IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
PROGRAMA DEL DIPLOMA DEL BI

MATHEMATICAL STUDIES
STANDARD LEVEL
PAPER 2

Friday 4 November 2005 (morning)

2 hours

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all five questions from Section A and one question from Section B.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.

Please start each question on a new page. You are advised to show all working, where possible. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

SECTION A

Answer all ***five*** *questions from this section.*

1. *[Maximum mark: 16]*

(i) A function is represented by the equation $f(x) = 3(2)^x + 1$.

The table of values of $f(x)$ for $-3 \le x \le 2$ is given below.

x	–3	–2	–1	0	1	2
$f(x)$	1.375	1.75	a	4	7	b

(a) Calculate the values for a and b. *[2 marks]*

(b) On graph paper, draw the graph of $f(x)$, for $-3 \le x \le 2$, taking 1 cm to represent 1 unit on both axes. *[4 marks]*

The domain of the function $f(x)$ is the real numbers, $\mathbb{R}$.

(c) Write down the range of $f(x)$. *[2 marks]*

(d) Using your graph, or otherwise, find the approximate value for x when $f(x) = 10$. *[2 marks]*

(ii) In triangle ABC, AB = 3.9 cm, BC = 4.8 cm and angle $\text{ABC} = 82^\circ$.

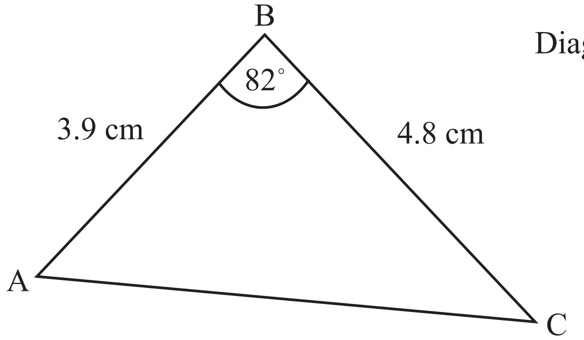


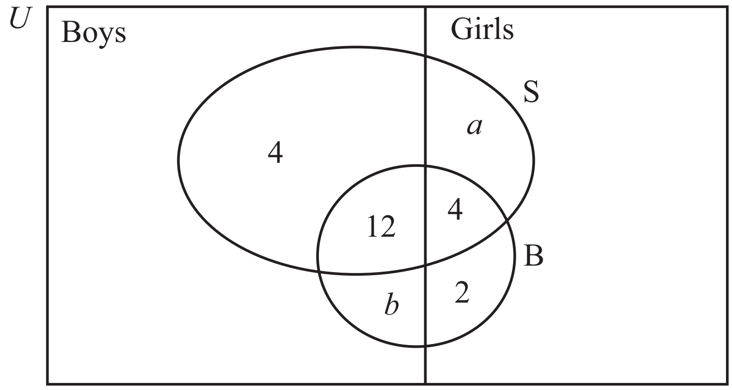
Diagram not to scale

(a) Calculate the length of AC. *[3 marks]*

(b) Calculate the size of angle ACB. *[3 marks]*

2. *[Maximum mark: 12]*

(i) Children in a class of 30 students are asked whether they can swim (S) or ride a bicycle (B).
There are 12 girls in the class. 8 girls can swim, 6 girls can ride a bicycle and 4 girls can do both.
16 boys can swim, 13 boys can ride a bicycle and 12 boys can do both.
This information is represented in a Venn diagram.



