IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
PROGRAMA DEL DIPLOMA DEL BI

N04/5/MATSD/SP2/ENG/TZ0/XX/M+

MARKSCHEME

November 2004

MATHEMATICAL STUDIES

Standard Level

Paper 2

18 pages

Paper 2 Markscheme

Instructions to Examiners

1 Method of marking

(a) All marking must be done using a **red** pen.

(b) Marks should be noted on candidates' scripts as in the markscheme:

- show the breakdown of individual marks using the abbreviations ***(M1)***, ***(A2)*** *etc*.
- write down each part mark total, indicated on the markscheme (for example, ***[3 marks]***) – it is suggested that this be written at the end of each part, and underlined;
- write down and circle the total for each question at the end of the question.

2 Abbreviations

The markscheme may make use of the following abbreviations:

M Marks awarded for **Method**

A Marks awarded for an **Answer** or for **Accuracy**

G Marks awarded for correct solutions, generally obtained from a **Graphic Display Calculator**, irrespective of working shown

R Marks awarded for clear **Reasoning**

AG **Answer Given** in the question and consequently marks are **not** awarded

3 Follow Through (ft) Marks

Errors made at any step of a solution can affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks should be awarded. The procedures for awarding these marks require that all examiners:

(i) penalise an error when it **first occurs**;

(ii) **accept the incorrect answer** as the appropriate value or quantity to be used in all subsequent working;

(iii) award ***M*** marks for a correct method, and ***A*****(ft)** marks if the subsequent working contains no further errors.

Follow through procedures may be applied repeatedly throughout the same problem.

The following illustrates a use of the **follow through** procedure:

Markscheme		Candidate's Script	Marking	
\$ 600 × 1.02	***M1***	Amount earned = \$ 600 × 1.02	√	***M1***
= \$ 612	***A1***	= \$602	×	***A0***
\$ (306 × 1.02) + (306 × 1.04)	***M1***	Amount = 301 × 1.02 + 301 × 1.04	√	***M1***
= \$ 630.36	***A1***	= \$ 620.06	√	***A1*(ft)**

Note that the candidate made an arithmetical error at line 2; the candidate used a correct method at lines 3, 4; the candidate's working at lines 3, 4 is correct.

However, if a question is transformed by an error into a **different, much simpler question** then:

(i) **fewer** marks should be awarded at the discretion of the Examiner;

(ii) marks awarded should be followed by "**(d)**" (to indicate that these marks have been awarded at the **discretion** of the Examiner);

(iii) a brief **note** should be written on the script explaining **how** these marks have been awarded.

4 Using the Markscheme

(a) This markscheme presents a particular way in which each question may be worked and how it should be marked. **Alternative methods** have not always been included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method in a manner which is consistent with the markscheme.

In this case:

(i) a mark should be awarded followed by "**(d)**" (to indicate that these marks have been awarded at the **discretion** of the Examiner);

(ii) a brief **note** should be written on the script explaining **how** these marks have been awarded.

Where alternative methods for complete questions are included, they are indicated by **METHOD 1**, **METHOD 2**, *etc*. Other alternative solutions, including graphic display calculator alternative solutions are indicated by **OR**. For example:

Mean = 7906/134 ***(M1)***
= 59 ***(A1)***

OR

Mean = 59 ***(G2)***

(b) Unless the question specifies otherwise, accept **equivalent forms**. For example: $\frac{\sin\theta}{\cos\theta}$ for $\tan\theta$.

On the markscheme, these equivalent numerical or algebraic forms will generally be written in brackets after the required answer. Paper setters will indicate the required answer, by allocating full marks at that point. Further working should be ignored, even if it is incorrect. For example: if candidates are asked to factorize a quadratic expression, and they do so correctly, they are awarded full marks. If they then continue and find the roots of the corresponding equation, do not penalize, even if those roots are incorrect ie, once the correct answer is seen, ignore further working.

(c) As this is an international examination, all **alternative forms of notation** should be accepted. For example: 1.7 , $1\cdot 7$, 1,7 ; different forms of vector notation such as $\vec{u}$, $\overline{u}$, $\underline{u}$; $\tan^{-1} x$ for arctan x.

5 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy.

There are two types of accuracy error. Candidates should be penalized **once only IN THE PAPER** for an accuracy error **(AP)**.

Award the marks as usual then write –1**(AP)** against the answer and also on the **front** cover

Rounding errors: only applies to final answers not to intermediate steps.

