MARKSCHEME

May 2007

MATHEMATICS

Higher Level

Paper 1

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Instructions to Examiners

Abbreviations

M   Marks awarded for attempting to use a correct **Method**; working must be seen.

(M)   Marks awarded for **Method**; may be implied by **correct** subsequent working.

A   Marks awarded for an **Answer** or for **Accuracy**: often dependent on preceding **M** marks.

(A)   Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R   Marks awarded for clear **Reasoning**.

N   Marks awarded for **correct** answers if **no** working shown.

AG   Answer given in the question and so no marks are awarded.

Using the markscheme

1   General

Write the marks in red on candidates' scripts, in the right hand margin.

- Show the **breakdown** of individual marks awarded using the abbreviations *M1*, *A1*, *etc*.
- Write down the total for each **question** (at the end of the question) and **circle** it.

2   Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award *M0* followed by *A1*, as *A* mark(s) depend on the preceding *M* mark(s), if any.
- Where *M* and *A* marks are noted on the same line, *e.g.* *M1A1*, this usually means *M1* for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and *A1* for using the **correct** values.
- Where the markscheme specifies *(M2)*, *N3*, *etc.*, do **not** split the marks.
- Once a correct answer to a question or part-question is seen, ignore further working.

3   *N* marks

*Award N marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of *N* and other marks.
- There may be fewer *N* marks available than the total of *M*, *A* and *R* marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4   Implied marks

*Implied marks appear in **brackets e.g. (M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5   Follow through marks

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s). To award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer **FT** marks.
- If the error leads to an inappropriate value (*e.g.* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent** *A* marks can be awarded, but *M* marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6   Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). Apply a **MR** penalty of 1 mark to that question. Award the marks as usual and then write –1(**MR**) next to the total. Subtract 1 mark from the total for the question. A candidate should be penalized only once for a particular mis-read.*

- If the question becomes much simpler because of the **MR**, then use discretion to award fewer marks.
- If the **MR** leads to an inappropriate value (*e.g.* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7   Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. The mark should be labelled **(d)** and a brief **note** written next to the mark explaining this decision.*

8   Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9   Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

  Example: for differentiating $f(x) = 2\sin(5x - 3)$, the markscheme gives:

  $$f'(x) = \left(2\cos(5x-3)\right)5 \quad \left(= 10\cos(5x-3)\right) \qquad\qquad \textbf{\textit{A1}}$$

  Award **A1** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

10   Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy.

- **Rounding errors**: only applies to final answers not to intermediate steps.
- **Level of accuracy**: when this is not specified in the question the general rule applies: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

*Candidates should be penalized **once only IN THE PAPER** for an accuracy error (AP). Award the marks as usual then write (AP) against the answer. On the **front** cover write –1(AP). Deduct 1 mark from the total for the paper, not the question.*

- If a final correct answer is incorrectly rounded, apply the *AP*.
- If the level of accuracy is not specified in the question, apply the *AP* for correct answers not given to three significant figures.

If there is no working shown, and answers are given to the correct two significant figures, apply the *AP*. However, do **not** accept answers to one significant figure without working.