(a) Find the values of *a* and *b*. *[2 marks]*

(b) Calculate the number of students who can do neither. *[2 marks]*

(c) Write down the probability that a student chosen at random can swim. *[2 marks]*

(d) Given that the student can ride a bicycle, write down the probability that the student is a girl. *[2 marks]*

(ii) Consider the following logic statements:

$p : x$ is a factor of 6
$q : x$ is a factor of 24

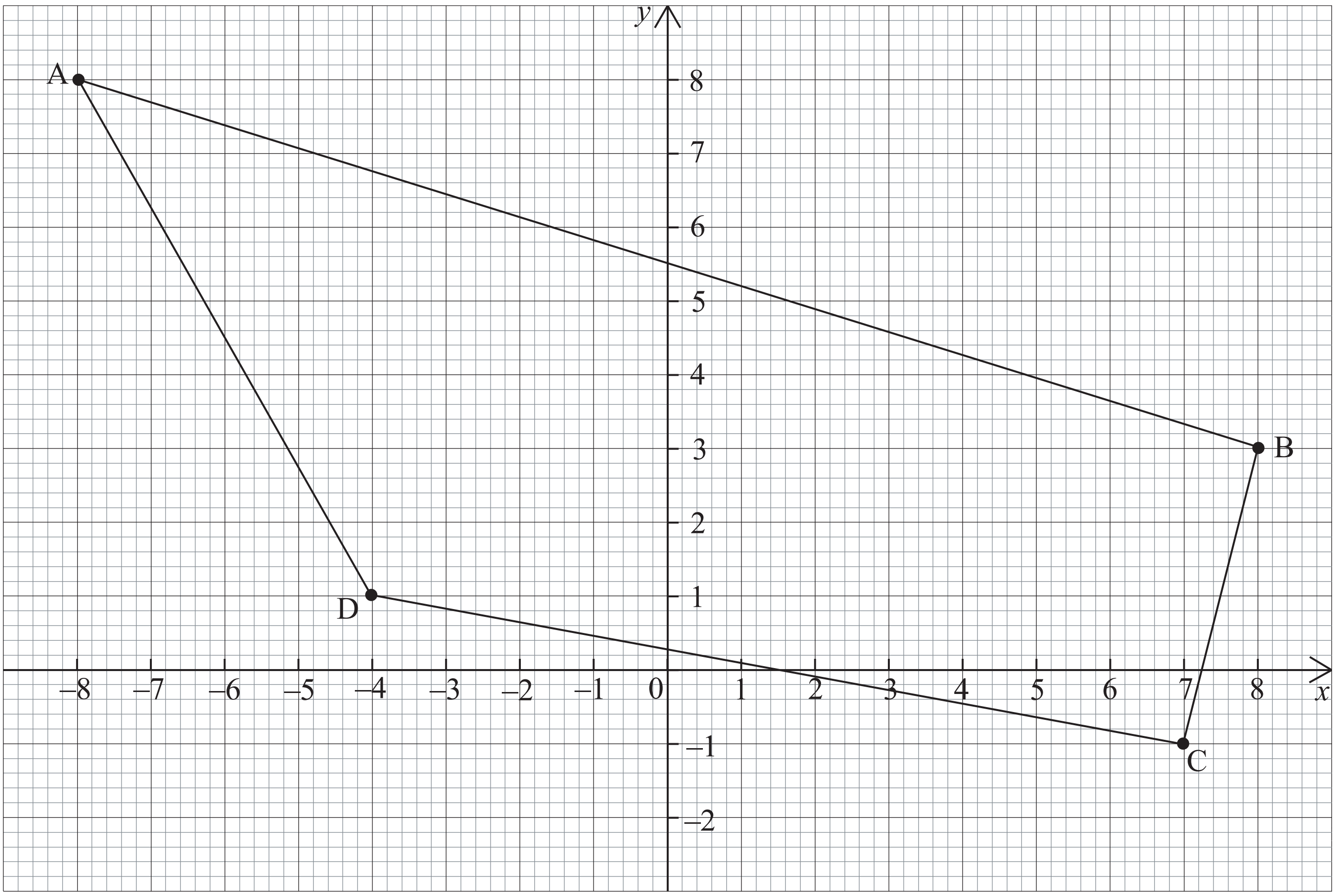
(a) Write $p \Rightarrow q$ in words. *[1 mark]*

(b) Write the converse of $p \Rightarrow q$. *[1 mark]*

(c) State if the converse is true or false and give an example to justify your answer. *[2 marks]*

3. *[Maximum mark: 13]*

The vertices of quadrilateral ABCD as shown in the diagram are $\mathrm{A}(-8, 8)$, $\mathrm{B}(8, 3)$, $\mathrm{C}(7, -1)$ and $\mathrm{D}(-4, 1)$.



