MATHEMATICAL METHODS SL

This was the last May session of the current course. Teachers are reminded that the new courses will be examined for the first time in May 2006, and they should make sure they are preparing their students from the new subject guide. Note that Mathematical Methods SL will be renamed Mathematics SL.

Overall grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-15	16-29	30-46	47-58	59-70	71-82	93-100

Portfolio

Teachers are reminded that both the requirements for the portfolio and the criteria are changed for assesment in 2006. Details are provided in the Subject Guide. Current tasks are not likely to be suitable for the new course.

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 4	5 – 6	7 – 9	10 – 12	13 – 14	15 – 16	17 – 20

Examiners reported some excellent work but also a range of administrative problems. Teachers should note that all of the following information will continue to be required:

- Dates of setting and submission of work
- A clean copy of the statement of the task
- Original student work, not photocopies
- Specific comments either on form 5/PFCS or Form B (from the Teacher Support Material) and/or on the student work itself

The following information is increasingly provided by schools and is extremely helpful in the moderation process:

- Background information on students' prior knowledge and the role of the assignment in the teaching programme
- Clear marking of student work to show correct/incorrect answers and conclusions
- Solutions/marking key for teacher designed tasks

Comments relating to the assessment criteria

The comments below will also be applicable to the new course

Notation and Terminology

Teachers must inform students who word process their work that notation such as ^ for squared and * for multiply are not acceptable and will be penalised in criterion A. For sections heavy on the use of symbols students should either use a facility such as Equation Editor in Word or write in the algebra by hand.

Communication

Graphs must be carefully drawn and fully labelled. This may be done by hand on computer printout if necessary. Explanations and linking sentences must be provided so that it is possible to read through the work without constantly referring to the statement of the task. It is not sufficient to copy each question and then add a response. A brief introduction should set the scene and make the overall task clear. Brief explanation should be given as to why any calculation or process is being carried out. Definitions of terms and routine operations should be concise. It is unlikely that very lengthy paragraphs of explanation will enhance communication.

Use of Technology

Students should be encouraged to use computer software as well as the graphic display calculator (GDC) when possible. Details of any software package used should be provided. Students should also, wherever possible, provide some printout of their work. Level 3 is often awarded too generously. Merely using some technology is not enough and may only achieve level 1. The key to this criterion is whether the use made enhances the mathematics; for instance, by enabling some analysis which would not otherwise be feasible.

Paper 1

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-16	17-32	33-48	49-58	59-69	70-79	8-90

G2 summaries

- **Comparison with last year's paper:**

Much easier	A little easier	Similar standard	A little more difficult	Much more difficult
4	64	130	14	2

- **Suitability of question paper:**

	Too easy	Appropriate	Too Difficult
Level of difficulty	20	265	1
	Poor	**Satisfactory**	**Good**
Syllabus coverage	2	140	146
Clarity of wording	5	110	173
Presentation of paper	2	77	209

General comments

Most candidates appeared to find the paper accessible, enabling them to show what they knew and could do. Details of common errors are given below. Few candidates appeared to have made full use of their graphic display calculator. Examiners reported a frequent lack of basic algebra skills. This was particularly evident in Questions 7, 9 and 12, where many candidates had a correct strategy but did not achieve full marks.

Question 1 Arithmetic sequences

Many candidates set $u_2 = S_2 - S_1 = d$ but were able to follow through correctly in part (c).

Question 2 Gradient and equation of a line

The most common error was to find the perpendicular line rather than the parallel one.

Question 3 Binomial expansion

A significant number of candidates tried to multiply out the expression rather than use the binomial formula. Many otherwise successful solutions missed the minus sign.

Question 4 Arc length and sector area

Missing units were often penalised here. If candidates knew the relevant formulae they usually found the question straightforward. Some tried to use degrees.

Question 5 Probability

Part (c) was a major source of errors. Many candidates assumed *E* and *F* were independent. Candidates who drew a table of outcomes were usually successful.

Question 6 Chain rule and integration

Many partially correct answers with one or more coefficients missing. A few candidates tried to expand the expression and a few others confused the notation for derivative with that for the inverse function.

Question 7 Exponential equation

Few students converted to a common base. Those who attempted to use logarithms made many basic algebraic errors. A graphical approximation would have been a useful check on the answer found by an algebraic approach.

Question 8 Anti-differentiation with a boundary condition

Some candidates did not realise the need to integrate and instead found the tangent line. Those who did integrate omitted the constant of integration and so made no further progress.

Question 9 Logarithmic equation

Most candidates attempted to solve this algebraically. There were many errors when combining the logarithms, which made solving the equation more difficult. A common mistake was to set $\frac{x}{x-5} = 1$.

Question 10 Angle between two vectors

Most used the scalar product formula but with many errors. Many candidates went on to find the angle, which was not required.

Question 11 Means

This question was mostly done well. Many answers incurred the accuracy penalty.

