



**MATHEMATICAL STUDIES
STANDARD LEVEL
PAPER 2**

Thursday 4 November 2004 (morning)

2 hours

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all five questions from Section A and one question from Section B.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.
- Indicate the make and model of your calculator in the appropriate box on your cover sheet.

Please start each question on a new page. You are advised to show all working, where possible. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

SECTION A

Answer all **five** questions from this section.

1. [Maximum mark: 14]

A school jazz band contains three different musical instruments – saxophone (S), clarinet (C) and drums (D). Students in the band are able to play one, two or three different instruments.

In a class of 40 IB students, 25 belong to the jazz band. Out of these 25

3 can play all three instruments

5 can play the saxophone and clarinet **only**

5 can play **at least** the clarinet and drums

7 can play **at least** the saxophone and drums

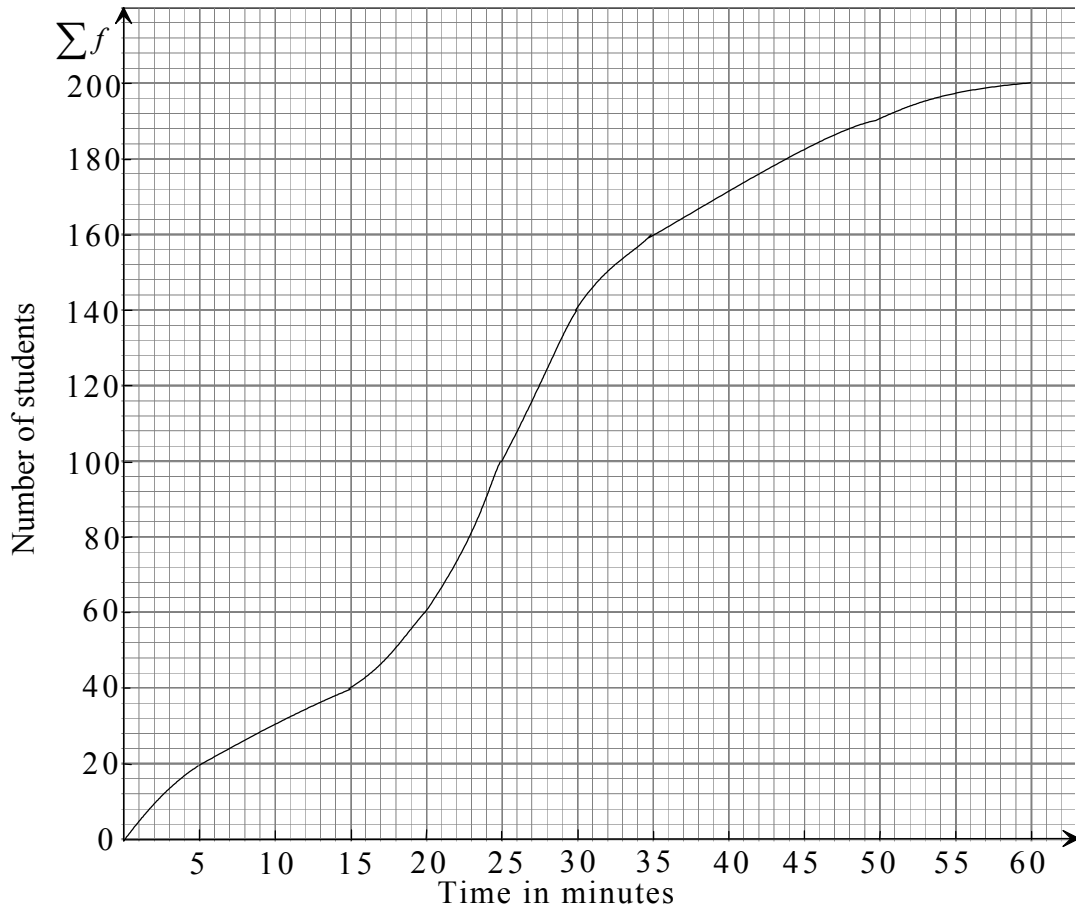
16 can play the saxophone

12 can play the clarinet.