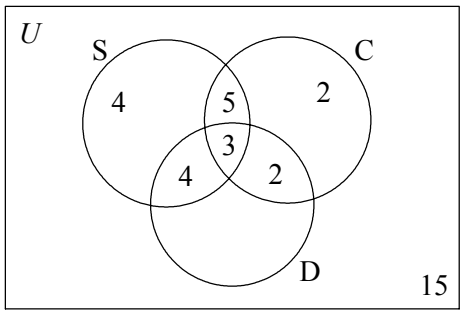
Level of accuracy: when this is not specified in the question the general rule *unless otherwise stated in the question all numerical answers must be given exactly or to three significant figures* applies.

- If a final correct answer is incorrectly rounded, apply the **AP**
  OR
- If the level of accuracy is not specified in the question, apply the **AP** for answers not given to 3 significant figures. (Please note that this has changed from May 2003).

6 Graphic Display Calculators

Many candidates will be obtaining solutions directly from their calculators, often without showing any working. They have been advised that they must use mathematical notation, not calculator commands when explaining what they are doing. Incorrect answers without working will receive no marks. However, if there is written evidence of using a graphic display calculator correctly, method marks may be awarded. Where possible, examples will be provided to guide examiners in awarding these method marks.

QUESTION 1



(a) 3 intersecting circles and rectangle. ***(A1)***
correct numbers ***(A4)***

Note:	Award ***(A4)*** for all 7 numbers correct, ***(A3)*** for 6 correct, ***(A2)*** for 5 correct, ***(A1)*** for 4 correct. (Do **not** count the number in D only on the Venn Diagram at this stage.)

[5 marks]

(b) $4+5+3+4+2+2+15+x=40$ ***(M1)***

$35+x=40$

$x=5$ ***(A1)***

Therefore, five play drums only. ***(AG)***

[2 marks]

(c) $\frac{4}{40}\left(\frac{1}{10}, 10\,\%, 0.1\right)$ ***(A2)***

Note:	Award ***(A1)*** for 4, ***(A1)*** for 40.

[2 marks]

(d) $\frac{21}{40}\left(52.5\,\%, 0.525\right)$ ***(A2)***

Note:	Award ***(A1)*** for 21, ***(A1)*** for 40.

[2 marks]

(e) $\frac{8}{16}\left(\frac{1}{2}, 50\,\%, 0.5\right)$ ***(A3)***

Note:	Award ***(A1)*** for 8, ***(A2)*** for 16. Do **not** separate ***(A2)***.

[3 marks]

Total [14 marks]

QUESTION 2

(a) (i) 25 minutes (± 2 minutes) ***(A2)***

(ii) Lower quartile $= 18$ (± 1 minute) ***(A1)***
Upper quartile $= 32$ (± 1 minute) ***(A1)***
Interquartile range $= 32 - 18 = 14$ minutes (± 2 minutes) ***(A1)***

OR

Accept [18 to 32] as interval for the interquartile range. ***(A3)***

[5 marks]

(b) $p = 20$ ***(A1)***
$q = 30$ ***(A1)***

[2 marks]

(c)

Midpoint	**Frequency**	$M \times f$
2.5	20	50
10	20	200
17.5	20	350
22.5	40	900
30	60	1800
42.5	30	1275
55	10	550
(A1)	Total = 200	Total = 5125

(A1)

Mean $= \frac{5125}{200} = 25.625$ (exact) or 25.6 (3 significant figures) ***(M1)(A1)***

Note: Not every step needs to be seen to get the marks.