Question 12 Inverse function

Most candidates understood that x and y must be exchanged. After that there were many problems with manipulating the algebra.

Question 13 Transformations of graphs

Some candidates were successful with at least one of the transformations. Some candidates' graphs were poorly drawn and so did not gain full credit.

Question 14 Trigonometric equations

Few candidates who used an algebraic approach were fully successful, often giving values outside the domain or missing a solution. The small number who used a graphical approach usually gained full marks.

Question 15 Evaluating general calculus expressions

This question proved to be a challenging test of understanding of concepts. Many candidates substituted before (or without) differentiating or integrating.

Paper 2

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-12	13-25	26-42	43-54	55-67	68-79	80-100

G2 summaries

- **Comparison with last year's paper:**

Much easier	A little easier	Similar standard	A little more difficult	Much more difficult
0	35	147	26	2

- **Suitability of question paper:**

	Too easy	Appropriate	Too Difficult
Level of difficulty	8	172	4
	Poor	**Satisfactory**	**Good**
Syllabus coverage	8	135	142
Clarity of wording	22	139	123
Presentation of paper	1	105	179

The areas of the programme that proved difficult for candidates

Two general areas of weakness continue to be vectors and probability. Many candidates missed even the most basic parts of the vector question and very few candidates were able to show mathematically that two events were not independent in the probability question. Trigonometry posed problems in the areas of solving trigonometric equations with a restricted domain and solving ambiguous triangles. Writing an expression for the area under the curve split into two regions proved very difficult for most candidates. In the option question on statistics, calculating the regression line from basic statistics was poorly done. In the option question on further calculus, use of the trapezium rule caused problems as did justifying the properties of curves on the basis of the first and second derivatives.

The levels of knowledge, understanding and skill demonstrated

In general, candidates did well when working with a quadratic function and its graph and finding the equation of a normal. Finding simple probabilities using a table was a strong point. Candidates demonstrated skill in using the GDC to find intersection points of curves. In the option question on statistics, candidates were strong in basic applications of the normal distribution. In the option question on further calculus, they were strong in using the quotient rule and were mixed in using integration by substitution.

The strengths and weaknesses of candidates in the treatment of individual questions

Question 1: *Quadratic Functions, Graphs and Normals*

Most candidates were able to write the equation of the quadratic function given the x-intercepts and one other point. They were able to find the derivative and the equation of the tangent line. Many had difficulty in writing an equation of the line perpendicular to the tangent at B(4,0). Two common errors were to simply write the equation of the tangent line or to assume that the gradient of the tangent was 7 from the previous part. Candidates did well in finding the intersection of the line and the parabola. However, if they find the intersection point with the GDC instead of algebraically, they need to show a sketch of the graph to support their method.

Question 2: *Probability*

Most candidates were able to find the basic probabilities when given a table of values. There was little success in justifying mathematically that the events were not independent. Many students confused independent and mutually exclusive events, while others simply talked about the two events having an effect on each other. The simple conditional probability in part (b) was often well done by those simply using the values from the table; others who used the formula were often unsuccessful. The final part on the probability of combined events was seldom totally correct; candidates either did not realise that there were two ways the combined event could occur or neglected to use conditional probability when selecting the second student.

Question 3: *Trigonometry: Solution of Triangles*

A number of candidates used right-angled triangle trigonometry throughout the problem garnering few, if any, marks. Many candidates were successful in using the cosine rule in the first part to find BC and in deciding to use the sine rule in the second part. However, candidates had difficulty interpreting the question and often used the length of BC as 8.24 throughout the remainder of the question. Other candidates seemed to think that the length of AC was fixed throughout the problem and were confused on how to proceed. Candidates made good attempts at finding the area. The third part of the question which involved the ambiguous case of the sine rule was poorly done. Those who were successful used a variety of valid approaches often beginning by finding BC = 9.02. The final part in recognizing that the shortest distance was the perpendicular distance was perhaps the easiest part of the question and unfortunately was not attempted by many candidates who had floundered in the parts before it.

Question 4: *Calculus of Trigonometric Functions*

Generally candidates were successful in finding the derivative Factorizing a rather basic quadratic trigonometric expression proved much more difficult than expected. Generally candidates did not use the factorization to find the solutions to the derivative being zero, but rather used the GDC. The answers were often given in degrees which were not appropriate, given the domain of the function. Many candidates found the solution $\frac{\pi}{6}$, but neglected to find $\frac{5\pi}{6}$ and $\frac{3\pi}{2}$ and incorrectly included $-\frac{\pi}{2}$. Finding the *x*-coordinate of the maximum was well done and most candidates knew that an integral was needed to find the area of the shaded region. However, very few could correctly write an expression for this area; many simply treated it as the integral from *a* to *b*, obtaining a value of zero for the area.