- (a) Draw a Venn Diagram and clearly indicate the numbers in each region. [5 marks]
- (b) Show that the number of students who play the drums **only** is 5. [2 marks]
- (c) Find the probability that a student chosen at random from the IB class plays only the saxophone. [2 marks]
- (d) Find the probability that a student chosen at random from the IB class plays either the clarinet or drums or both. [2 marks]
- (e) Given that a student plays the saxophone, find the probability that he also plays the clarinet. [3 marks]

2. [Maximum mark: 11]

The cumulative frequency graph has been drawn from a frequency table showing the time it takes a number of students to complete a computer game.



- (a) From the graph find
- (i) the median time;
 - (ii) the interquartile range.

[5 marks]

(This question continues on the following page)

(Question 2 continued)

The graph has been drawn from the data given in the table below.

Time in minutes	Number of students
$0 < x \leq 5$	20
$5 < x \leq 15$	20
$15 < x \leq 20$	p
$20 < x \leq 25$	40
$25 < x \leq 35$	60
$35 < x \leq 50$	q
$50 < x \leq 60$	10

- (b) Using the graph, find the values of p and q . [2 marks]
- (c) Calculate an estimate of the mean time taken to finish the computer game. [4 marks]

3. [Maximum mark: 17]

Two functions are defined as follows

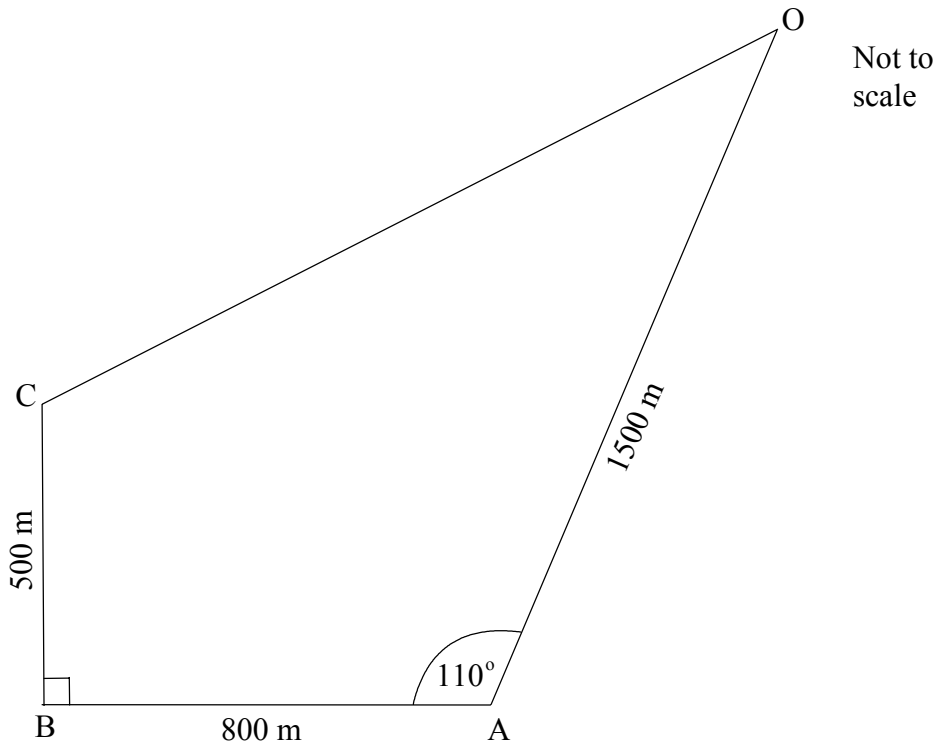
$$f(x) = \begin{cases} 6-x & \text{for } 0 \leq x < 6 \\ x-6 & \text{for } x \geq 6 \end{cases}$$

