IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
PROGRAMA DEL DIPLOMA DEL BI

22077402

MATHEMATICAL STUDIES
STANDARD LEVEL
PAPER 2

Tuesday 8 May 2007 (morning)

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Please start each question on a new page. You are advised to show all working, where possible. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. *[Maximum mark: 14]*

The figure below shows the lengths in centimetres of fish found in the net of a small trawler.
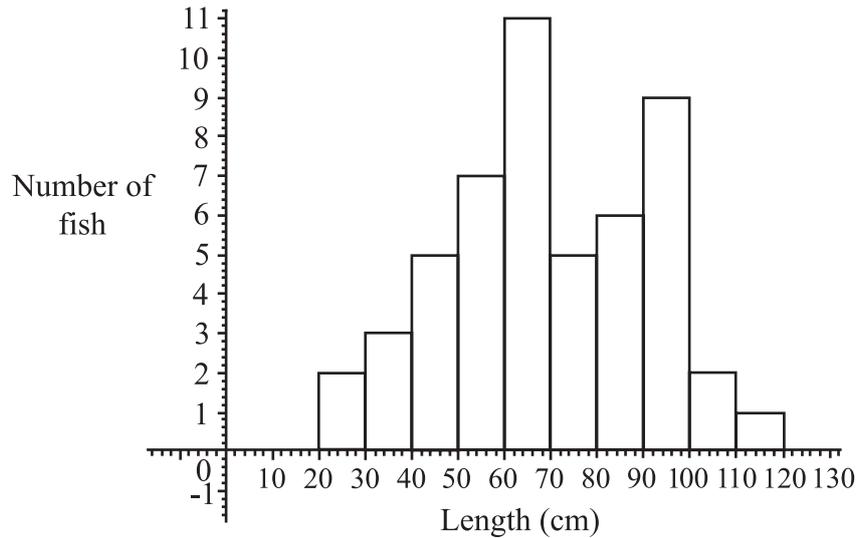
(a) Find the total number of fish in the net. *[2 marks]*

(b) Find (i) the modal length interval,

(ii) the interval containing the median length,

(iii) an estimate of the mean length. *[5 marks]*

(c) (i) Write down an estimate for the standard deviation of the lengths.

(ii) How many fish (if any) have length **greater than** three standard deviations **above** the mean? *[3 marks]*
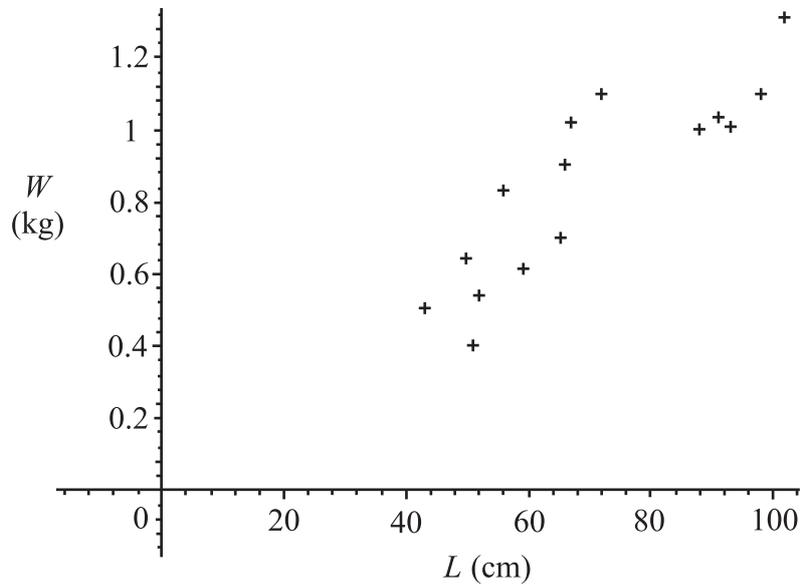
The fishing company must pay a fine if more than 10% of the catch have lengths less than 40cm.

(d) Do a calculation to decide whether the company is fined. *[2 marks]*

(This question continues on the following page)

(Question 1 continued)

A sample of 15 of the fish was weighed. The weight, *W* was plotted against length, *L* as shown below.

(e)   Exactly **two** of the following statements about the plot could be correct. Identify the two correct statements. *[2 marks]*

Note: You do **not** need to enter data in a GDC **or** to calculate *r* exactly.

(i)   The value of *r*, the correlation coefficient, is approximately 0.871.

(ii)   There is an exact linear relation between *W* and *L*.

(iii)   The line of regression of *W* on *L* has equation $W = 0.012L + 0.008$.

(iv)   There is negative correlation between the length and weight.

(v)   The value of *r*, the correlation coefficient, is approximately 0.998.

