

MARKSCHEME

November 2005

MATHEMATICAL STUDIES

Standard Level

Paper 2

*This markscheme is **confidential** and for the exclusive use of examiners in this examination session.*

*It is the property of the International Baccalaureate and must **not** be reproduced or distributed to any other person without the authorization of IBCA.*

Paper 2 Markscheme

Instructions to Examiners

1 Method of marking

- (a) All marking must be done using a **red** pen.
- (b) Marks should be noted on candidates' scripts as in the markscheme:
 - show the breakdown of individual marks using the abbreviations (*M1*), (*A2*) etc.
 - write down each part mark total, indicated on the markscheme (for example, [*3marks*]) – it is suggested that this be written at the end of each part, and underlined;
 - write down and circle the total for each question at the end of the question.

2 Abbreviations

The markscheme may make use of the following abbreviations:

M Marks awarded for **Method**

A Marks awarded for an **Answer** or for **Accuracy**

G Marks awarded for correct solutions, generally obtained from a **Graphic Display Calculator**, irrespective of working shown

R Marks awarded for clear **Reasoning**

AG **Answer Given** in the question and consequently marks are **not** awarded

3 Follow Through (ft) Marks

Errors made at any step of a solution can affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks should be awarded. The procedures for awarding these marks require that all examiners:

- (i) penalize the error when it **first occurs**;
- (ii) **accept the incorrect answer** as the appropriate value or quantity to be used in all subsequent working;
- (iii) award **M** marks for a correct method and **A(ft)** marks if the subsequent working contains no further errors.

Follow through procedures may be applied repeatedly throughout the same problem.

The following illustrates a use of the **follow through** procedure.

Markscheme		Candidate's Script	Marking	
$\$ 600 \times 1.02$	<i>MI</i>	Amount earned = $\$ 600 \times 1.02$	✓	<i>MI</i>
= $\$ 612$	<i>AI</i>	= $\$ 602$	×	<i>A0</i>
$\$ (306 \times 1.02) + (306 \times 1.04)$	<i>MI</i>	Amount = $301 \times 1.02 + 301 \times 1.04$	✓	<i>MI</i>
= $\$ 630.36$	<i>AI</i>	= $\$ 620.06$	✓	<i>AI(ft)</i>

Note that the candidate made an arithmetical error at line 2; the candidate used a correct method at lines 3, 4; the candidate's working at lines 3, 4 is correct.

However, if a question is transformed by an error into a **different, much simpler question** then:

- (i) **fewer** marks should be awarded at the discretion of the Examiner;
- (ii) marks awarded should be followed by “(d)” (to indicate that these marks have been awarded at the **discretion** of the Examiner);
- (iii) a brief **note** should be written on the script explaining **how** these marks have been awarded.

4 Using the Markscheme

- (a) This markscheme presents a particular way in which each question may be worked and how it should be marked. **Alternative methods** have not always been included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method in a manner which is consistent with the markscheme.

In this case:

- (i) a mark should be awarded followed by “(d)” (to indicate that the marks have been awarded at the **discretion** of the Examiner);
- (ii) a brief **note** should be written on the script explaining **how** these marks have been awarded.

Where alternative methods for complete questions are included, they are indicated by **METHOD 1**, **METHOD 2**, *etc.* Other alternative solutions, including graphic display calculator alternative solutions are indicated by **OR**. For example:

$$\begin{aligned} \text{Mean} &= 7906/134 && \text{(M1)} \\ &= 59 && \text{(A1)} \end{aligned}$$

OR

$$\text{Mean} = 59 \qquad \qquad \qquad \text{(G2)}$$

- (b) Unless the question specifies otherwise, accept **equivalent forms**. For example: $\frac{\sin \theta}{\cos \theta}$ for $\tan \theta$.
On the markscheme, these equivalent numerical or algebraic forms will be written in brackets after the required answer. Paper setters will indicate the required answer, by allocating full marks at that point. Further working should be ignored, even if it is incorrect. For example: if candidates are asked to factorize a quadratic expression, and they do so correctly, they are awarded full marks. If they then continue and find the roots of the corresponding equation, do not penalize, even if those roots are incorrect *i.e.*, once the correct answer is seen, ignore further working.
- (c) As this is an international examination, all **alternative forms of notation** should be accepted. For example: 1.7, 1·7, 1,7 ; different forms of vector notation such as \vec{u} , \bar{u} , \underline{u} ; $\tan^{-1} x$ for $\arctan x$.