$$g(x) = \frac{1}{2}x$$

- (a) Draw the graphs of the functions f and g in the interval $0 \leq x \leq 14, 0 \leq y \leq 8$ using a scale of 1 cm to represent 1 unit on both axes. [5 marks]
- (b) (i) Mark the intersection points A and B of $f(x)$ and $g(x)$ on the graph.
- (ii) Write down the coordinates of A and B. [3 marks]
- (c) (i) If N is the point where $f(x)$ meets the x -axis write down the coordinates of N.
- (ii) Show that angle ANB is a right angle. [3 marks]
- (d) Calculate the midpoint M of the line AB. [2 marks]
- (e) Find the equation of the straight line which joins the points M and N. [4 marks]

4. [Maximum mark: 16]

A cross-country running course is given in the diagram below. Runners start and finish at point O.



- (a) Show that the distance CA is 943 m correct to 3 s.f. [2 marks]
- (b) Show that angle BCA is 58.0° correct to 3 s.f. [2 marks]
- (c) (i) Calculate the angle CAO.
- (ii) Calculate the distance CO. [5 marks]
- (d) Calculate the area enclosed by the course OABC. [4 marks]
- (e) Gonzales runs at a speed of 4 ms^{-1} . Calculate the time, in minutes, taken for him to complete the course. [3 marks]

5. [Maximum mark: 12]

A small company produces two models of mobile phone, the *SpeakEasy* and the *ClearTalk*.

Each *SpeakEasy* takes six hours to make and each *ClearTalk* takes ten hours to make. There is a total number of 900 hours available per week for making the two models.

Let x be the number of *SpeakEasy* mobile phones produced per week and y be the number of *ClearTalk* mobile phones produced per week.

- (a) Show that an inequality for the above information is $3x + 5y \leq 450$. [1 mark]

Due to the difference in demand for the two models the marketing costs are \$ 20 for each *SpeakEasy* phone and \$ 10 for each *ClearTalk* phone. The company can spend up to a total of \$ 1600 each week on marketing.

- (b) Write down a second inequality found from this information. [1 mark]

- (c) Draw the graph representing these two inequalities taking 1 cm to represent 10 units on each axis. [5 marks]

- (d) Given that $x \geq 0$ and $y \geq 0$ indicate clearly, by shading, the region represented by all four inequalities. [1 mark]

The profit for each *SpeakEasy* is \$ 20 and for each *ClearTalk* is \$ 30.

- (e) Write down an equation for the weekly profit, P . [1 mark]

- (f) Find the maximum profit and the number of each model made to give this profit. [3 marks]

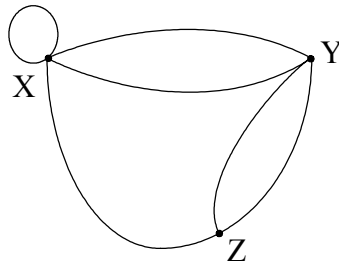
SECTION B

Answer **one** question from this section.

Matrices and Graph Theory

6. [Maximum mark: 30]

(i) A graph of a road joining villages X, Y and Z is shown below.



(a) Write down the number of vertices shown in the graph. [1 mark]

(b) Write down the number of edges shown in the graph. [1 mark]

(c) State the degree of each vertex. [3 marks]

(ii) An adjacency matrix is given for the roads between the towns A, B, C and D. Draw a directed graph to represent these roads.

