COPY to Mr Anil

**International Baccalaureate®**
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22087305

# MATHEMATICS
# STANDARD LEVEL
# PAPER 1

Wednesday 7 May 2008 (afternoon)

1 hour 30 minutes

Candidate session number

| 0 | 0 | | | | | | | |
|---|---|---|---|---|---|---|---|---|

## INSTRUCTIONS TO CANDIDATES

* Write your session number in the boxes above.
* Do not open this examination paper until instructed to do so.
* You are not permitted access to any calculator for this paper.
* Section A:   answer all of Section A in the spaces provided.
* Section B:   answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
* At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
* Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

*Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.*

## SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1.  *[Maximum mark: 7]*

    A box contains 100 cards. Each card has a number between one and six written on it. The following table shows the frequencies for each number.
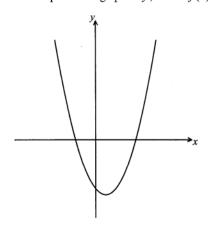
    | Number | 1 | 2 | 3 | 4 | 5 | 6 |
    |---|---|---|---|---|---|---|
    | Frequency | 26 | 10 | 20 | $k$ | 29 | 11 |

    (a)  Calculate the value of $k$.                                           *[2 marks]*

    (b)  Find

        (i)   the median;

        (ii)  the interquartile range.                                        *[5 marks]*

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**2.** *[Maximum mark: 6]*

The following diagram shows part of the graph of $f$, where $f(x) = x^2 - x - 2$.



(a) Find both $x$-intercepts. [4 marks]

(b) Find the $x$-coordinate of the vertex. [2 marks]

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3.     *[Maximum mark: 6]*

Let $M = \begin{pmatrix} 2 & 1 \\ 2 & -1 \end{pmatrix}$.

    (a)   Write down the determinant of $M$.              *[1 mark]*

    (b)   Write down $M^{-1}$.              *[2 marks]*

    (c)   **Hence** solve $M \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$.        *[3 marks]*

**4.** *[Maximum mark: 6]*

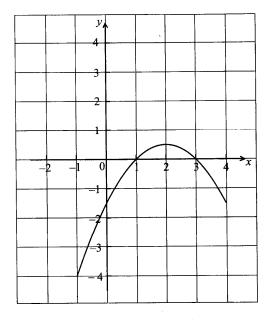(a) Given that $\cos A = \dfrac{1}{3}$ and $0 \le A \le \dfrac{\pi}{2}$, find $\cos 2A$.

*[3 marks]*

(b) Given that $\sin B = \dfrac{2}{3}$ and $\dfrac{\pi}{2} \le B \le \pi$, find $\cos B$.

*[3 marks]*

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**Turn over**

**5.** *[Maximum mark: 6]*

Part of the graph of a function $f$ is shown in the diagram below.



(a) On the same diagram sketch the graph of $y = -f(x)$. *[2 marks]*

(b) Let $g(x) = f(x+3)$.

    (i) Find $g(-3)$.

    (ii) Describe **fully** the transformation that maps the graph of $f$ to the graph of $g$. *[4 marks]*

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**6.** *[Maximum mark: 7]*

There are 20 students in a classroom. Each student plays only one sport. The table below gives their sport and gender.

|  | Football | Tennis | Hockey |
|---|---|---|---|
| **Female** | 5 | 3 | 3 |
| **Male** | 4 | 2 | 3 |

(a) One student is selected at random.

    (i) Calculate the probability that the student is a male or is a tennis player.

    (ii) Given that the student selected is female, calculate the probability that the student does not play football. *[4 marks]*

(b) Two students are selected at random. Calculate the probability that neither student plays football. *[3 marks]*

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**7.** *[Maximum mark: 7]*

Let $\int_{1}^{5} 3f(x)\,dx = 12$ .

(a) Show that $\int_{5}^{1} f(x)\,dx = -4$ . *[2 marks]*

(b) Find the value of $\int_{1}^{2} \left(x + f(x)\right)\,dx + \int_{2}^{5} \left(x + f(x)\right)\,dx$ . *[5 marks]*

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**SECTION B**

*Answer **all** the questions on the answer sheets provided.  Please start each question on a new page.*

8.  *[Maximum mark: 13]*

    Consider the points $A(1, 5, 4)$, $B(3, 1, 2)$ and $D(3, k, 2)$, with $(AD)$ perpendicular to $(AB)$.

    (a)  Find

         (i)   $\overrightarrow{AB}$;

         (ii)  $\overrightarrow{AD}$, giving your answer in terms of $k$.          *[3 marks]*

    (b)  Show that $k = 7$.                                                        *[3 marks]*

    The point C is such that $\overrightarrow{BC} = \dfrac{1}{2}\overrightarrow{AD}$.

    (c)  Find the position vector of C.                                           *[4 marks]*

    (d)  Find $\cos A\hat{B}C$.                                                    *[3 marks]*

9.  *[Maximum mark: 14]*

    Let $f : x \mapsto \sin^3 x$.

    (a)  (i)   Write down the range of the function $f$.

         (ii)  Consider $f(x) = 1$, $0 \le x \le 2\pi$.  Write down the number of solutions to this equation.  Justify your answer.          *[5 marks]*

    (b)  Find $f'(x)$, giving your answer in the form $a \sin^p x \cos^q x$ where $a, p, q \in \mathbb{Z}$.          *[2 marks]*
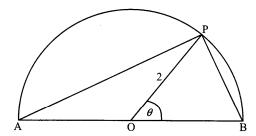
    (c)  Let $g(x) = \sqrt{3} \sin x (\cos x)^{\frac{1}{2}}$ for $0 \le x \le \dfrac{\pi}{2}$.  Find the volume generated when the curve of $g$ is revolved through $2\pi$ about the $x$-axis.          *[7 marks]*

**10.** *[Maximum mark: 18]*

The following diagram shows a semicircle centre O, diameter [AB], with radius 2.
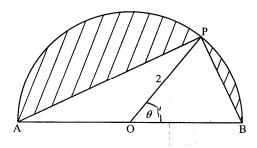Let P be a point on the circumference, with $P\hat{O}B = \theta$ radians.



(a) Find the area of the triangle OPB, in terms of $\theta$. *[2 marks]*

(b) Explain why the area of triangle OPA is the same as the area triangle OPB. *[3 marks]*

Let $S$ be the total area of the two segments shaded in the diagram below.



(c) Show that $S = 2(\pi - 2\sin\theta)$. *[3 marks]*

(d) Find the value of $\theta$ when $S$ is a local minimum, justifying that it is a minimum. *[8 marks]*

(e) Find a value of $\theta$ for which $S$ has its greatest value. *[2 marks]*