OR

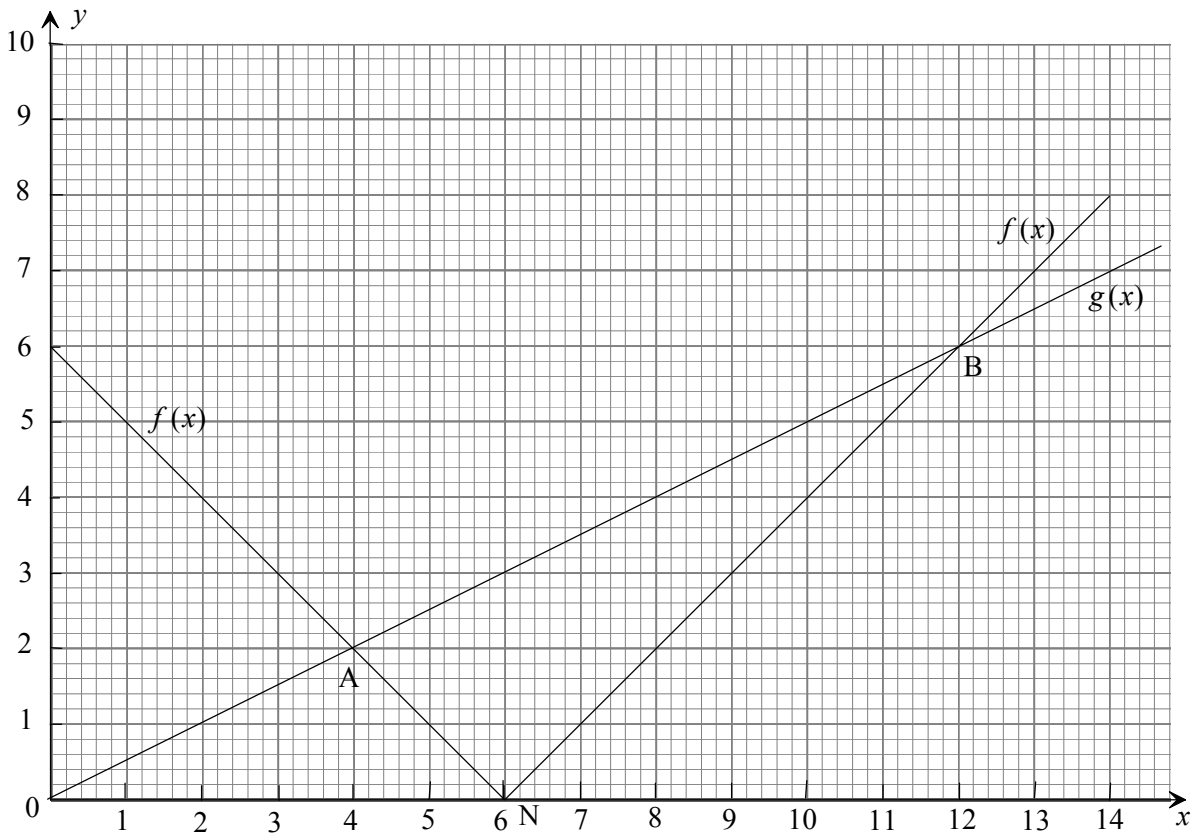
Mean = 25.625 or 25.6 (using GDC) ***(G4)***

[4 marks]

Total [11 marks]

QUESTION 3

(a)



(A5)

Note:	Award ***(A1)*** for a correctly labelled graph, ***(A1)*** for correct scales, ***(A1)*** for line $f(x) = 6 - x$ drawn correctly, ***(A1)*** for line $f(x) = x - 6$ drawn correctly, ***(A1)*** for $g(x) = \frac{1}{2}x$ drawn correctly.

[5 marks]

(b) (i) Points named on the graph (A and B can be inversed) ***(A1)***

(ii) $A(4, 2), B(12, 6)$ ***(A1)(A1)***

[3 marks]

(c) (i) (6, 0) ***(A1)***

(ii) Gradient of $AN = -1$, gradient of $NB = +1$. ***(A1)***

Therefore, $-1 \times 1 = -1$. Hence lines are perpendicular. ***(A1)***

Note:	Accept any other valid mathematical calculation.

[3 marks]

continued…

Question 3 continued

(d) Midpoint $= \left(\frac{12+4}{2}, \frac{6+2}{2} \right)$ ***(M1)***

$= (8, 4)$ ***(A1)***

Note: Allow ***(A2)*** for reading from the graph but **both** coordinates must be correct.

[2 marks]

(e) Gradient $= \frac{4-0}{8-6} = 2$ ***(A1)***

$y = mx + c$

$0 = 2 \times 6 + c$ ***(M1)***

$c = -12$ ***(A1)***

Equation is $y = 2x - 12$ (or correct alternatives). **Ft** from candidate's previous work. ***(A1)***

[4 marks]

Total [17 marks]

QUESTION 4

(a) $CA^2 = 800^2 + 500^2$ ***(M1)(A1)***

$CA = 943$ ***(AG)***

[2 marks]

(b) $\tan BCA = \frac{800}{500}$ ***(M1)(A1)***

$B\hat{C}A = 58.0^\circ$ (Allow 58°) ***(AG)***

[2 marks]

(c) (i) $C\hat{A}B = 180 - 90 - 58 = 32^\circ$ ***(M1)***

$C\hat{A}O = 110 - 32 \quad = 78^\circ$ ***(A1)***

(ii) $CO^2 = 1500^2 + 943^2 - 2 \times 1500 \times 943 \times \cos 78^\circ$ ***(M1)(A1)***

$CO = 1597$

$= 1600$ m (3 s.f.) ***(A1)***

[5 marks]

(d) Area of triangle $OAC = \frac{1}{2} \times 1500 \times 943 \times \sin 78^\circ$ (or more accurate values) ***(M1)***

$= 691794.89$ m^2 (692000) ***(A1)***

Area of triangle $ABC = \frac{1}{2} \times 800 \times 500 = 200000$ m^2 ***(A1)***

Total area $= 691795 + 200000$

$= 892000$ m^2 (**ft** from candidate's angle CAO) ***(A1)***

[4 marks]

(e) Time in seconds $= \frac{(1500 + 800 + 500 + 1597)}{4}$ ***(A1)(A1)***

Note: Award ***(A1)*** for numerator, ***(A1)*** for 4 in denominator.

