M08/5/MATME/SP2/ENG/TZ2/XX

22087306

International Baccalaureate®
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MATHEMATICS
STANDARD LEVEL
PAPER 2

Thursday 8 May 2008 (morning)

1 hour 30 minutes

Candidate session number

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 6]*

Consider the infinite geometric sequence $3000, -1800, 1080, -648, \ldots$.

(a) Find the common ratio. *[2 marks]*

(b) Find the 10^{th} term. *[2 marks]*

(c) Find the **exact** sum of the infinite sequence. *[2 marks]*

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2. *[Maximum mark: 5]*

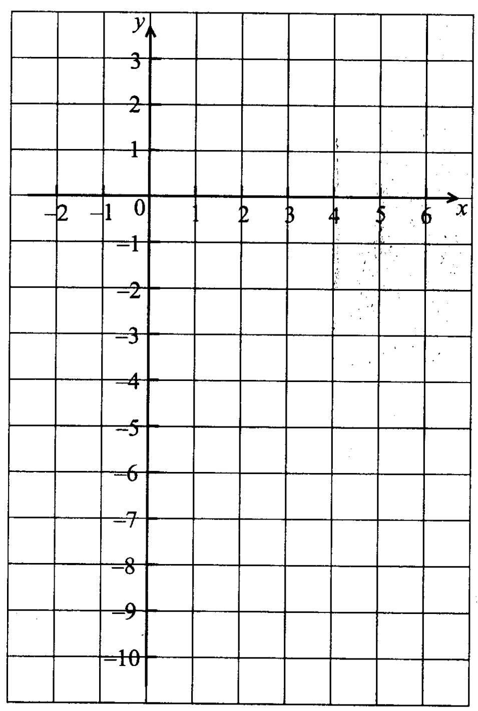
Find the term in x^3 in the expansion of $\left(\frac{2}{3}x-3\right)^8$.

3. *[Maximum mark: 7]*

Let $f(x) = 3x - e^{x-2} - 4$, for $-1 \le x \le 5$.

(a) Find the x-intercepts of the graph of f. *[3 marks]*

(b) On the grid below, sketch the graph of f. *[2 marks]*



(This question continues on the following page)

(Question 3 continued)

(c) Write down the gradient of the graph of f at $x = 2$. *[2 marks]*

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4. *[Maximum mark: 7]*

The following table shows the probability distribution of a discrete random variable X.

x	-1	0	2	3
$\text{P}(X = x)$	0.2	$10k^2$	0.4	$3k$

(a) Find the value of k. *[4 marks]*

(b) Find the expected value of X. *[3 marks]*

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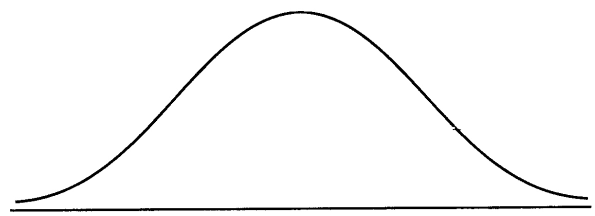
5. *[Maximum mark: 7]*

The heights of certain plants are normally distributed. The plants are classified into three categories.

The shortest 12.92 % are in category A.
The tallest 10.38 % are in category C.
All the other plants are in category B with heights between r cm and t cm.

(a) Complete the following diagram to represent this information. *[2 marks]*



(b) Given that the mean height is 6.84 cm and the standard deviation 0.25 cm, find the value of r and of t. *[5 marks]*

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6. *[Maximum mark: 7]*

Paula goes to work three days a week. On any day, the probability that she goes on a red bus is $\frac{1}{4}$.

(a) Write down the expected number of times that Paula goes to work on a red bus in one week. *[2 marks]*

In one week, find the probability that she goes to work on a red bus

(b) on exactly two days; *[2 marks]*

(c) on at least one day. *[3 marks]*

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7. *[Maximum mark: 6]*

The line L_1 is represented by $\boldsymbol{r}_1 = \begin{pmatrix} 2 \\ 5 \\ 3 \end{pmatrix} + s \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and the line L_2 by $\boldsymbol{r}_2 = \begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix} + t \begin{pmatrix} -1 \\ 3 \\ -4 \end{pmatrix}$.

The lines L_1 and L_2 intersect at point T. Find the coordinates of T.