5 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy.

There are two types of accuracy error. Candidates should be penalized once only **IN THE PAPER** for an accuracy error (**AP**). Award the marks as usual then write $-1(\mathbf{AP})$ against the answer and also on the **front** cover.

Rounding errors: only applies to final answers not to intermediate steps.

Level of accuracy: when this is not specified in the question the general rule *unless otherwise stated in the question all numerical answers must be given exactly or to three significant figures* applies.

- If a final correct answer is incorrectly rounded, apply the AP
- OR**
- If the level of accuracy is not specified in the question, apply the AP for answers not given to 3 significant figures. (Please note that this has changed from May 2003).

6 Graphic Display Calculators

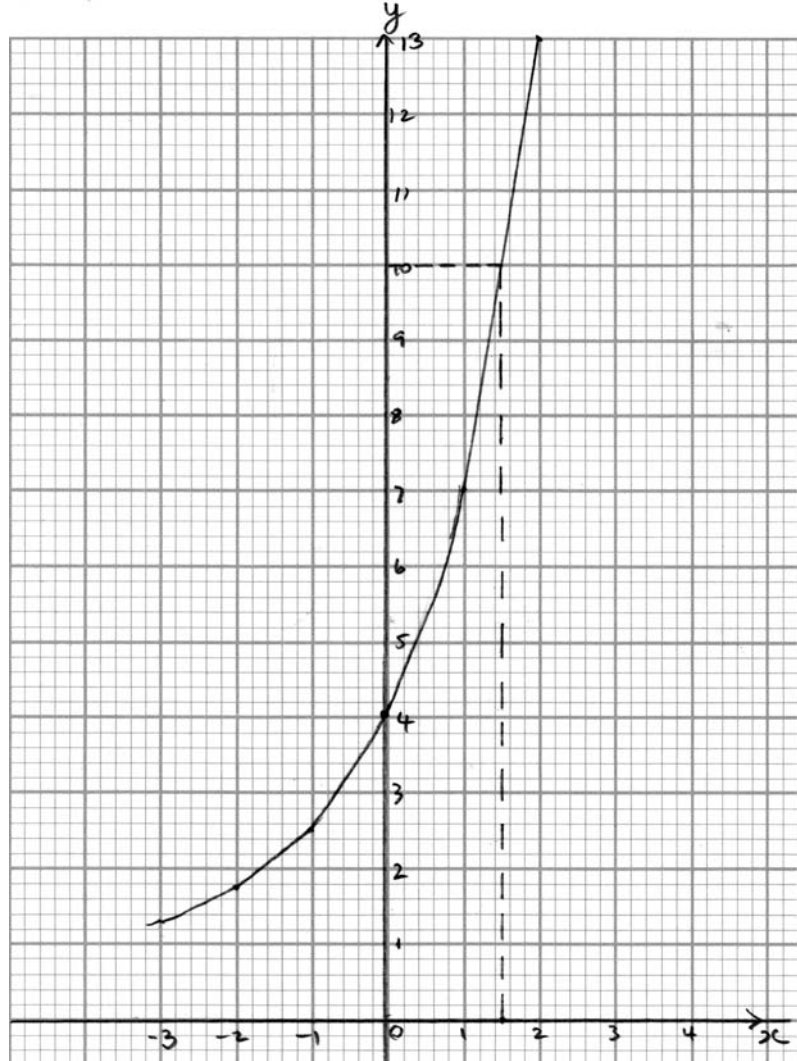
Many candidates will be obtaining solutions directly from their calculators, often without showing any working. They have been advised that they must use mathematical notation, not calculator commands when explaining what they are doing. Incorrect answers without working will receive no marks. However, if there is written evidence of using a graphic display calculator correctly, method marks may be awarded. Where possible, examples will be provided to guide examiners in awarding these method marks