11   Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

QUESTION 1

(a) $1.19 = \dfrac{1}{2}(1.74)(\sin 42°)\,AC$ *M1*

$AC = 2.044...$

$AC = 2.04 \;\text{(cm)}$ *A1*   *N1*

(b) $AB^2 = 2.044...^2 + 1.74^2 - 2(1.74)(2.044...)\cos 42°$ *M1A1*

$AB^2 = 1.919...$ (or $1.913...$)   *(A1)*

$AB = 1.39 \;\text{(cm)}$ (accept 1.38 cm or 1.40 cm)   *A1*   *N2*

QUESTION 2

(a)   (i)   Use of $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ *M1*

$0.6 = 0.5 + 0.3 - P(A \cap B)$

$P(A \cap B) = 0.2$ *A1*   *N1*

(ii) $P(A)P(B) = 0.15 \neq P(A \cap B)$ *R1*

Hence not independent   *AG*

(b)   Use of $P(B \mid A) = \dfrac{P(B \cap A)}{P(A)}$ *M1*

$= \dfrac{0.2}{0.5}$ *(A1)*

$= 0.4$ *A1*   *N0*

QUESTION 3

(a) X has a binomial distribution   *(M1)*

$0.4 = np$ $0.36 = np(1-p)$ *A1A1*

Attempting to solve.   *(M1)*

$p = 0.1$ *A1*   *N3*

(b) $n = 4$ *A1*   *N1*

| **Note:**   Allow *FT* only on integer values of n for p between 0 and 1. |

QUESTION 4

  (a) **METHOD 1**

Interchanging x and y, $x = \dfrac{3y-4}{y+2}$ *M1*

$yx + 2x = 3y - 4$ *(A1)*

simplifying $y(x-3) = -2x - 4$ *(M1)*

expressing y in terms of x, $y = \dfrac{2x+4}{3-x}$ *(A1)*

$f^{-1}(x) = \dfrac{2x+4}{3-x}$ *A1*   *N2*

        METHOD 2

$y = \dfrac{3x-4}{x+2}$

$xy + 2y = 3x - 4$ *A1*

simplifying $x(y-3) = -2y - 4$ *(M1)*

expressing y in terms of x, $x = \dfrac{2y+4}{3-y}$ *(A1)*

interchanging x and y, $y = \dfrac{2x+4}{3-x}$ *(M1)*

$f^{-1}(x) = \dfrac{2x+4}{3-x}$ *A1*   *N2*

> **Note:**   Only 2 of these marks can be implied.

  (b) Domain $x \neq 3$, ($x \in \mathbb{R}$ not required)   *A1*

QUESTION 5

Area of sector $= \dfrac{\theta}{2}$ *(A1)*

Area of triangle $= \dfrac{\sin\theta}{2}$ *(A1)*

$\dfrac{\sin\theta}{2} = 3\left(\dfrac{\theta}{2} - \dfrac{\sin\theta}{2}\right)$ *M1A1*

$3\theta = 4\sin\theta$ *(A1)*

$\theta = 1.28$ radians   *A1*   *N2*

> **Note:** Accept $73.1°$.

QUESTION 6

(a)　　$f'(x) = \ln x + x\left(\dfrac{1}{x}\right) - 1$　　　　　　　　　　　　　　　　***(M1)***

　　　　　　$= \ln x$　　　　　　　　　　　　　　　　　　　　　　***A1***　　　***N2***

(b)　　Using integration by parts

　　　METHOD 1

　　　$\displaystyle\int (\ln x)^2\, dx = (\ln x)^2 \bullet x - \int x \bullet \dfrac{2}{x}(\ln x)\, dx$　　　　　　***A1A1***

　　　　　　　　$= x(\ln x)^2 - 2\displaystyle\int (\ln x)\, dx$　　　　　　　　***(A1)***

　　　　　　　　$= x(\ln x)^2 - 2(x\ln x - x) + C$　　　　　　　***A1***

　　　　　　　　$\left(= x(\ln x)^2 - 2x\ln x + 2x + C\right)$

　　　METHOD 2

　　　$\displaystyle\int (\ln x)^2\, dx = x(\ln x)^2 - x\ln x - \int (\ln x - 1)\, dx$　　　***A1A1A1***

　　　　　　　　$= x(\ln x)^2 - x\ln x - (x\ln x - x - x) + C$　　　***A1***

　　　　　　　　$\left(= x(\ln x)^2 - 2x\ln x + 2x + C\right)$

> **Note:**　Do not penalize the absence of $+ C$.

QUESTION 7

(a)　　$z = 4\left(-\dfrac{1}{2} + i\dfrac{\sqrt{3}}{2}\right) + 4\sqrt{3}\left(\dfrac{\sqrt{3}}{2} + \dfrac{i}{2}\right)$　　　　　　　　***A1A1***

　　　　$= 8\left(\dfrac{1}{2} + \dfrac{i\sqrt{3}}{2}\right) = 4 + 4i\sqrt{3}$

　　　$r = 8$　　　　　　　　　　　　　　　　　　　　　　　　***A1***

　　　$\theta = \dfrac{\pi}{3}$ or $60°$　　　　　　　　　　　　　　　　　　***A1***

　　　$\left(z = 8e^{i\left(\frac{\pi}{3}\right)}\right)$　　　　　　　　　　　　　　　　　　　　***N4***

(b)　　$z^{\frac{1}{3}} = 2e^{i\left(\frac{\pi}{9}\right)}\ (= 2e^{i20°})$　　　　　　　　　　　　　　　***A1***

　　　$z^{\frac{1}{3}} = 2e^{i\left(\frac{7\pi}{9}\right)}\ (= 2e^{i140°}),\quad z^{\frac{1}{3}} = 2e^{i\left(\frac{13\pi}{9}\right)}\ (= 2e^{i260°})$　　***A1***

> **Notes:**　Do not allow any form other than $re^{i\theta}$.　Both answers must be given for final ***A1***.

