

## MATHEMATICAL STUDIES <br> STANDARD LEVEL <br> PAPER 1

Tuesday 3 May 2005 (afternoon)
1 hour

Candidate session number


## INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.

Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Working may be continued below the box, if necessary. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. The first five terms of an arithmetic sequence are shown below.

$$
2, \quad 6, \quad 10, \quad 14, \quad 18
$$

(a) Write down the sixth number in the sequence.
(b) Calculate the $200^{\text {th }}$ term.
(c) Calculate the sum of the first 90 terms of the sequence.

## Working:

Answers:
(a)
(b)
(c)
$\qquad$
$\qquad$
$\qquad$
2. The numbers of games played in each set of a tennis tournament were

$$
\begin{aligned}
& 9,7,8,11,9,6,10,8,12,6,8,13,7,9,10,9,10,11, \\
& 12,8,7,13,10,7,7 .
\end{aligned}
$$

The raw data has been organised in the frequency table below.

| games | frequency |
| :---: | :---: |
| 6 | 2 |
| 7 | 5 |
| 8 | $n$ |
| 9 | 4 |
| 10 | 4 |
| 11 | 2 |
| 12 | 2 |
| 13 | 2 |

(a) Write down the value of $n$.
(b) Calculate the mean number of games played per set.
(c) What percentage of the sets had more than 10 games?
(d) What is the modal number of games?

## Working:

Answers:
(a)
(b) $\qquad$
(c)
(d) $\qquad$
3.

(a) On the grid above, draw a straight line with a gradient of -3 that passes through the point $(-2,0)$.
(b) Find the equation of this line.

## Working:

Answer:
(b)
4. (a) Convert 0.001673 litres to millilitres (ml). Give your answer to the nearest ml .

The SI unit for energy is Joules. An object with mass $m$ travelling at speed $v$ has energy given by $\frac{1}{2} m v^{2}$ (Joules).
(b) Calculate the energy of a comet of mass 351223 kg travelling at speed $176.334 \mathrm{~m} / \mathrm{sec}$. Give your answer correct to six significant figures.

In the SI system of units, distance is measured in metres ( m ), mass in kilograms ( kg ) and time in seconds (s). The momentum of an object is given by the mass of the object multiplied by its speed.
(c) Write down the correct combination of SI units ( $\mathrm{m}, \mathrm{kg}, \mathrm{s}$ ) for momentum.

## Working:

Answers:
(a)
(b) $\qquad$
(c) $\qquad$
5. The equation $M=90 \times 2^{-t / 20}$ gives the amount, in grams, of radioactive material held in a laboratory over $t$ years.
(a) What was the original mass of the radioactive material?

The table below lists some values for $M$.

| $t$ | 60 | 80 | 100 |
| :---: | :---: | :---: | :---: |
| $M$ | 11.25 | $v$ | 2.8125 |

(b) Find the value of $v$.
(c) Calculate the number of years it would take for the radioactive material to have a mass of 45 grams.

## Working:

Answers:
(a)
(b)
(c)
c)
$\qquad$
$\qquad$
6. The diagram below shows the graph of $y=c+k x-x^{2}$, where $k$ and $c$ are constants.

(a) Find the values of $k$ and $c$.
(b) Find the coordinates of Q , the highest point on the graph.

## Working:

Answers:
(a)
(b) $\qquad$
7. The local council has been monitoring the number of cars parked near a supermarket on an hourly basis. The results are displayed below.

| Parked Cars/Hour | Frequency | Cumulative Frequency |
| :---: | :---: | :---: |
| $0-19$ | 3 | 3 |
| $20-39$ | 15 | 18 |
| $40-59$ | 25 | $w$ |
| $60-79$ | 35 | 78 |
| $80-99$ | 17 | 95 |

(a) Write down the value of $w$.
(b) Draw and label the Cumulative Frequency graph for this data.

(This question continues on the following page)

## (Question 7 continued)

(c) Determine the median number of cars per hour parked near the supermarket.

Working:

Answers:
(a)
(c)
8. The graphs of three trigonometric functions are drawn below. The $x$ variable is measured in degrees, with $0 \leq x \leq 360$. The amplitude ' $a$ ' is a positive constant with $0<a \leq 1$.