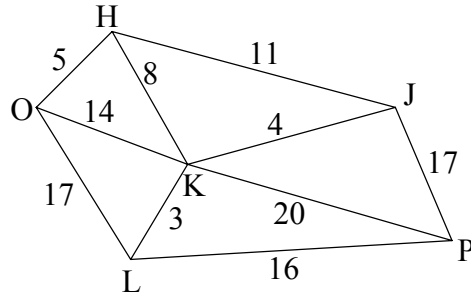
$$\begin{array}{c}
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 0 & 0 & 1 & 2 \\
 1 & 1 & 0 & 1 \\
 2 & 1 & 1 & 0
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[4 marks]

(This question continues on the following page)

(Question 6 continued)

- (iii) Given the graph below to show the distances in kilometres between the towns O, H, J, K, L and P calculate the shortest distance between O and P given that the road must pass through H and K. (It could also pass through other towns.)



[2 marks]

- (iv) A man either drives his car (d) or takes a train (t) to work each day. He never takes the train two days in a row. If he drives to work, then the next day the probability that he will take the train is the same as the probability he will drive.

The transition matrix for the above information is shown below.

$$\begin{matrix} & t & d \\ t & \begin{pmatrix} 0 & 1 \end{pmatrix} \\ d & \begin{pmatrix} a & b \end{pmatrix} \end{matrix}$$

- (a) Explain why there is a 1 in the row for t . [1 mark]
- (b) Calculate the values of a and b . [2 marks]

(This question continues on the following page)

(Question 6 continued)

- (v) Four teams Alpha, Beta, Gamma and Delta play a football tournament. They can win and receive 3 points, draw and each team receives 1 point or lose and receive 0 points.

- (a) Write down a 3 by 1 column matrix to indicate possible points scored per match, by each team. *[1 mark]*

- (b) The results of the matches are given in the matrix below.

	Win	Draw	Lose
Alpha	4	0	2
Beta	1	4	1
Gamma	2	2	2
Delta	0	6	0

- (i) Write down the two matrices that will be multiplied together to work out the total number of points scored by each team. *[2 marks]*

- (ii) **Hence** calculate the total number of points scored by each team. *[2 marks]*

- (vi) A matrix A is given by $\begin{pmatrix} 1 & x \\ 2 & 3 \end{pmatrix}$.

- (a) Write down the transpose of matrix A . *[1 mark]*

- (b) Calculate AA^T . *[4 marks]*

(This question continues on the following page)

(Question 6 continued)

- (vii) Two players Stephen and Jane take part in a game. They each, independently, pick either +1 or –1. After their choices are revealed Stephen pays to Jane the sum of the numbers picked. The possible outcomes are shown below (the payoff matrix for Jane).

$$\begin{array}{c} \text{Stephen} \\ \quad 1 \quad -1 \\ \text{Jane } 1 \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix} \\ -1 \end{array}$$

- (a) What is Jane’s play safe strategy? *[1 mark]*
- (b) What is Stephen’s play safe strategy? *[1 mark]*
- (c) In the case when they both use their play safe strategy what do they each gain? *[1 mark]*
- (d) If Stephen believes that Jane will play her optimum strategy what should he play? *[1 mark]*
- (e) Is this game fair? Give a reason for your answer. *[2 marks]*

Further Statistics and Probability

7. [Maximum mark: 30]

- (i) The following table of observed results gives the number of candidates taking a Mathematics examination classified by gender and grade obtained.

		Grade			Total
		5, 6 or 7	3 or 4	1 or 2	
Gender	Males	5000	3400	600	9000
	Females	6000	4000	1000	11000
	Total	11000	7400	1600	20000

The question posed is whether gender and grade obtained are independent.

