IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
PROGRAMA DEL DIPLOMA DEL BI

M05/5/MATME/SP2/ENG/TZ0/XX+

22057302

MATHEMATICAL METHODS
STANDARD LEVEL
PAPER 2

Wednesday 4 May 2005 (morning)

2 hours

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all five questions from Section A and one question from Section B.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.

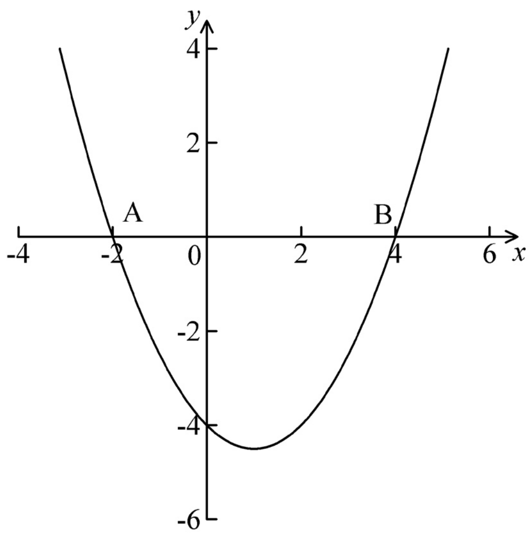
Please start each question on a new page. You are advised to show all working, where possible. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

SECTION A

Answer all ***five*** *questions from this section.*

1. *[Maximum mark: 15]*

The equation of a curve may be written in the form $y = a(x - p)(x - q)$. The curve intersects the x-axis at A(–2, 0) and B(4, 0). The curve of $y = f(x)$ is shown in the diagram below.



(a) (i) Write down the value of p and of q.

(ii) Given that the point (6, 8) is on the curve, find the value of a.

(iii) Write the equation of the curve in the form $y = ax^2 + bx + c$. *[5 marks]*

(b) (i) Find $\frac{dy}{dx}$.

(ii) A tangent is drawn to the curve at a point P. The gradient of this tangent is 7. Find the coordinates of P. *[4 marks]*

(c) The line L passes through B(4, 0), and is perpendicular to the tangent to the curve at point B.

(i) Find the equation of L.

(ii) Find the x-coordinate of the point where L intersects the curve again. *[6 marks]*

2. *[Maximum mark: 12]*

The table below shows the subjects studied by 210 students at a college.

	Year 1	Year 2	Totals
History	50	35	85
Science	15	30	45
Art	45	35	80
Totals	110	100	210

(a) A student from the college is selected at random.

Let A be the event the student studies Art.

Let B be the event the student is in Year 2.

(i) Find $\mathrm{P}(A)$.

(ii) Find the probability that the student is a Year 2 Art student.

(iii) Are the events A and B independent? Justify your answer. *[6 marks]*

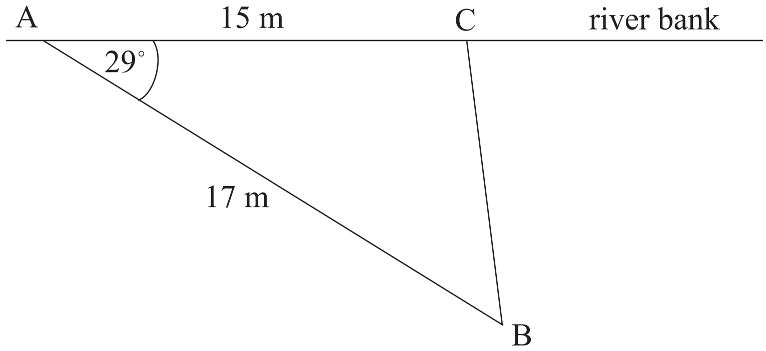
(b) Given that a History student is selected at random, calculate the probability that the student is in Year 1. *[2 marks]*

(c) Two students are selected at random from the college. Calculate the probability that one student is in Year 1, and the other in Year 2. *[4 marks]*

3. *[Maximum mark: 14]*

The diagram shows a triangular region formed by a hedge [AB], a part of a river bank [AC] and a fence [BC]. The hedge is 17 m long and $\mathrm{B\hat{A}C}$ is 29°. The end of the fence, point C, can be positioned anywhere along the river bank.

(a) Given that point C is 15 m from A, find the length of the fence [BC]. *[3 marks]*



(b) The farmer has another, longer fence. It is possible for him to enclose two different triangular regions with this fence. He places the fence so that $\mathrm{A\hat{B}C}$ is 85°.