The gradient of the line AB is $-\frac{5}{16}$.

(a) Calculate the gradient of the line DC. *[2 marks]*

(b) State whether or not DC is parallel to AB and give a reason for your answer. *[2 marks]*

The equation of the line through A and C is $3x + 5y = 16$.

(c) Find the equation of the line through B and D expressing your answer in the form $ax + by = c$, where a, b and $c \in \mathbb{Z}$. *[5 marks]*

The lines AC and BD intersect at point T.

(d) Calculate the coordinates of T. *[4 marks]*

4. *[Maximum mark: 17]*

Ali, Bob and Connie each have 3 000 USD (US dollar) to invest.

Ali invests his 3 000 USD in a firm that offers simple interest at 4.5 % per annum. The interest is added at the end of each year.
Bob invests his 3 000 USD in a bank that offers interest compounded annually at a rate of 4 % per annum. The interest is added at the end of each year.
Connie invests her 3 000 USD in another bank that offers interest compounded half-yearly at a rate of 3.8 % per annum. The interest is added at the end of each half year.

(a) Calculate how much money Ali and Bob have at the **beginning** of year 7. *[6 marks]*

(b) Show that Connie has 3 760.20 USD at the beginning of year 7. *[3 marks]*

(c) Calculate how many years it will take for Bob to have 6 000 USD in the bank. *[3 marks]*

At the beginning of year 7, Connie moves to England.
She transfers her money into a Bank there at an exchange rate of 1 USD = 0.711 GBP (British pounds)
The bank charges 2 % commission.

(d) (i) Calculate, in USD, the commission that the bank charges.

(ii) Calculate the amount of money, in GBP, that Connie transfers to the bank in England. *[5 marks]*

5. *[Maximum mark: 12]*

Jenny has 80 compact discs and 60 videos to sell.
She sells no more than 120 items in total.

Let x be the number of compact discs and y be the number of videos she sells.

(a) Given that, $x \geq 0$, $y \geq 0$ and $x \leq 80$ write down two more inequalities to represent the information above. *[2 marks]*

(b) Using a scale of 1cm for 10 compact discs on the x-axis and 1 cm for 10 videos on the y-axis, draw the graph to represent all these inequalities. *[5 marks]*

(c) Indicate clearly, with R, the region which satisfies all the inequalities. *[1 mark]*

Jenny makes a profit of \$ 1.50 on each compact disc that she sells and \$ 2 on each video.

(d) Write down an equation to calculate the profit, P, that Jenny can make. *[1 mark]*

(e) Calculate the number of compact discs and videos that Jenny must sell to make a maximum profit. *[2 marks]*

(f) Write down this maximum profit. *[1 mark]*

SECTION B

*Answer **one** question from this section.*

Matrices and Graph Theory

6. *[Maximum mark: 30]*

(i) Consider the following matrices:

$$\boldsymbol{A} = \begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix} \quad \boldsymbol{B} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \quad \boldsymbol{C} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\boldsymbol{D} = \begin{pmatrix} 6 & -8 \\ -3 & 4 \end{pmatrix} \quad \boldsymbol{E} = \begin{pmatrix} 2 & -4 & 3 \end{pmatrix} \quad \boldsymbol{F} = \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix}$$

$$\boldsymbol{G} = \begin{pmatrix} 3 & 2 \\ -4 & 3 \end{pmatrix} \quad \boldsymbol{H} = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} \quad \boldsymbol{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