QUESTION 1

(i) (a) $a = 2.5, b = 13$

(A1)(A1)
[2 marks]

(b)



(A4)

Note: Award (A1) for scales and labels, (A2) for all points accurate ((A1) for 5 correct), (A1) for smooth curve.

[4 marks]

(c) Range $f(x) > 1$
($y > 1$)

(A2)

Note: Award (A1) for $f(x) >$, (A1) for 1.

[2 marks]

(d) $x = 1.6 (\pm 0.1)$

(M1)(A1) (or (G2))

Note: Answer by calculation is 1.58.

[2 marks]

Question 1 continued

(ii) (a) $AC^2 = 3.9^2 + 4.8^2 - 2 \times 3.9 \times 4.8 \times \cos 82^\circ$ (M1)(A1)
 $AC^2 = 33.04$
 $AC = 5.75$ (A1)

[3 marks]

(b) $\frac{3.9}{\sin C} = \frac{\sqrt{33.04}}{\sin 82^\circ}$ (M1)(A1)
 $\sin C = 0.671889$
 $C = 42.2^\circ$ (A1)

[3 marks]

Total [16 marks]

QUESTION 2

(i) (a) $a = 4, b = 1$ *(A1)(A1)*
[2 marks]

(b) $30 - (4 + 12 + 1 + 2 + 4 + 4) = 3$ *(M1)(A1) (or (A2))*
[2 marks]

(c) $\frac{24}{30} \left(= \frac{4}{5} \right)$ *(A1)(A1)*

Note: Award <i>(A1)</i> for numerator, <i>(A1)</i> for denominator.
--

[2 marks]

(d) $\frac{6}{19}$ *(A1)(A1)*

Note: Award <i>(A1)</i> for numerator, <i>(A1)</i> for denominator.
--

[2 marks]

(ii) (a) If x is a factor of 6 then x is a factor of 24 *(A1)*
[1 mark]

(b) If x is a factor of 24 then x is a factor of 6 (or $q \Rightarrow p$) *(A1)*
[1 mark]

(c) False *(R1)*
4, 8, 12 are all factors of 24 but not of 6 (any one of the three factors will do) *(A1)*

[2 marks]

Total [12 marks]

QUESTION 3

(a) Gradient of DC = $-\frac{2}{11}$ (A2)

Note: Award (A1) for sign, (A1) for $\frac{2}{11}$.

[2 marks]

(b) No. The gradients are not equal. (A1)(R1)

[2 marks]

(c) Gradient = $\frac{2}{12} = \frac{1}{6}$ (A1)

$$y = \frac{1}{6}x + c$$
(A1)

$$3 = \frac{1}{6}(8) + c$$

$$18 = 8 + 6c$$

$$10 = 6c$$

$$c = \frac{10}{6} = \frac{5}{3}$$
(A1)

$$y = \frac{1}{6}x + \frac{5}{3}$$

$$6y = x + 10$$
(M1)

$$x - 6y = -10$$
(A1)

[5 marks]

(d) $3x + 5y = 16$
 $x - 6y = -10$ (M1)

$$3x - 18y = -30$$
(M1)

$$23y = 46$$

$$y = 2$$
(A1)

$$x = 2 \Rightarrow T(2, 2)$$
(A1)

Note: Award (A2) for the answers only.