QUESTION 8

(a) x-intercepts are $x = 1$ and $x = \pi$ (accept 3.14)   ***A1A1***

(b)   Attempting to find the area of two regions   ***M1***

$B = \int_{0.5}^{1} \sin x \ln x \, dx + \int_{\pi}^{3.5} \sin x \ln x \, dx$

$\quad = -(0.09310... + 0.07736...)$

$B = 0.1704...$ ***(A1)***

$A = \int_{1}^{\pi} \sin x \ln x \, dx = 0.8809...$ ***(A1)***

$0.8809 = k \times 0.1704$

$\quad\quad k = 5.17$ ***A1***   ***N2***

> **Notes:** Accept values for A and B rounded to at least two decimal places.
>
> Accept only 5.17 for final ***A1***.
>
> Do not penalize if a negative value of B is used to yield a negative value of k.

QUESTION 9

$\mathrm{E}(X) = \int_{0}^{\ln 2} x\mathrm{e}^{x} \, dx$ ***M1A1***

Attempting to use integration by parts   ***(M1)***

$\int x\mathrm{e}^{x} \, dx = x\mathrm{e}^{x} - \int \mathrm{e}^{x} \, dx$ ***A1***

$\mathrm{E}(X) = \left[x\mathrm{e}^{x} - \mathrm{e}^{x} \right]_{0}^{\ln 2}$ ***A1***

$\quad = 2\ln 2 - 1 \quad = \left(\ln 4 - 1, \; \ln \dfrac{4}{e} \right)$ ***A1***   ***N3***

> **Note:**   If no working shown, award ***N1*** for the answer 0.386.

QUESTION 10

$$a \cdot b = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}$$

$$= 3 \hspace{6cm} \textbf{\textit{(A1)}}$$

$$s = 3 \begin{pmatrix} 3 \\ 1 \\ \lambda \end{pmatrix} + \begin{pmatrix} \mu \\ -2 \\ 1 \end{pmatrix} \hspace{5cm} \textbf{\textit{M1}}$$

Note: Allow *FT* on $a \cdot b$ provided $a \cdot b$ is scalar.

$$s = \begin{pmatrix} 9 + \mu \\ 1 \\ 3\lambda + 1 \end{pmatrix} \hspace{5cm} \textbf{\textit{A1}}$$

$$s \cdot a = 0 \Rightarrow \begin{pmatrix} 9 + \mu \\ 1 \\ 3\lambda + 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} = 0 \hspace{3cm} \textbf{\textit{(M1)}}$$

Note: Allow *FT* for *s*.

$$18 + 2\mu + 3 - 3\lambda - 1 = 0 \quad (20 + 2\mu = 3\lambda) \hspace{2.5cm} \textbf{\textit{A1}}$$

$$\lambda = \frac{20 + 2\mu}{3} \hspace{5cm} \textbf{\textit{A1}} \hspace{2cm} \textbf{\textit{N3}}$$

QUESTION 11

$$y = x\arcsin x$$

$$\frac{dy}{dx} = \arcsin x + \frac{x}{\sqrt{1-x^2}}$$ *M1A1*

$$\frac{d}{dx}\left(\frac{x}{\sqrt{1-x^2}}\right) = \frac{1\left(\sqrt{1-x^2}\right) + x^2(1-x^2)^{-\frac{1}{2}}}{1-x^2}$$ *M1A1*

$$\frac{d^2y}{dx^2} = \frac{1}{\sqrt{1-x^2}} + \frac{1\left(\sqrt{1-x^2}\right) + x^2(1-x^2)^{-\frac{1}{2}}}{1-x^2}$$

$$= \frac{1}{(1-x^2)^{\frac{1}{2}}} + \frac{1}{(1-x^2)^{\frac{1}{2}}} + \frac{x^2}{(1-x^2)^{\frac{3}{2}}}$$ *A1*

$$= \frac{2}{(1-x^2)^{\frac{1}{2}}} + \frac{x^2}{(1-x^2)^{\frac{3}{2}}}$$

$$= \frac{2\left(1-x^2\right) + x^2}{(1-x^2)^{\frac{3}{2}}}$$ *A1*

$$= \frac{2-x^2}{(1-x^2)^{\frac{3}{2}}}$$ *AG*

Note: The final *A1A1* are for equivalent algebraic manipulations leading to *AG*.