(i) Find the distance from A to C.

(ii) Find the area of the region ABC with the fence in this position. *[5 marks]*

(c) To form the second region, he moves the fencing so that point C is closer to point A. Find the new distance from A to C. *[4 marks]*

(d) Find the minimum length of fence [BC] needed to enclose a triangular region ABC. *[2 marks]*

4. *[Maximum mark: 12]*

Let $f(x)=\frac{1}{2}\sin 2x+\cos x$ for $0\le x\le 2\pi$.

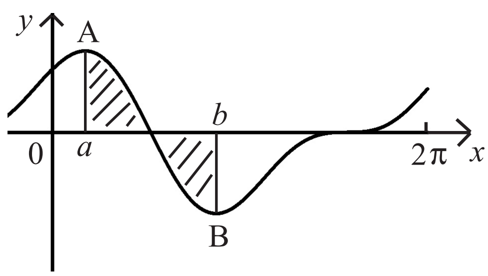
(a) (i) Find $f'(x)$.

One way of writing $f'(x)$ is $-2\sin^2 x-\sin x+1$.

(ii) Factorize $2\sin^2 x+\sin x-1$.

(iii) Hence or otherwise, solve $f'(x)=0$. *[6 marks]*

The graph of $y=f(x)$ is shown below.



There is a maximum point at A and a minimum point at B.

(b) Write down the x-coordinate of point A. *[1 mark]*

(c) The region bounded by the graph, the x-axis and the lines $x=a$ and $x=b$ is shaded in the diagram above.

(i) Write down an expression that represents the area of this shaded region.

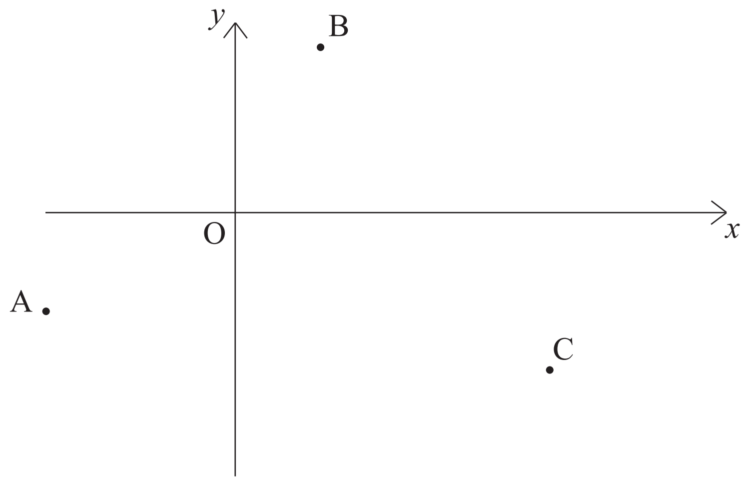
(ii) Calculate the area of this shaded region. *[5 marks]*

5. *[Maximum mark: 17]*

In this question the vector $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ represents a displacement of 1 km east,

and the vector $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ represents a displacement of 1 km north.

The diagram below shows the positions of towns A, B and C in relation to an airport O, which is at the point (0, 0). An aircraft flies over the three towns at a constant speed of $250\ \text{km}\,\text{h}^{-1}$.



Town A is 600 km west and 200 km south of the airport.
Town B is 200 km east and 400 km north of the airport.
Town C is 1 200 km east and 350 km south of the airport.

(a) (i) Find $\overrightarrow{\text{AB}}$.

(ii) Show that the vector of length one unit in the direction of $\overrightarrow{\text{AB}}$ is $\begin{pmatrix} 0.8 \\ 0.6 \end{pmatrix}$. *[4 marks]*

An aircraft flies over town A at 12:00, heading towards town B at $250\ \text{km}\,\text{h}^{-1}$.

Let $\begin{pmatrix} p \\ q \end{pmatrix}$ be the velocity vector of the aircraft. Let t be the number of hours in flight after 12:00. The position of the aircraft can be given by the vector equation

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -600 \\ -200 \end{pmatrix} + t\begin{pmatrix} p \\ q \end{pmatrix}.$$

(This question continues on the following page)

(Question 5 continued)

(b) (i) Show that the velocity vector is $\begin{pmatrix} 200 \\ 150 \end{pmatrix}$.

(ii) Find the position of the aircraft at 13:00.