Question 5: *Vectors*

Candidates were generally successful in finding vector $\vec{AB}$ but were unable to find the unit vector requested. Since this was a "show that" question, many candidates did not receive marks for simply dividing by 1000 without indicating where this number came from. Many candidates stated that "$1 \text{ km} = 1000 \text{ m}$" as faulty justification for dividing by 1000 while others tried to work in reverse which is not acceptable. In part (b), candidates had difficulty with the velocity vector but were able to find the position of the aircraft at 13:00. Most could find when the aircraft was flying over town B, although there was some difficulty with the 24-hour clock. There were some good, succinct solutions to the warning light problem but many candidates found themselves wandering in a circuitous route of computations and getting nowhere.

Question 6: *Statistical Methods Option*

Though the formula is given in the information booklet, few candidates seemed to know how to apply it to find the least squares regression line. Much the same could be said for the correlation coefficient, though it was encouraging to see reasonably good recognition of the significance of the value of r by a number of candidates. The work on the normal distribution was done quite well, and there were some very good solutions to the hypothesis testing section. However, many showed very little idea of how to proceed with such testing. Two common errors occurred in the question on the confidence interval for the mean savings. Many used the theoretical mean of \$ 3000 instead of the sample mean of \$ 2950, and many calculated a 98 % confidence interval using $z = 2.327$ instead of the 99 % interval using $z = 2.576$. Most did correctly use the standard error of the mean.

Question 7: *Further Calculus Option*

Candidates generally did well on part (i) in writing the equation of the vertical asymptote and in using the quotient rule. In part (ii), most of the candidates were able to identify the value of the *x* coordinate where the function had a maximum or an inflexion point and also give the intervals on which the function was decreasing. They were less successful in justifying their responses. Very few candidates stated that the concavity had to change for an inflexion point to exist and simply said that the second derivative being zero was justification enough. Many good graphs were sketched but others had discontinuous functions or non-differentiable functions or extra points of inflexion near $x = 0.5$. The integration by substitution saw some good results but too often candidates retained *x* as well as *u* in their integrals. They were, however, able to gain some follow through marks in the successive parts. In part (iv), the application of the trapezium rule was not well done. Many candidates sketched rectangles instead of trapezia and many others neglected to draw the trapezia between $x = 2$ and $x = 4$. The calculation of the area with the trapezium rule was poorly done; even those who correctly set it up often did not obtain the correct answer. Most candidates earned the final mark for explaining why the trapezium rule gave an under estimation. There were many candidates who did a very fine job with this question.

Question 8: *Further Geometry Option*

Several candidates did good work on this question. Others who opted to do this question were obviously unprepared for it and could not do even the simplest of the parts. Candidates did well on the first part of the question on matrix algebra. In part (ii), candidates could often answer the part concerning the enlargement and the rotation, but stumbled on the shear transformation question. In part (iii) many of the candidates could find the value of h but few could find the image of the line under $\boldsymbol{T}$. There was little success in the final part in finding the transformation matrix and working with the area factor.

Recommendations and guidance for the teaching of future candidates

Candidates must be aware of the change to the rubrics for papers in 2006. For paper 1, working must be clearly shown and correct answers with no working may not gain full marks. The time allowed is increased to one and a half hours. For paper 2, there will be 5 questions and the time allowed is one and a half hours.

Candidates need to have an understanding of underlying concepts in addition to an ability to carry out standard processes.

Candidates need to have more practice with basic algebra skills so that they can complete solutions correctly.

Candidates need more guidance on when and how to use their graphical display calculator in answering questions. There were several questions where a graphical solution would be appropriate but was seldom seen, e.g. paper 1, Questions 7 (as a check), 9(b) and 14. Where the calculator is used to solve a problem graphically, a clear sketch should be provided.

Candidates need practice in giving explanations for results and in justifying their answers.

Candidates need practice with questions that request they "Show that…" something is true. Generally, these types of questions cannot be done with the GDC. It is also important that candidates do not simply verify that the answer is correct by working in reverse.

Candidates would benefit from guidance and practice on ways of checking answers.

There is a continuing need to pay attention to details:

- avoid premature rounding of answers which leads to inaccuracy in the final result
- give numerical answers to three significant figures not three decimal places
- use the given domain, e.g. paper 1, Question 14
- interpret notation correctly e.g. paper 1, Question 6, Question 12
- read the question carefully and answer what is asked e.g. paper 1, Question 10
- draw any graphs or diagrams carefully e.g. paper 1, Question 14
- always state any units in the answer
- consider when it is appropriate to use degrees and when it is appropriate to use radians.

Good examination preparation should include becoming familiar with the following:

- all parts of the syllabus and especially more work with vectors and probability

- the notation that will be used on the paper (see list in the subject guide)
- the contents of the formula booklet (renamed information booklet for 2006)
- terminology that may be used (see syllabus content)
- the command terms
- good presentation. (On paper 2 candidates should start each question on a new page and carefully label the part of the question that is being answered.)