$= 1099.25$

Time in minutes $= \frac{1099.25}{60}$

$= 18.3$ (**ft** from candidate's answer to (c)(ii)) ***(A1)***

[3 marks]

Total [16 marks]

QUESTION 5

(a) Time for *SpeakEasy* = $6x$.
Time for *ClearTalk* = $10y$.
900 hours. Hence $6x + 10y \le 900 \div 2$. ***(A1)***

$3x + 5y \le 450$ ***(AG)***

[1 mark]

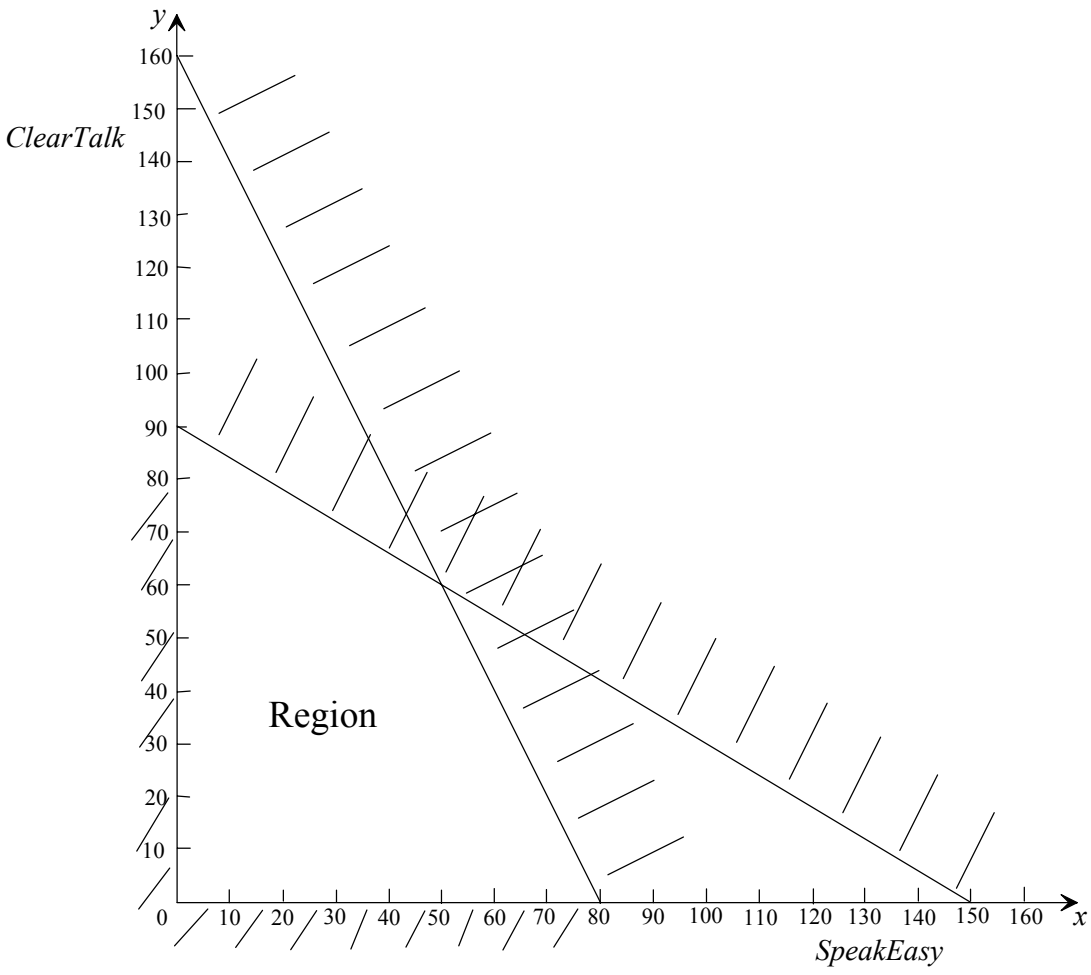
(b) $20x + 10y \le 1600$ or $2x + y \le 160$ ***(A1)***

[1 mark]

(c)

x	0	150
y	90	0

x	0	80
y	160	0



(A5)

Notes:	Award ***(A1)*** for two correct axes, ***(A1)*** for $3x + 5y = 450$ correctly plotted, ***(A1)*** for $2x + y = 160$ correctly plotted, ***(A1)(A1)*** for each inequality correctly shaded. The shading may be either side of the line but must be consistent.