(iii) At what time is the aircraft flying over town B? *[6 marks]*

Over town B the aircraft changes direction so it now flies towards town C. It takes five hours to travel the $1\,250$ km between B and C. Over town A the pilot noted that she had $17\,000$ litres of fuel left. The aircraft uses $1\,800$ litres of fuel per hour when travelling at $250\ \text{km}\,\text{h}^{-1}$. When the fuel gets below $1\,000$ litres a warning light comes on.

(c) How far from town C will the aircraft be when the warning light comes on? *[7 marks]*

SECTION B

*Answer **one** question from this section.*

Statistical Methods

6. *[Maximum mark: 30]*

(i) The number of hours a student spends studying for a particular examination is x. The score out of 100 the student receives is y. Pairs of (x, y) values are recorded for a class of students and the relationship between x and y is investigated. The results may be summarized in the following table.

	x	y
Mean	10	60
Standard deviation	3	15

The covariance of x and y is equal to 36.

(a) Find the equation of the least squares regression line of y on x, expressing your answer in the form $y = mx + c$. *[4 marks]*

(b) (i) Use your answer to part (a) to predict how many marks a student who studies for 20 hours would achieve.

(ii) A teacher wishes to explain to students why they cannot guarantee a score of 100 by studying for the hours calculated in part (b) (i). In order to do so, the value of the product-moment correlation coefficient, r, is to be used.

(a) Calculate r for the given data.

(b) Based on the value of r obtained, how reliable is the prediction of part (b) (i)? *[7 marks]*

(This question continues on the following page)

(Question 6 continued)

(ii) Residents of a small town have savings which are normally distributed with a mean of \$ 3 000 and a standard deviation of \$ 500.

(a) (i) What percentage of townspeople have savings greater than \$ 3 200?

(ii) Two townspeople are chosen at random. What is the probability that **both** of them have savings between \$ 2 300 and \$ 3 300?

(iii) The percentage of townspeople with savings less than d dollars is 74.22 %. Find the value of d. *[8 marks]*

(b) The local newspaper claims that the mean savings is less than \$ 3 000. To test this claim, they take a random sample of 100 townspeople and they find that the mean of this sample is \$ 2 950.

(i) State the null hypothesis and the alternative hypothesis.

(ii) Show there is not sufficient evidence to support the newspaper's claim at the 5 % level of significance.

(iii) Use the above random sample of 100 townspeople to calculate the 99 % confidence interval for the mean savings. *[11 marks]*

Further Calculus

7. *[Maximum mark: 30]*

(i) Let $f(x) = \dfrac{3x^2}{5x-1}$.

(a) Write down the **equation** of the vertical asymptote of $y = f(x)$. *[1 mark]*

(b) Find $f'(x)$. Give your answer in the form $\dfrac{ax^2+bx}{(5x-1)^2}$, where a and $b \in \mathbb{Z}$. *[4 marks]*

(ii) The function $g(x)$ is defined for $-3 \le x \le 3$. The behaviour of $g'(x)$ and $g''(x)$ is given in the tables below.

x	$-3 < x < -2$	-2	$-2 < x < 1$	1	$1 < x < 3$
$g'(x)$	negative	0	positive	0	negative

x	$-3 < x < -\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2} < x < 3$
$g''(x)$	positive	0	negative

Use the information above to answer the following. In each case, justify your answer.

(a) Write down the value of x for which g has a maximum. *[2 marks]*

(b) On which intervals is the value of g decreasing? *[2 marks]*

(c) Write down the value of x for which the graph of g has a point of inflexion. *[2 marks]*

(d) Given that $g(-3) = 1$, sketch the graph of g. On the sketch, clearly indicate the position of the maximum point, the minimum point, and the point of inflexion. *[3 marks]*

(This question continues on the following page)

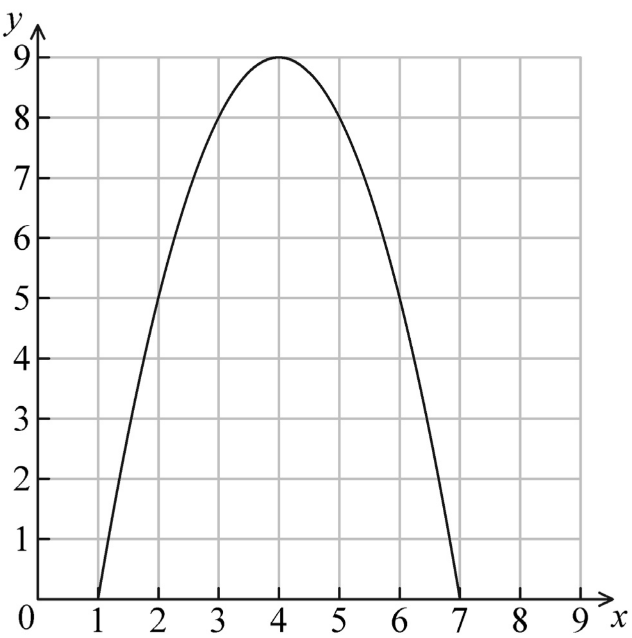
(Question 7 continued)