(vi)   The line of regression of *W* on *L* has equation $W = 63.5L + 16.5$.

Turn over

2. *[Maximum mark: 18]*

(i) Jenny has a circular cylinder with a lid. The cylinder has height 39 **cm** and diameter 65 **mm**.

(a) Calculate the volume of the cylinder **in cm³**. Give your answer correct to **two** decimal places. *[3 marks]*

The cylinder is used for storing tennis balls.
Each ball has a **radius** of 3.25 cm.

(b) Calculate how many balls Jenny can fit in the cylinder if it is filled to the top. *[1 mark]*

(c) (i) Jenny fills the cylinder with the number of balls found in part (b) and puts the lid on. Calculate the volume of air inside the cylinder in the spaces between the tennis balls.
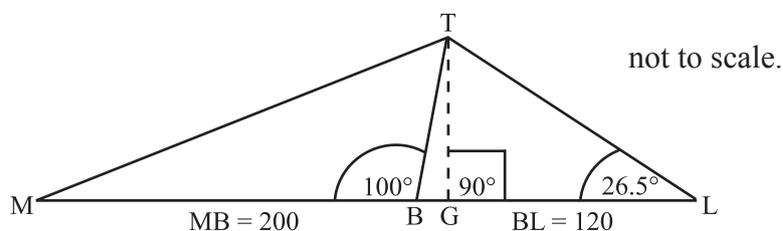
(ii) Convert your answer to (c) (i) into cubic metres. *[4 marks]*

(ii) An old tower (BT) leans at 10° away from the vertical (represented by line TG).

The base of the tower is at B so that $M\hat{B}T = 100°$.
Leonardo stands at L on flat ground 120 m away from B in the direction of the lean.
He measures the angle between the ground and the top of the tower T to be $B\hat{L}T = 26.5°$.

not to scale.

(a) (i) Find the value of angle $B\hat{T}L$.

(ii) Use triangle BTL to calculate the sloping distance BT from the base, B to the top, T of the tower. *[5 marks]*

(b) Calculate the vertical height TG of the top of the tower. *[2 marks]*

(c) Leonardo now walks to point M, a distance 200 m from B on the opposite side of the tower. Calculate the distance from M to the top of the tower at T. *[3 marks]*

3. *[Maximum mark: 21]*

Given $f(x) = x^2 - 3x^{-1}$, $x \in \mathbb{R}$, $-5 \le x \le 5$, $x \ne 0$,

(i) (a) Write down the equation of the vertical asymptote. *[1 mark]*

(b) Find $f'(x)$. *[2 marks]*

(c) Using your graphic display calculator or otherwise, write down the coordinates of any point where the graph of $y = f(x)$ has zero gradient. *[2 marks]*

(d) Write down all intervals in the given domain for which $f(x)$ is increasing. *[3 marks]*

(ii) A football is kicked from a point A $(a, 0)$, $0 < a < 10$ on the ground towards a goal to the right of A.

The ball follows a path that can be modelled by **part** of the graph

$$y = -0.021x^2 + 1.245x - 6.01, x \in \mathbb{R}, y \ge 0.$$

x is the horizontal distance of the ball from the origin
y is the height above the ground
Both x and y are measured in metres.

(a) Using your graphic display calculator or otherwise, find the value of a. *[1 mark]*

(b) Find $\dfrac{dy}{dx}$. *[2 marks]*

(c) (i) Use your answer to part (b) to calculate the horizontal distance the ball has travelled from A when its height is a maximum.

(ii) Find the maximum vertical height reached by the football. *[4 marks]*

(d) Draw a graph showing the path of the football from the point where it is kicked to the point where it hits the ground again. Use 1 cm to represent 5 m on the horizontal axis and 1 cm to represent 2 m on the vertical scale. *[4 marks]*

The goal posts are 35 m from **the point where the ball is kicked**.

(e) At what height does the ball pass over the goal posts? *[2 marks]*

4.   *[Maximum mark: 16]*

(i)   The natural numbers: 1, 2, 3, 4, 5… form an arithmetic sequence.

    (a)   State the values of u_1 and d for this sequence.   *[2 marks]*

    (b)   Use an appropriate formula to show that the sum of the natural numbers from 1 to n is given by $\frac{1}{2}n\,(n+1)$.   *[2 marks]*

    (c)   Calculate the sum of the natural numbers from 1 to 200.   *[2 marks]*

(ii)   A geometric progression G_1 has 1 as its first term and 3 as its common ratio.

