = \sin 40^\circ$ and $q = \cos 110^\circ$. Give your answers to the following in terms of p and/or q.

(a) Write down an expression for

(i) $\sin 140^\circ$;

(ii) $\cos 70^\circ$. *[2 marks]*

(b) Find an expression for $\cos 140^\circ$. *[3 marks]*

(c) Find an expression for $\tan 140^\circ$. *[1 mark]*

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3. *[Maximum mark: 6]*

Consider the arithmetic sequence $2, 5, 8, 11, \ldots$.

(a) Find u_{101}. *[3 marks]*

(b) Find the value of *n* so that $u_n = 152$. *[3 marks]*

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4. *[Maximum mark: 6]*

Consider $g(x) = 3\sin 2x$.

(a) Write down the period of g. *[1 mark]*

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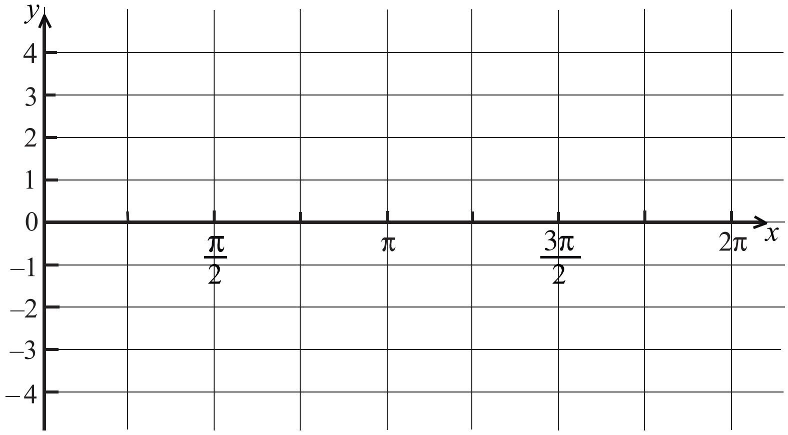
(b) On the diagram below, sketch the curve of g, for $0 \le x \le 2\pi$. *[3 marks]*



(c) Write down the number of solutions to the equation $g(x) = 2$, for $0 \le x \le 2\pi$. *[2 marks]*

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5. *[Maximum mark: 6]*

(a) Find $\int \frac{1}{2x+3}\,\mathrm{d}x$. *[2 marks]*

(b) Given that $\int_0^3 \frac{1}{2x+3}\,\mathrm{d}x = \ln\sqrt{P}$, find the value of P. *[4 marks]*

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6. *[Maximum mark: 7]*

A particle moves along a straight line so that its velocity, $v \text{ m s}^{-1}$ at time t seconds is given by $v = 6e^{3t} + 4$. When $t = 0$, the displacement, s, of the particle is 7 metres. Find an expression for s in terms of t.

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7. *[Maximum mark: 7]*

Let $f(x) = \ln(x+5) + \ln 2$, for $x > -5$.

(a) Find $f^{-1}(x)$. *[4 marks]*

Let $g(x) = \mathrm{e}^x$.

(b) Find $(g \circ f)(x)$, giving your answer in the form $ax + b$, where $a, b \in \mathbb{Z}$. *[3 marks]*

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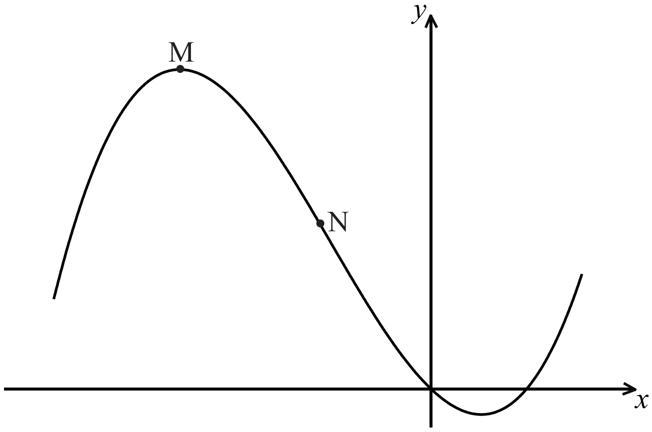
SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 14]*

Consider $f(x)=\frac{1}{3}x^3+2x^2-5x$. Part of the graph of f is shown below. There is a maximum point at M, and a point of inflexion at N.



(a) Find $f'(x)$. *[3 marks]*

(b) Find the x-coordinate of M. *[4 marks]*

(c) Find the x-coordinate of N. *[3 marks]*

(d) The line L is the tangent to the curve of f at $(3, 12)$. Find the equation of L in the form $y=ax+b$. *[4 marks]*

9. *[Maximum mark: 15]*

Let $f(x) = 3(x+1)^2 - 12$.

(a) Show that $f(x) = 3x^2 + 6x - 9$. *[2 marks]*

(b) For the graph of f

(i) write down the coordinates of the vertex;

(ii) write down the **equation** of the axis of symmetry;

(iii) write down the y-intercept;

(iv) find both x-intercepts. *[8 marks]*

(c) **Hence** sketch the graph of f. *[2 marks]*

(d) Let $g(x) = x^2$. The graph of f may be obtained from the graph of g by the two transformations:

a stretch of scale factor t in the y-direction

followed by

a translation of $\begin{pmatrix} p \\ q \end{pmatrix}$.

Find $\begin{pmatrix} p \\ q \end{pmatrix}$ and the value of t. *[3 marks]*

10. *[Maximum mark: 16]*

A **four-sided** die has three blue faces and one red face. The die is rolled.

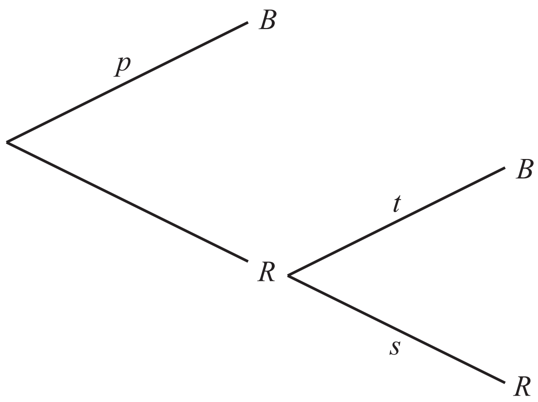
Let B be the event a blue face lands down, and R be the event a red face lands down.

(a) Write down

(i) $\mathrm{P}(B)$;

(ii) $\mathrm{P}(R)$. *[2 marks]*

(b) If the blue face lands down, the die is not rolled again. If the red face lands down, the die is rolled once again. This is represented by the following tree diagram, where p, s, t are probabilities.



Find the value of p, of s and of t. *[2 marks]*

Guiseppi plays a game where he rolls the die. If a blue face lands down, he scores 2 and is finished. If the red face lands down, he scores 1 and rolls one more time. Let X be the total score obtained.

(c) (i) Show that $\mathrm{P}(X=3)=\frac{3}{16}$.

(ii) Find $\mathrm{P}(X=2)$. *[3 marks]*

(d) (i) Construct a probability distribution table for X.

(ii) Calculate the expected value of X. *[5 marks]*

(e) If the total score is 3, Guiseppi wins \$ 10. If the total score is 2, Guiseppi gets nothing.

Guiseppi plays the game twice. Find the probability that he wins exactly \$ 10. *[4 marks]*