(iii) (a) Using the substitution $u = x^2 + 1$, or otherwise, find $\int 6x\sqrt{x^2+1}\,dx$. *[4 marks]*

(b) Evaluate $\int_0^{\sqrt{3}} 6x\sqrt{x^2+1}\,dx$. *[2 marks]*

(c) Let $\int_0^{\sqrt{3}} 6x\sqrt{x^2+1}\,dx = \int_0^k (2x+5)\,dx$. Find the two possible values of k. *[4 marks]*

(iv) The graph of $h(x) = -(x-1)(x-7) = -x^2 + 8x - 7$ is given below. The trapezium rule is used to estimate the value of $\int_2^4 (-x^2 + 8x - 7)\,dx$.



(a) (i) Sketch this graph on your answer paper.
On your sketch, draw in the **four** trapezia that are used to estimate $\int_2^4 (-x^2 + 8x - 7)\,dx$.

(ii) Use the trapezium rule to estimate $\int_2^4 (-x^2 + 8x - 7)\,dx$. *[5 marks]*

(b) Explain why the answer in part (a) (ii) underestimates the area under the curve between $x = 2$ and $x = 4$. *[1 mark]*

Further Geometry

8. *[Maximum mark: 30]*

(i) Let $\boldsymbol{C} = \begin{pmatrix} -2 & 4 \\ 1 & 7 \end{pmatrix}$ and $\boldsymbol{D} = \begin{pmatrix} 5 & 2 \\ -1 & a \end{pmatrix}$.

The 2×2 matrix $\boldsymbol{Q}$ is such that $3\boldsymbol{Q} = 2\boldsymbol{C} - \boldsymbol{D}$.

(a) Find $\boldsymbol{Q}$. *[3 marks]*

(b) Find $\boldsymbol{CD}$. *[4 marks]*

(c) Find $\boldsymbol{D}^{-1}$. *[2 marks]*

(ii) (a) Find the matrices representing the following transformations.

(i) The enlargement $\boldsymbol{E}$ such that $\boldsymbol{E}\begin{pmatrix} -1 \\ -2 \end{pmatrix} = \begin{pmatrix} -4 \\ -8 \end{pmatrix}$.

(ii) The rotation $\boldsymbol{R}$ such that $\boldsymbol{R}\begin{pmatrix} 5 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$.

(iii) The shear $\boldsymbol{H}$ in the x-direction such that $\boldsymbol{H}\begin{pmatrix} 5 \\ 5 \end{pmatrix} = \begin{pmatrix} 20 \\ 5 \end{pmatrix}$. *[9 marks]*

(b) Explain why the transformation $\boldsymbol{H}$ has an inverse. *[1 mark]*

(iii) (a) Let $\boldsymbol{v}$ and $\boldsymbol{w}$ be 2×1 non-zero vectors. Let $\boldsymbol{M}$ be a transformation such that the images of $\boldsymbol{v}$, $\boldsymbol{w}$ and $(2\boldsymbol{v} - 5\boldsymbol{w})$ under $\boldsymbol{M}$ are $\boldsymbol{M}(\boldsymbol{v}) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $\boldsymbol{M}(\boldsymbol{w}) = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ and $\boldsymbol{M}(2\boldsymbol{v} - 5\boldsymbol{w}) = \begin{pmatrix} 9 \\ h \end{pmatrix}$. Find h. *[2 marks]*

(b) Let $\boldsymbol{T} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$.

Find the image of the line $y = 2x + 1$ under $\boldsymbol{T}$. *[3 marks]*

(c) Let $\boldsymbol{S} = \begin{pmatrix} -1 & 2 \\ 1 & 4 \end{pmatrix}$.

(i) Find the matrix of the transformation $\boldsymbol{W}$ such that $\boldsymbol{TW} = \boldsymbol{S}$.

(ii) Let A' be the image under $\boldsymbol{S}$ of the square A with vertices at (0, 0), (0, 1), (1, 1), and (1, 0). Calculate the area of A'. *[6 marks]*