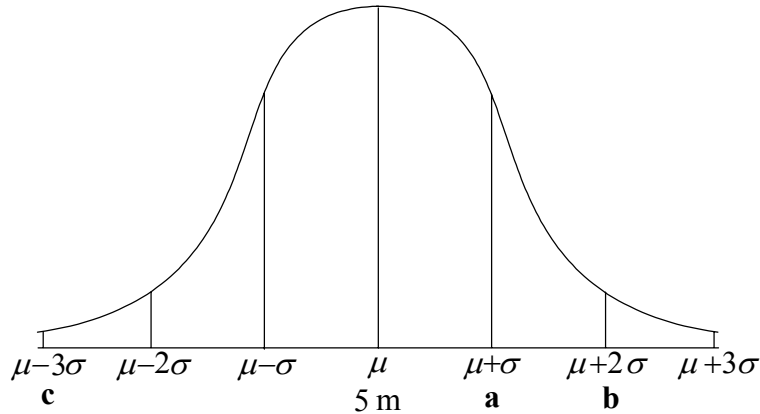
- (a) **Show clearly** that the expected number of males achieving a grade of 5, 6 or 7 is 4950. [2 marks]
- (b) A χ^2 test is set up.
- (i) State the Null hypothesis. [1 mark]
- (ii) State the number of degrees of freedom. [1 mark]
- (iii) The calculated χ^2 value at the 5 % test level is 39.957. Write down the critical value of χ^2 at the 5 % level of significance. [1 mark]
- (iv) What can you say about gender and grade obtained? [1 mark]

(This question continues on the following page)

(Question 7 continued)

(ii) A manufacturer makes wooden sticks with a mean length of 5 m. The lengths are normally distributed with a standard deviation of 10 cm.

(a) Calculate the values of **a**, **b** and **c** shown on the graph below.



[3 marks]

(b) What is the probability that a stick chosen at random will measure more than 4.85 m?

[3 marks]

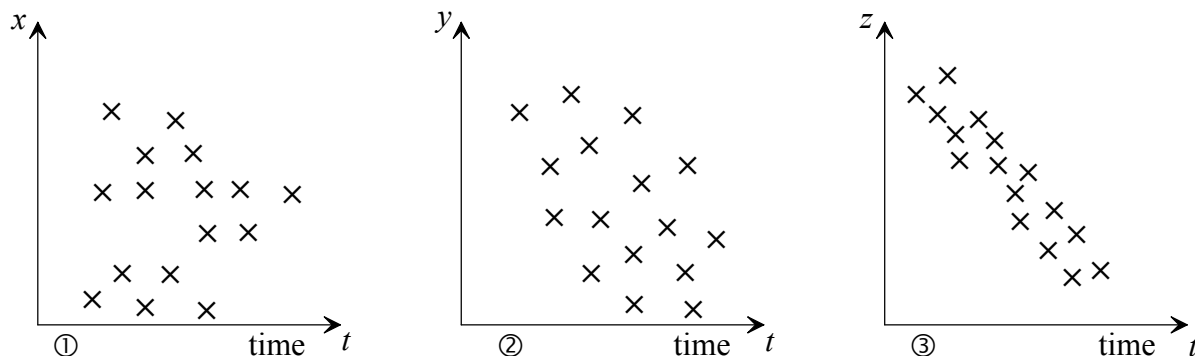
(c) The manufacturer sets the machine to make different sticks with a mean length of 3.5 m. It is known that 90 % of the sticks will be less than 3.8 m in length. What is the standard deviation of these lengths?

[4 marks]

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(Question 7 continued)