(a) Write down the letter that represents the identity matrix. *[1 mark]*

(b) Write down the letter that represents a row matrix. *[1 mark]*

(c) Write down a letter that represents a diagonal matrix. *[1 mark]*

(d) Write down the letter that represents a singular matrix. *[2 marks]*

(e) Calculate the determinant of matrix $\boldsymbol{G}$. *[2 marks]*

(f) Calculate $\boldsymbol{A} - 2\boldsymbol{D}$. *[2 marks]*

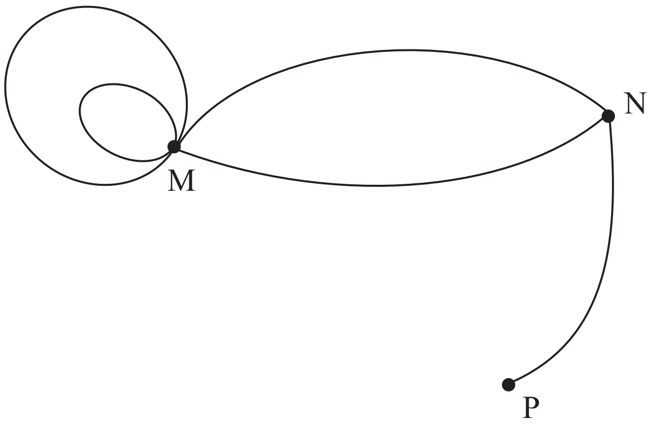
(g) Calculate $\boldsymbol{EF}$. *[2 marks]*

(h) Write down the transpose of matrix $\boldsymbol{D}$. *[1 mark]*

(This question continues on the following page)

(Question 6 continued)

(ii) The diagram below shows the number of routes connecting the three towns M, N and P.



(a) Construct an adjacency matrix, $\boldsymbol{T}$, to represent the routes connecting the three towns. *[3 marks]*

(b) Write down the degree of vertex N. *[1 mark]*

(c) Draw a subgraph of the diagram above connecting towns M, N and P. *[1 mark]*

The matrix $\boldsymbol{T}^2$ can be written as follows:

$$\boldsymbol{T}^2 = \begin{array}{c c} & \begin{array}{ccc} \mathrm{M} & \mathrm{N} & \mathrm{P} \end{array} \\ \begin{array}{c} \mathrm{M} \\ \mathrm{N} \\ \mathrm{P} \end{array} & \begin{pmatrix} 20 & 8 & 2 \\ 8 & 5 & 0 \\ 2 & 0 & 1 \end{pmatrix} \end{array}$$

(d) What do the entries of the matrix $\boldsymbol{T}^2$ represent? *[2 marks]*

(This question continues on the following page)

(Question 6 continued)

(iii) In a school with 1 000 students a mathematical model was used to examine the pattern of the number of students absent (A) and present (P).

Every Monday the number of students absent was noted. The following pattern was found:
81 % of those students who were absent on a particular Monday were present the following Monday.
7 % of those students who were present on a particular Monday were absent the following Monday.

(a) Copy and complete the transition matrix to represent the above information.

$$\text{Following Monday} \quad \begin{array}{c c} & \begin{array}{cc} & \text{Monday} \\ \boldsymbol{A} & \boldsymbol{P} \end{array} \\ \begin{array}{c} \boldsymbol{A} \\ \boldsymbol{P} \end{array} & \begin{pmatrix} & \\ 0.81 & \end{pmatrix} \end{array}$$

[3 marks]

On Monday 12th July there were 100 students absent and 900 students present.

(b) Calculate the number of students absent the following Monday. *[2 marks]*

The records show that 250 students were absent and 750 students were present on Monday 5th July.

(c) Check, showing your working clearly, if this information is true or false. *[2 marks]*

(iv) Tom and Jerry play a 2 person zero sum game. The matrix, $\boldsymbol{J}$, below shows Jerry's winnings.

$$\boldsymbol{J} = \begin{array}{c c} & \begin{array}{cc} t_1 & t_2 \end{array} \\ \begin{array}{c} j_1 \\ j_2 \end{array} & \begin{pmatrix} -2 & 3 \\ 4 & -3 \end{pmatrix} \end{array}$$

(a) What is Jerry's play safe strategy? *[1 mark]*

(b) What is Tom's play safe strategy? *[1 mark]*

(c) What is the outcome for Tom and for Jerry if they each play their play safe strategy? *[2 marks]*

Further Statistics and Probability

7. *[Maximum mark: 30]*

(i) 300 contestants enter a swimming competition. The 60 contestants with the fastest times go through to the semi-finals.