[4 marks]

Total [13 marks]

QUESTION 4

(a) Ali : $\frac{(3\,000 \times 4.5 \times 6)}{100} + 3\,000 = \$3\,810$ **(M1)(A1)(A1)**

Note: Award **(M1)** for correct formula, **(A1)** for correct numbers in formula, **(A1)** for correct answer.

Bob: $3\,000 \left(1 + \frac{4}{100}\right)^6 = \$3\,795.96$ (= \$ 3 800) **(M1)(A1)(A1) or (G3)**

Note: Award **(M1)** for correct formula, **(A1)** for correct numbers in formula, **(A1)** for correct answer.

[6 marks]

(b) Connie: $3\,000 \left(1 + \frac{3.8}{200}\right)^{12} = \$3\,760.204(482)$ **(M1)(A1)(A1)**

Note: Award **(M1)** for correct formula, **(A1)** for correct numbers in formula, **(A1)** for full correct answer.

= \$ 3 760.20 **(AG)**
[3 marks]

(c) $3\,000(1.04)^n = 6\,000$ **(M1)(A1)**
 $1.04^n = 2$
 $n = 18$ (by trial and error or solver or by using logarithms) **(A1) or (G3)**
 (accept 17.7) **[3 marks]**

(d) (i) $2\% \text{ of } 3\,760.20 = 75.20 \text{ USD}$ **(M1)(A1)**
[2 marks]

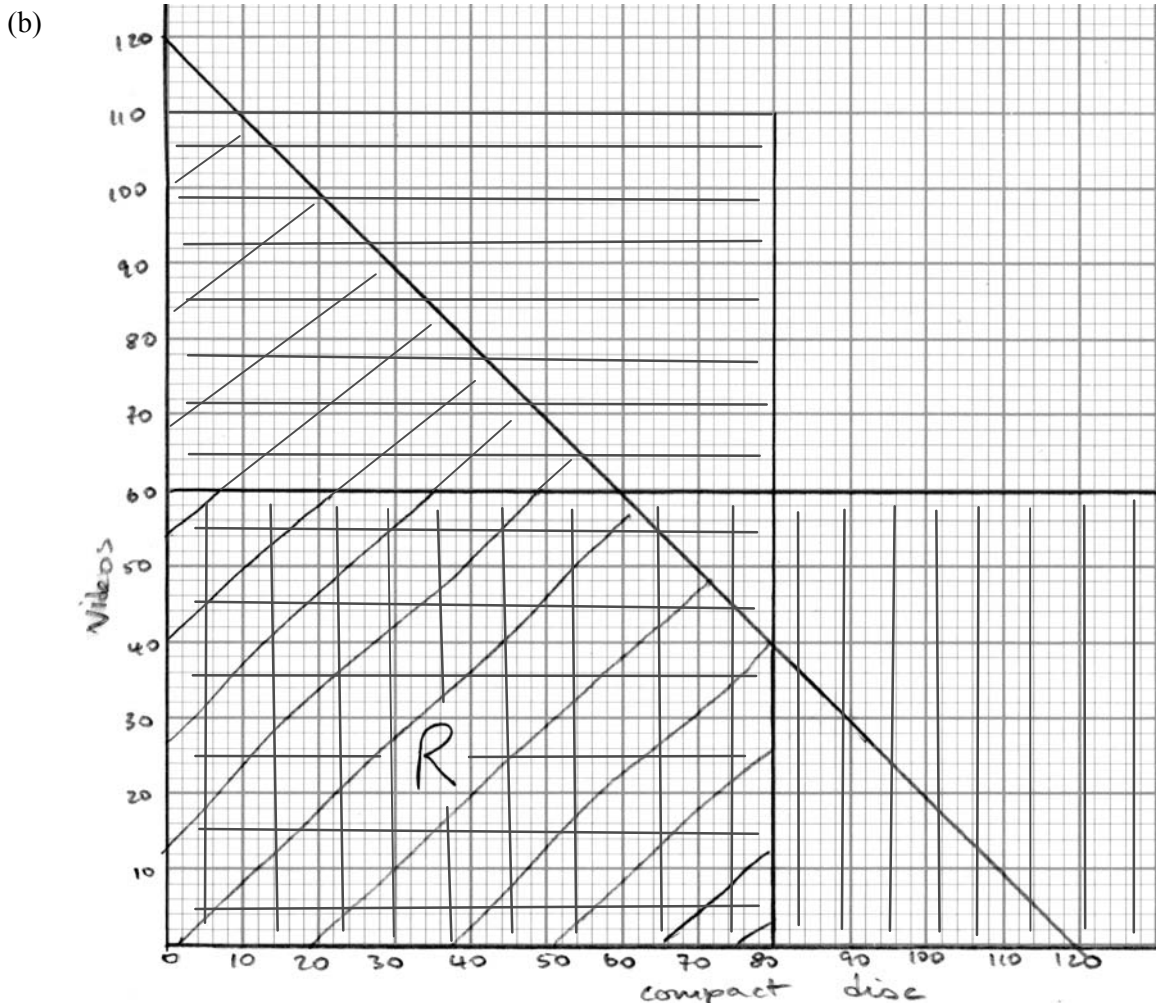
(ii) $3\,760.20 - 75.20 = 3\,685$ **(A1)**
 $3\,685 \times 0.711 = 2\,620.035 \text{ GBP}$ (accept 2620.04, 2620.03, 2620) **(M1)(A1)**
[3 marks]