QUESTION 12

Using $b^2 - 4ac = (k-3)^2 - 4k(k-8)$ *M1A1*

$-3k^2 + 26k + 9 = 0$

$\Rightarrow k = -\frac{1}{3}, \ k = 9$ *(A1)*

Using $b^2 - 4ac < 0$ *R1*

$-3k^2 + 26k + 9 < 0 \quad (3k^2 - 26k - 9 > 0)$

$k < -\frac{1}{3} \quad \text{or} \quad k > 9$ *A1A1*   *N2N2*

Notes: Award *A1A0* for ($k \leq -\frac{1}{3}$ or $k \geq 9$).

Award *A1FT A1FT* for correct inequalities from incorrect values of k.

If no working shown, award *N3* ($k \leq -\frac{1}{3}$ or $k \geq 9$).

QUESTION 13

METHOD 1

$$\begin{pmatrix} 1 & 6 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+6y \\ 4x+3y \end{pmatrix}$$ *M1*

$$\Rightarrow \begin{cases} x+6y = kx \\ 4x+3y = ky \end{cases}$$ *A1*

Getting 1 variable $x+6y = kx \Rightarrow y = \dfrac{(k-1)x}{6}$ *M1*

Substituting correctly in other equation

$$4x + \frac{3(k-1)x}{6} = \frac{k(k-1)x}{6}$$ *A1*

Finding equation in k

$24 + 3(k-1) = k^2 - k \quad (0 = k^2 - 4k - 21)$ *A1*

$k = 7$ or $k = -3$ *A1*

METHOD 2

$$\begin{pmatrix} 1-k & 6 \\ 4 & 3-k \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 0$$ *M1A1*

Determinant $= 0$ *M1*

$(1-k)(3-k) - 24 = 0$ *A1*

$3 - 4k + k^2 - 24 = 0$

$k^2 - 4k - 21 = 0$ *A1*

$(k-7)(k+3) = 0$

$k = 7$ or $k = -3$ *A1*

Note: Award *A2* if candidate finds only one value of k by any method.

QUESTION 14

(a) $r = 4 - 3x \implies$

$\qquad |4 - 3x| < 1$ *M1*

$\qquad -1 < 4 - 3x < 1$

$\qquad 1 < x < \dfrac{5}{3}$ *A1*   *N1*

(b) $x = 1.2$

$\qquad \implies a = 0.8 \quad r = 0.4$ *(A1)*

$\qquad S_n = \dfrac{0.8(1 - 0.4^n)}{0.6}$ *A1*

$\qquad$ So $\dfrac{0.8(1 - 0.4^n)}{0.6} > 1.328$

$\qquad$ Solving gives $n > 6.02$ *(A1)*

$\qquad$ 7 terms are needed   *A1*   *N4*

> **Note:** Generating terms of the series to find that 7 terms are needed is an alternative method.

QUESTION 15

$e^{xy} - y^2 \ln x = 1$

Differentiating implicitly   *(M1)*

$e^{xy}\left(y + x\dfrac{dy}{dx} \right) - \left(2y\dfrac{dy}{dx}\ln x + \dfrac{y^2}{x} \right) = 0$ *A1A1*

> **Notes:** Award *A1* for $e^{xy}\left(y + x\dfrac{dy}{dx} \right)$, *A1* for $-\left(2y\dfrac{dy}{dx}\ln x + \dfrac{y^2}{x} \right)$ **and** 0.
>
> $\qquad$ **N.B.** Incorrect manipulation of e^{xy} can lead to the correct final result.

EITHER

Collecting terms $e^{xy}y - \dfrac{y^2}{x} = (2y\ln x - xe^{xy})\dfrac{dy}{dx}$ *(M1)*

$\dfrac{dy}{dx} = \dfrac{ye^{xy} - \dfrac{y^2}{x}}{2y\ln x - xe^{xy}}$ *(A1)*

$x = 1 \quad y = 1 \implies \dfrac{dy}{dx} = \dfrac{1 - e}{e}$ *A1*

OR

Substituting $x = 1 \quad y = 1$ *(M1)*

$e\left(1 + \dfrac{dy}{dx} \right) - \left(2\dfrac{dy}{dx}\ln 1 + 1 \right) = 0$ *(A1)*