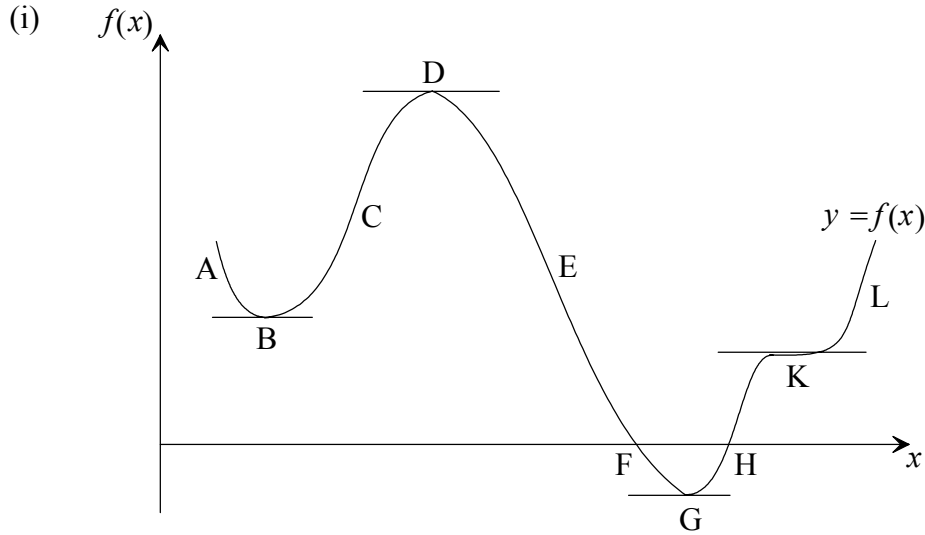
- (iii) The sketches below represent scatter diagrams for the way in which variables x , y and z change over time, t , in a given chemical experiment. They are labelled ①, ② and ③.



- (a) State which of the diagrams indicate that the pair of variables
- is not correlated. [1 mark]
 - shows strong linear correlation. [1 mark]
- (b) A student is given a piece of paper with five numbers written on it. She is told that three of these numbers are the product moment correlation coefficients for the three pairs of variables shown above. The five numbers are
- 0.9, -0.85, -0.20, 0.04, 1.60
- For each sketch above state which of these five numbers is the most appropriate value for the correlation coefficient. [3 marks]
 - For the two remaining numbers, state why you reject them for this experiment. [2 marks]
- (c) Another variable, w , over time, t , gave the following information
- $$\sum t = 124 \quad \sum w = 250 \quad s_t = 6.08 \quad s_w = 10.50 \quad s_{tw} = 55.00$$
- for 20 data points.
- Calculate
- the product moment correlation coefficient for this data. [2 marks]
 - the equation of the regression line of w on t in the form $w = at + b$. [5 marks]

Introductory Differential Calculus

8. [Maximum mark: 30]



Given the graph of $f(x)$ state

- (a) the intervals from A to L in which $f(x)$ is increasing. [1 mark]
- (b) the intervals from A to L in which $f(x)$ is decreasing. [1 mark]
- (c) a point that is a maximum value. [1 mark]
- (d) a point that is a minimum value. [1 mark]
- (e) the name given to point K where the gradient is zero. [1 mark]

(ii) Consider the function $g(x) = x^4 + 3x^3 + 2x^2 + x + 4$.

Find

- (a) $g'(x)$ [3 marks]
- (b) $g'(1)$ [2 marks]

(This question continues on the following page)

(Question 8 continued)

(iii) A function is given as $y = ax^2 + bx + 6$.

(a) Find $\frac{dy}{dx}$. [2 marks]

(b) If the gradient of this function is 2 when x is 6 write an equation in terms of a and b . [2 marks]

(c) If the point $(3, -15)$ lies on the graph of the function find a second equation in terms of a and b . [2 marks]

(iv) The cost of producing a mathematics textbook is \$ 15 (US dollars) and it is then sold for \$ x .

(a) Find an expression for the profit made on each book sold. [1 mark]

A total of $(100000 - 4000x)$ books is sold.

(b) Show that the profit made on all the books sold is $P = 160000x - 4000x^2 - 1500000$. [3 marks]

(c) (i) Find $\frac{dP}{dx}$. [2 marks]

(ii) Hence calculate the value of x to make a maximum profit. [2 marks]

(d) Calculate the number of books sold to make this maximum profit. [2 marks]

(v) A particle moving in a straight line passes a fixed point O with a velocity of 20 ms^{-1} . It has an acceleration $a \text{ ms}^{-2}$ given by $a = 2t - 8$ where t seconds is the time after the particle passes the fixed point, O.

Write an expression for v , the velocity of the particle, at time t seconds after passing O. [4 marks]