Total [17 marks]

QUESTION 5

(a) $y \leq 60, x + y \leq 120$

(A1)(A1)
[2 marks]



(A5)

Note: Award (A1) for scales and labels, (A1) for $x = 80$, (A1) for $y = 60$, (A1) for $x + y = 120$, (A1) for shading in or out.

[5 marks]

(c) Region indicated clearly with R

(A1)
[1 mark]

(d) $(P =) 1.50x + 2y$

(A1)
[1 mark]

(e) $(80, 40) P = 1.50 \times 80 + 2 \times 40 = 200$
 $(60, 60) P = 1.50 \times 60 + 2 \times 60 = 210$

(M1)

Note: Award the (M1) for putting numbers in their formula.

Jenny must sell 60 compact discs and 60 videos.

(A1)

Note: If no working shown, award (A2) for 60, 60.

[2 marks]

(f) \$ 210

(A1)
[1 mark]

Total [12 marks]

QUESTION 6

(i) (a) I (A1)
[1 mark]

(b) E (A1)
[1 mark]

(c) H or I (A1)
[1 mark]

(d) D (A2)
[2 marks]

(e) $9 - (-8) = 17$ (M1)(A1) (or (A2))
[2 marks]

(f)
$$\begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix} - \begin{pmatrix} 12 & -16 \\ -6 & 8 \end{pmatrix}$$
$$= \begin{pmatrix} -10 & 19 \\ 9 & -6 \end{pmatrix}$$
 (A2)

Note: Award (A2) for 4 correct entries, (A1) for 3 correct entries.

[2 marks]

(g)
$$(2 \quad -4 \quad 3) \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix}$$
$$= (10 - 4 + 9)$$
$$= (15)$$
 (M1)(A1) (or (A2))
[2 marks]

(h)
$$\begin{pmatrix} 6 & -3 \\ -8 & 4 \end{pmatrix}$$
 (A1)
[1 mark]

continued...

Question 6 continued

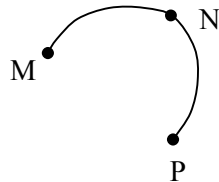
(ii) (a)
$$T = \begin{matrix} & \begin{matrix} M & N & P \end{matrix} \\ \begin{matrix} M \\ N \\ P \end{matrix} & \begin{pmatrix} 4 & 2 & 0 \\ 2 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \end{matrix}$$
 (A3)

Note: Award (A1) for each row.