$\dfrac{dy}{dx} = \dfrac{1 - e}{e}$ *A1*

QUESTION 16

$$\frac{\mathrm{d}y}{y} = \frac{x\,\mathrm{d}x}{x^2+1}$$ *M1*

$$\int \frac{\mathrm{d}y}{y} = \ln y$$ *A1*

$$\int \frac{x}{x^2+1}\,\mathrm{d}x = \frac{1}{2}\ln(x^2+1)$$ *A1*

EITHER

$$\ln y = \frac{1}{2}\ln(x^2+1) + \ln C$$

$$\ln y = \ln C\sqrt{x^2+1}$$

$$1 = C\sqrt{2}\ \text{ for substituting } x=1,\ y=1$$ *M1*

$$C = \frac{1}{\sqrt{2}}$$ *A1*

$$y = \sqrt{\frac{x^2+1}{2}}$$ *A1*

OR

$$\ln y = \frac{1}{2}\ln(x^2+1) + A$$

$$\ln 1 = \frac{1}{2}\ln 2 + A\ \text{ for substituting } x=1,\ y=1$$ *M1*

$$A = -\frac{1}{2}\ln 2$$ *A1*

$$\ln y = \frac{1}{2}\ln(x^2+1) - \frac{1}{2}\ln 2\ \left(= \frac{1}{2}\ln(x^2+1) - 0.347\right)$$ *A1*

$$\left(\ln y = \ln\sqrt{\frac{x^2+1}{2}}\right)$$

$$\left(y = \sqrt{\frac{x^2+1}{2}}\right)$$

QUESTION 17

$(f \circ g)(x) = 2\left(\dfrac{x}{x+1}\right) - 1$ *(A1)*

$(g \circ f)(x) = \dfrac{2x-1}{2x}$ *(A1)*

EITHER

$\dfrac{2x}{x+1} - 1 \le \dfrac{2x-1}{2x}$ *M1*

Getting 0 on one side   *(M1)*

$\dfrac{2x}{x+1} - 1 - \dfrac{1-3x}{2x} \le 0 \quad \left(\dfrac{1-3x}{2x(x+1)} \le 0\right)$

$-1 < x < 0 \text{ or } x \ge \dfrac{1}{3}$ *A1A1*

OR

For attempting to graph $(f \circ g)(x)$ and $(g \circ f)(x)$ or $(f \circ g)(x) - (g \circ f)(x)$ *M1*
For an accurate graph.   *A1*

$-1 < x < 0 \text{ or } x \ge \dfrac{1}{3}$ *A1A1*

> **Notes:** The inequality sign **and** the values of x must both be correct.
> Accept ($-1 < x < 0$ **and** $x \ge \dfrac{1}{3}$).

QUESTION 18

(a)   derivative $= \dfrac{3f'(x)[g(x)-1] - 3f(x)g'(x)}{[g(x)-1]^2}$ *M1*

       when $x = 0 \Rightarrow$ derivative $= \dfrac{3(1)(-4-1) - 3(4)(5)}{(-4-1)^2}$ *(A1)*

                   $= -3$ *A1*   *N0*

(b)   derivative $= f'(g(x)+2x)(g'(x)+2)$ *M1*

       when $x = 1$ derivative $= f'(-1+2)(2+2)$ *(A1)*

            $= (3)(4)$

            $= 12$ *A1*   *N0*

QUESTION 19

$2^{2x+3} - 2^{x+1} - 3 = 0$

Let $p = 2^x$ *(M1)*

$8p^2 - 2p - 3 = 0$ *A1*

$(2p+1)(4p-3) = 0$

$p = -\dfrac{1}{2}$ or $p = \dfrac{3}{4}$ *A1*

$2^x = \dfrac{3}{4}$ *A1*

$x = \log_2 3 - \log_2 4$

$\quad = -2 + \log_2 3 \quad (a = -2, b = 3)$ *A1A1* **N0**

Note: Award no marks if candidates take $\log_2$ of each term, even though this gives the correct answer.	

QUESTION 20

$81\pi = \dfrac{4}{3}\pi(3)^3 + \pi(3)^2 h$ *(M1)*

$81\pi = 36\pi + 9\pi h$

$\quad h = 5 \text{ (cm)}$ *A1*

$V = \dfrac{4}{3}\pi r^3 + \pi r^2 h$

Attempt to differentiate with respect to time. *M1*

$\dfrac{dV}{dt} = 4\pi r^2 \dfrac{dr}{dt} +$ *A1*

$\quad\quad 2\pi rh \dfrac{dr}{dt} + \pi r^2 \dfrac{dh}{dt}$ *A1*

$204\pi = 4\pi(3)^2(2) + 2\pi(3)(5)(2) + \pi(3)^2 \dfrac{dh}{dt}$

$204\pi = 72\pi + 60\pi + 9\pi \dfrac{dh}{dt}$

$72\pi = 9\pi \dfrac{dh}{dt}$

$\dfrac{dh}{dt} = 8 \text{ (cm/min)}$ *A1* **N2**