(a) Calculate the percentage who go through to the semi-finals. *[1 mark]*

The times are approximately normally distributed with a mean of 204 seconds and a standard deviation of 6 seconds.

(b) Represent this information on a normal distribution graph, indicating clearly the mean and percentage who reach the semi-finals. *[3 marks]*

(c) Calculate the time of the slowest contestant to reach the semi-finals. *[4 marks]*

(ii) In another competition the number of males and females taking part in different swimming races is given in the table of observed values below.

	Backstroke (100m)	Freestyle (100m)	Butterfly (100m)	Breaststroke (100m)	Relay (4×100m)
Male	30	90	31	29	20
Female	28	63	20	37	12

The Swimming Committee decides to perform a χ^2 test at the 5 % significance level in order to test if the number of entries for the various strokes is related to gender.

(a) State the null hypothesis. *[1 mark]*

(b) Write down the number of degrees of freedom. *[1 mark]*

(c) Write down the critical value of χ^2. *[1 mark]*

The expected values are given in the table below:

	Backstroke (100m)	Freestyle (100m)	Butterfly (100m)	Breaststroke (100m)	Relay (4×100m)
Male	32	a	28	37	18
Female	26	68	23	b	14

(d) Calculate the values of a and b. *[2 marks]*

(e) Calculate the χ^2 value. *[3 marks]*

(f) State whether or not you accept your null hypothesis and give a reason for your answer. *[2 marks]*

(This question continues on the following page)

(Question 7 continued)

(iii) It is thought that the breaststroke time for 200 m depends on the length of the arm of the swimmer.
Eight students swim 200 m breaststroke. Their times (y) in seconds and arm lengths (x) in cm are shown in the table below.

	1	2	3	4	5	6	7	8
Length of arm, x cm	79	74	72	70	77	73	64	69
Breaststroke, y seconds	135.1	135.7	139.3	141.0	132.8	137.0	152.9	144.0

(a) Calculate the mean and standard deviation of x and y. *[4 marks]*

(b) Given that $s_{xy} = -24.82$, calculate the correlation coefficient, r. *[2 marks]*

(c) Comment on your value for r. *[2 marks]*

(d) Calculate the equation of the regression line of y on x. *[3 marks]*

(e) Using your regression line, estimate how many seconds it will take a student with an arm length of 75 cm to swim the 200 m breaststroke. *[1 mark]*

Introductory Differential Calculus

8. *[Maximum mark: 30]*

(i) Consider the function $f(x) = x^3 + 7x^2 - 5x + 4$.

(a) Differentiate $f(x)$ with respect to x. *[3 marks]*

(b) Calculate $f'(x)$ when $x = 1$. *[2 marks]*

(c) Calculate the values of x when $f'(x) = 0$. *[3 marks]*

(d) Calculate the coordinates of the local maximum and the local minimum points. *[2 marks]*

(e) On graph paper, taking axes $-6 \le x \le 3$ and $0 \le y \le 80$, draw the graph of $f(x)$ indicating clearly the local maximum, local minimum and y-intercept. *[4 marks]*

(ii) An object is thrown vertically upwards from a point O on the ground.

The height, h, of the object after t seconds is given by the equation

$$h = 25t - 5t^2$$

(a) Calculate the height after 2 seconds. *[2 marks]*

(b) Find an expression, in terms of t, for the velocity, v, of the object. *[2 marks]*

(c) Calculate the initial velocity. *[2 marks]*

(d) Calculate the time when the velocity is zero. *[2 marks]*

(e) Calculate the height reached by the object at the time when the velocity is zero. *[2 marks]*

(iii) A curve passes through the point with coordinates (2, 3).
The gradient of the tangent to the curve is given by the equation

$$\frac{dy}{dx} = 3x^2 - 2x - 1.$$

Calculate the equation of the curve. *[6 marks]*