Graph B


> Graph C

(a) Write the letter of the graph next to the function representing that graph in the box below.

| FUNCTION | GRAPH |
| :---: | :---: |
| $y=a \cos (x)$ |  |
| $y=a \sin (2 x)$ |  |
| $y=2+a \sin (x)$ |  |

(b) State the period of the function shown in graph B.
(c) State the range of the function $2+a \sin (x)$ in terms of the constant $a$.

## (Question 8 continued)

Working:

Answers:
(b)
(c)
9. The vectors $\boldsymbol{a}, \boldsymbol{b}$ and $\boldsymbol{c}$ are shown below.

(a) Write an equation connecting $\boldsymbol{a}, \boldsymbol{b}$ and $\boldsymbol{c}$.
(b) (i) If $\boldsymbol{a}=\binom{5}{-2}$ and $\boldsymbol{b}=\binom{4}{5}$ express $\boldsymbol{c}$ as a column vector.
(ii) Calculate the magnitude of $\boldsymbol{c}$.

## Working:

Answers:
(a)
(b) (i) $\qquad$
(ii) $\qquad$
10. Sandra is attempting an exam question. She has to choose two correct statements from a list of five. Below is a tree diagram showing Sandra's possible choices. One of the probability values is missing.

(a) Fill in the missing probability value on the diagram.
(b) (i) If Sandra makes two guesses, what is the probability that she will get only one of them correct?
(ii) Sandra definitely knows the first correct statement but has to guess the second. What is the probability that she will answer both correctly?

## Working:

Answers:
(b) (i) $\qquad$
(ii) $\qquad$
11. A family in Malaysia received a gift of AUD $\$ 4000$ from a cousin living in Australia.

The money was converted to Malaysian Ringgit. One Ringgit can be exchanged for 0.4504 AUD.
(a) Calculate the amount of Ringgit received.

The money was invested for 2 years and 6 months at $5.2 \%$ p.a. compounding monthly.
(b) Calculate the amount of interest earned from this investment. Give your answer to the nearest Ringgit.

Working:

Answers:
(a)
(b) $\qquad$
12. A swimming pool is to be built in the shape of a letter $L$. The shape is formed from two squares with side dimensions $x$ and $\sqrt{x}$ as shown.

(a) Write down an expression for the area $A$ of the swimming pool surface.
(b) The area $A$ is to be $30 \mathrm{~m}^{2}$. Write a quadratic equation that expresses this information.
(c) Find both the solutions of your equation in part (b).
(d) Which of the solutions in part (c) is the correct value of $x$ for the pool? State briefly why you made this choice.

## Working:

Answers:
(a) $\qquad$
(b)
(c) $\qquad$
(d) $\qquad$
$\qquad$
$\qquad$
$\qquad$
13. Peter has marked 80 exam scripts. He has calculated the mean mark for the scripts to be 62.1. Maria has marked 60 scripts with a mean mark of 56.8.
(a) Peter discovers an error in his marking. He gives two extra marks each to eleven of the scripts. Calculate the new value of the mean for Peter's scripts.
(b) After the corrections have been made and the marks changed, Peter and Maria put all their scripts together. Calculate the value of the mean for all the scripts.

Working:

Answers:
(a)
(b)
14. (a) Given $x=2.6 \times 10^{4}$ and $y=5.0 \times 10^{-8}$, calculate the value of $w=x \times y$. Give your answer in the form $a \times 10^{k}$ where $1 \leq a<10$ and $k \in \mathbb{Z}$.
(b) Which two of the following statements about the nature of $x, y$ and $w$ above are incorrect?
(i) $x \in \mathbb{N}$
(ii) $y \in \mathbb{Z}$
(iii) $y \in \mathbb{Q}$
(iv) $w<y$
(v) $x+y \in \mathbb{R}$
(vi) $\frac{1}{w}<x$

Working:

Answers:
(a)
(b) $\qquad$
15. Omar is standing at a point $O$, coordinates $(0,0,0)$. He is flying a kite at point $K$, at a height $h$ metres above the flat ground. The $x$ and $y$ axes are on the ground and the $z$ axis is vertical.

Omar's dog is on the ground at $\mathrm{D}=(12,16,0)$ vertically below the kite. The situation is shown in the diagram below using a 3-dimensional axis system, $x, y, z$.

(a) Write down the coordinates of the kite K .
(b) Calculate the distance OD from Omar to his dog.
(c) If the length of the string OK is 30 m , find $h$.
(d) Find the angle DÔK.

## Working:

Answers:
(a)
(b) $\qquad$
(c)
(d) $\qquad$