[3 marks]

(b) 3 (A1)
[1 mark]

(c) this, or any other correct subgraph



(A1)
[1 mark]

(d) The number of ways to get from one town to another (or return to the same town) via another town (or even via the same town). (A1)(A1)
[2 marks]

(iii) (a)
$$\begin{pmatrix} 0.19 & 0.07 \\ 0.81 & 0.93 \end{pmatrix}$$
 (A1)(A1)(A1)
[3 marks]

(b)
$$\begin{pmatrix} 0.19 & 0.07 \\ 0.81 & 0.93 \end{pmatrix} \begin{pmatrix} 100 \\ 900 \end{pmatrix} = \begin{pmatrix} 82 \\ 918 \end{pmatrix}$$

Therefore 82 absent (the answer can be implied from the matrix multiplication). (M1)(A1)
[2 marks]

(c)
$$\begin{pmatrix} 0.19 & 0.07 \\ 0.81 & 0.93 \end{pmatrix} \begin{pmatrix} 250 \\ 750 \end{pmatrix} = \begin{pmatrix} 100 \\ 900 \end{pmatrix}$$

The records are correct. (M1)(A1)
[2 marks]

(iv) (a) j_1 (A1)
[1 mark]

(b) t_2 (A1)
[1 mark]

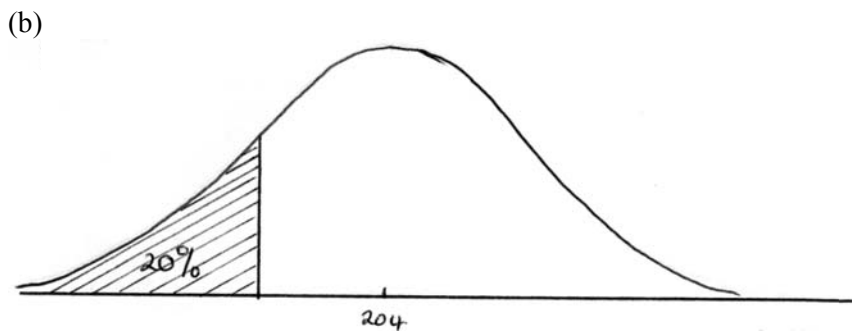
(c) Jerry wins 3 and Tom loses 3. (A1)(A1)
[2 marks]

Total [30 marks]

QUESTION 7

(i) (a) $\frac{60}{300} = 20\%$ (A1)

[1 mark]



(A3)

Note: Award (A1) for 204, (A1) for 20 %, (A1) if on correct side.

[3 marks]

(c) $z = -0.8416$ (A1)

$-0.8416 = \frac{(x-204)}{6}$ (M1)(A1)

$x = 199$ (A1) (or (G4))

[4 marks]

(ii) (a) H_0 : number of entries is independent of gender. (A1)

[1 mark]

(b) 4 (A1)

[1 mark]

(c) 9.488 (A1)

[1 mark]

(d) $a = 85, b = 29$ (A1)(A1)

[2 marks]

(e) $\frac{(30-32)^2}{32} + \dots$ (M1)(A1)

$= 6.10$ (using given values) (A1)

OR

5.80 (from calculator) (G3)

[3 marks]

(f) Do not reject the null hypothesis as the χ^2 value is less than the critical value.
So, gender and stroke are independent. (A1)(R1)

(Also allow "accept"). [2 marks]

continued...

Question 7 continued

- (iii) (a) mean of $x = 72.25$ (A1)
 sd of $x = 4.41$ (A1)
 mean of $y = 139.7$ (140) (A1)
 sd of $y = 5.99$ (A1)
[4 marks]

- (b) $r = -0.940$ (G2)

OR

$$r = \frac{-24.82}{(4.41 \times 5.99)}$$

$$= -0.9396 (= -0.94)$$

(M1)(A1)
[2 marks]

- (c) strong, negative correlation (A2)

Note: Award (A1) for negative, (A1) for strong.