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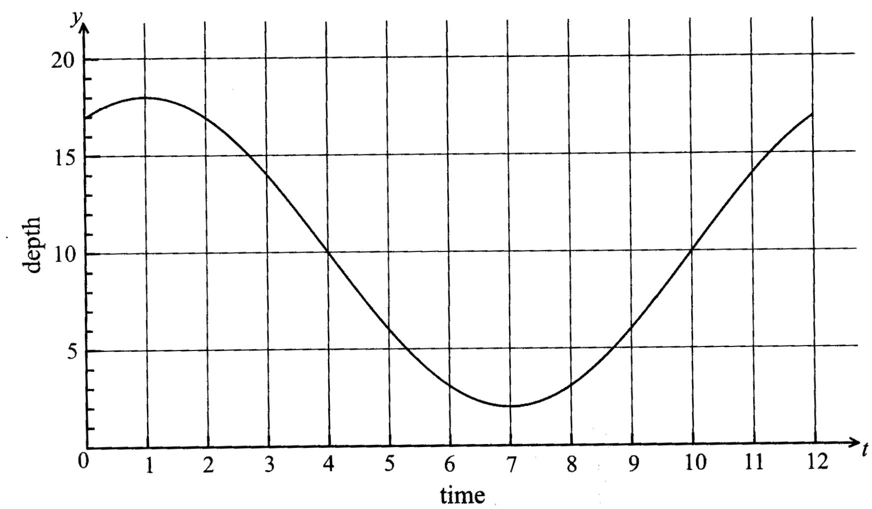
SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 11]*

The following graph shows the depth of water, y metres, at a point P, during one day. The time t is given in hours, from midnight to noon.



(a) Use the graph to write down an estimate of the value of t when

(i) the depth of water is minimum;

(ii) the depth of water is maximum;

(iii) the depth of the water is increasing most rapidly. *[3 marks]*

(b) The depth of water can be modelled by the function $y = A\cos(B(t-1)) + C$.

(i) Show that $A = 8$.

(ii) Write down the value of C.

(iii) Find the value of B. *[6 marks]*

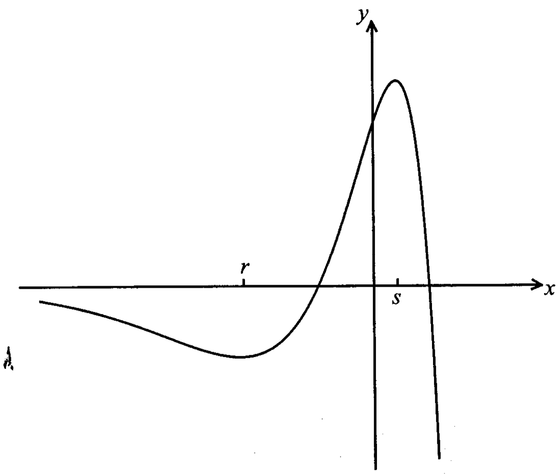
(c) A sailor knows that he cannot sail past P when the depth of the water is less than 12 m. Calculate the values of t between which he cannot sail past P. *[2 marks]*

9. *[Maximum mark: 17]*

Let $f(x) = \mathrm{e}^x(1 - x^2)$, $-6 \le x \le 2$.

(a) Show that $f'(x) = \mathrm{e}^x(1 - 2x - x^2)$. *[3 marks]*

The graph of $y = f(x)$ is shown below. The x-coordinates of the local minimum and maximum points are r and s respectively.



(b) Write down the **equation** of the horizontal asymptote. *[1 mark]*

(c) Write down the value of r and of s. *[4 marks]*

(d) Let L be the normal to the curve of f at P(0, 1). Show that L has equation $x + y = 1$. *[4 marks]*

(e) Let R be the region enclosed by the curve $y = f(x)$ and the line L.

(i) Find an expression for the area of R.

(ii) Calculate the area of R. *[5 marks]*

10. *[Maximum mark: 17]*

A city is concerned about pollution, and decides to look at the number of people using taxis. At the end of the year 2000, there were 280 taxis in the city. After n years the number of taxis, T, in the city is given by

$$T = 280 \times 1.12^n .$$

(a) (i) Find the number of taxis in the city at the end of 2005.

(ii) Find the year in which the number of taxis is double the number of taxis there were at the end of 2000. *[6 marks]*

(b) At the end of 2000 there were 25 600 people in the city who used taxis. After n years the number of people, P, in the city who used taxis is given by

$$P = \frac{2\,560\,000}{10 + 90e^{-0.1n}} .$$

(i) Find the value of P at the end of 2005, giving your answer to the nearest whole number.

(ii) After seven complete years, will the value of P be double its value at the end of 2000? Justify your answer. *[6 marks]*

(c) Let R be the ratio of the number of people using taxis in the city to the number of taxis. The city will reduce the number of taxis if $R < 70$.

(i) Find the value of R at the end of 2000.

(ii) After how many complete years will the city first reduce the number of taxis? *[5 marks]*