    (a)   The sum of the first n terms of G_1 is 29 524. Find n.   *[3 marks]*

A second geometric progression G_2 has the form $1,\ \dfrac{1}{3},\ \dfrac{1}{9},\ \dfrac{1}{27}\ldots$

   (b)   State the common ratio for G_2.   *[1 mark]*

    (c)   Calculate the sum of the first 10 terms of G_2.   *[2 marks]*

    (d)   Explain why the sum of the first 1000 terms of G_2 will give the same answer as the sum of the first 10 terms, when corrected to three significant figures.   *[1 mark]*

    (e)   Using your results from parts (a) to (c), or otherwise, calculate the sum of the first 10 terms of the sequence $2,\ 3\,\dfrac{1}{3},\ 9\,\dfrac{1}{9},\ 27\,\dfrac{1}{27}\ldots$
Give your answer **correct to one decimal place**.   *[3 marks]*

5. *[Maximum mark: 21]*

(i) Neil has three dogs. Two are brown and one is grey. When he feeds the dogs, Neil uses three bowls and gives them out randomly. There are two red bowls and one yellow bowl. This information is shown on the tree diagram below.
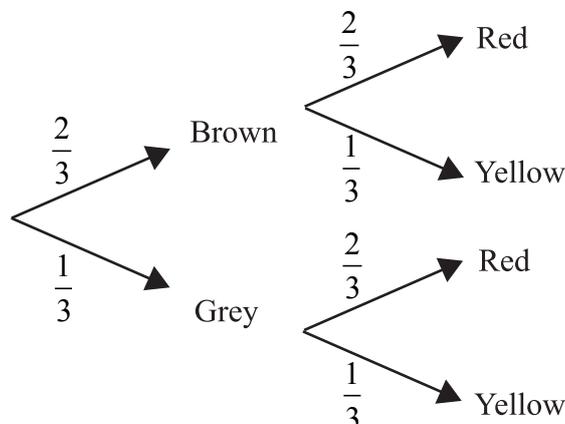
(a) One of the dogs is chosen at random.

(i) Find P (the dog is grey and has the yellow bowl).

(ii) Find P (the dog does not get the yellow bowl). *[3 marks]*

(b) Neil often takes the dogs to the park after they have eaten. He has noticed that the grey dog plays with a stick for a quarter of the time and both brown dogs play with sticks for half of the time. This information is shown on the tree diagram below.
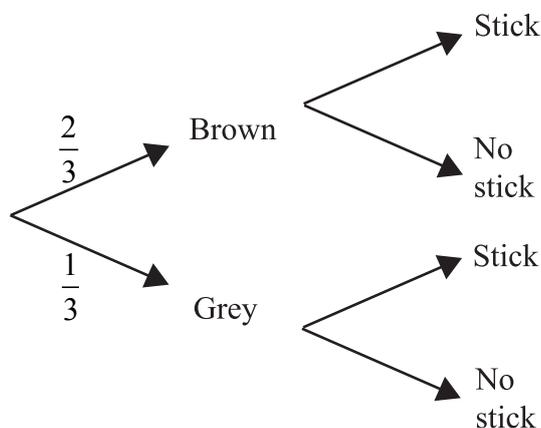
(i) Copy the tree diagram and add the four missing probability values on the branches that refer to playing with a stick.

During a trip to the park, one of the dogs is chosen at random.

(ii) Find P (the dog is grey or is playing with a stick, but not both).

(iii) Find P (the dog is grey given that the dog is playing with a stick).

(iv) Find P (the dog is grey and was fed from the yellow bowl and is not playing with a stick). *[9 marks]*

(This question continues on the following page)

(Question 5 continued)

(ii)   There are 49 mice in a pet shop.
        30 mice are white.
        27 mice are male.
        18 mice have short tails.
        8 mice are white and have short tails.
        11 mice are male and have short tails.
        7 mice are male but neither white nor short-tailed.
        5 mice have all three characteristics and
        2 have none.

        Copy the diagram below to your examination script.
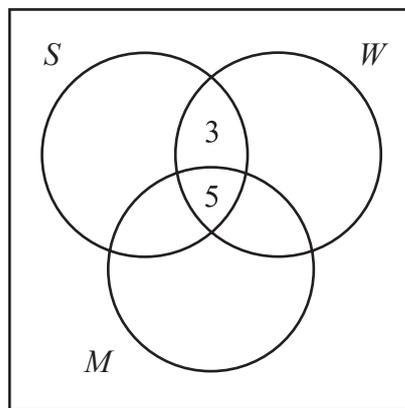
W represents white mice.
M represents male mice.
S represents short-tailed mice.

(a)   Complete the diagram, using the information given in the question.   *[4 marks]*

(b)   Find (i) $n(M \cap W)$

             (ii) $n(M' \cup S)$ *[3 marks]*

Two mice are chosen without replacement.

(c)   Find P (both mice are short-tailed).   *[2 marks]*