[2 marks]

- (d) $y = 232 - 1.28x$ (G3)

OR

$$(y - 139.7) = -\frac{24.82}{4.41^2}(x - 72.25)$$

$$Y = -1.28x + 232$$

(M1)(A1)(A1)
[3 marks]

- (e) $y = 232 - 1.28 \times 75 = 136$ seconds (A1)

[1 mark]

Total [30 marks]

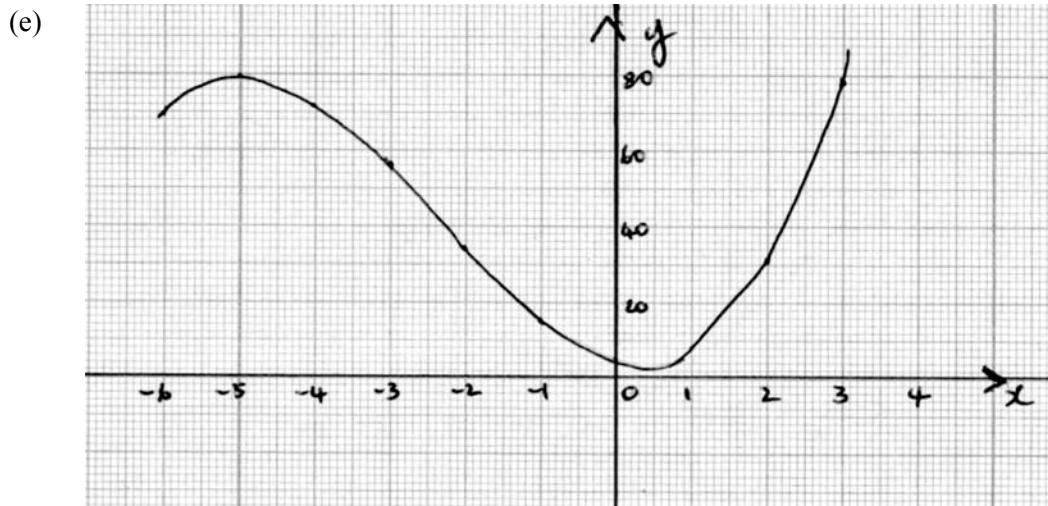
QUESTION 8

(i) (a) $f'(x) = 3x^2 + 14x - 5$ (A1)(A1)(A1)
[3 marks]

(b) $f'(1) = 3 + 14 - 5 = 12$ (M1)(A1)
[2 marks]

(c) $3x^2 + 14x - 5 = 0$ (M1)
 $(3x - 1)(x + 5) = 0$
 $x = \frac{1}{3}$ or -5 (A1)(A1) (or (G3))
[3 marks]

(d) $\left(\frac{1}{3}, 3.15\right)$ $(-5, 79)$ (A1)(A1) (or (G2))
[2 marks]



(A4)

Note: Award (A1) for axes labelled, (A1) for maximum, (A1) for minimum, (A1) for y-intercept.

[4 marks]

continued...

Question 8 continued

- (ii) (a) $h = 50 - 5 \times 4 = 30$ *(M1)(A1)*
[2 marks]
- (b) $v = 25 - 10t$ *(A1)(A1)*
[2 marks]
- (c) $v = 25 - 0 = 25$ *(M1)(A1)*
[2 marks]
- (d) $25 - 10t = 0$
 $t = 2.5$ *(M1)(A1)*
[2 marks]
- (e) $h = 25 \times 2.5 - 5 \times 2.5^2$
 $h = 31.25$ *(M1)(A1)*
[2 marks]
- (iii) $y = x^3 - x^2 - x + c$ *(A1)(A1)(A1)*
 $3 = 8 - 4 - 2 + c$ *(M1)*
 $1 = c$ *(A1)*
 $y = x^3 - x^2 - x + 1$ *(A1)*
[6 marks]
- Total [30 marks]***
-