[5 marks]

(d) Region clearly indicated. ***(A1)***

[1 mark]

continued…

Question 5 continued

(e) $P = 20x + 30y$ ***(A1)***

Note: Accept $20x + 30y$.

[1 mark]

(f) Feasible points (0, 90), (80, 0) and (50, 60) ***(A1)***
Maximum profit $= \$\,2800$ ***(A1)***

Note: Award ***(A2)*** for correct answer without the first line shown.

50 *SpeakEasy* and 60 *ClearTalk* mobile phones. ***(A1)***

[3 marks]

Total [12 marks]

QUESTION 6

(i) (a) 3 ***(A1)***

[1 mark]

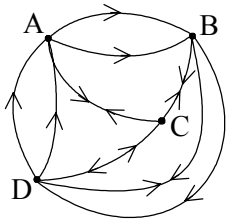
(b) 6 ***(A1)***

[1 mark]

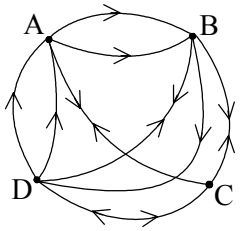
(c) X = 5, Y = 4, Z = 3 ***(A1)(A1)(A1)***

[3 marks]

(ii)



OR



(A4)

Note:	Award ***(A4)*** for fully correct, labelled, arrowed graph. **OR** if there are errors: award ***(A1)*** for drawing a labelled graph with some attempt to interpret the numbers in the matrix, ***(A1)*** for attempting to distinguish 1-way from 2-way using arrows at least on 1-way, ***(A1)*** for correct numbers of paths between vertices, consistent with the candidate's arrows shown.

[4 marks]

(iii) $O \to H \to K \to L \to P$ ***(M1)***

$5+8+3+16=32$ ***(A1)***

OR

32 ***(A2)***

[2 marks]

(iv) (a) He never takes the train two days in a row and so he will always drive the day after he takes the train. ***(R1)***

[1 mark]

(b) $a=\frac{1}{2}$ ***(A1)***

$b=\frac{1}{2}$ ***(A1)***

[2 marks]

(v) (a) $\begin{pmatrix}3\\1\\0\end{pmatrix}$ ***(A1)***

Note:	Accept the numbers in any order at this stage. Award ***(A0)*** for a row matrix.

[1 mark]

continued…

Question 6 (v) continued

(b) (i) $\begin{pmatrix} 4 & 0 & 2 \\ 1 & 4 & 1 \\ 2 & 2 & 2 \\ 0 & 6 & 0 \end{pmatrix}\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$ ***(A1)(A1)***

Note: Award ***(A1)*** for correct 4×3 matrix, ***(A1)*** for correct 3×1 matrix.

(ii) $\begin{pmatrix} 12 \\ 7 \\ 8 \\ 6 \end{pmatrix}$ ***(A2)***

Note: Award ***(A1)*** for one error, ***(A0)*** for 2 or more errors.

[4 marks]

(vi) (a) $\boldsymbol{A}^{\mathrm{T}} = \begin{pmatrix} 1 & 2 \\ x & 3 \end{pmatrix}$ ***(A1)***

[1 mark]

(b) $\boldsymbol{A}\boldsymbol{A}^{\mathrm{T}} = \begin{pmatrix} 1 & x \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ x & 3 \end{pmatrix}$

$= \begin{pmatrix} 1+x^2 & 2+3x \\ 2+3x & 13 \end{pmatrix}$ ***(A4)***

Note: Award ***(A1)*** for each correct entry.
Award ***(A3)*** for

$\begin{pmatrix} 1 & 2 \\ x & 3 \end{pmatrix}\begin{pmatrix} 1 & x \\ 2 & 3 \end{pmatrix}$

$= \begin{pmatrix} 5 & x+6 \\ x+6 & x^2+9 \end{pmatrix}$

[4 marks]

(vii) (a) +1 ***(A1)***

[1 mark]

(b) –1 ***(A1)***

[1 mark]

(c) Zero gain ***(A1)***

[1 mark]

(d) –1 ***(A1)***

[1 mark]

(e) Yes. The value of the game is zero. ***(A1)(R1)***

[2 marks]

Total [30 marks]

QUESTION 7

(i) (a) $\text{Males} = \dfrac{9000 \times 11000}{20000}$ ***(M1)(A1)***

$= 4950$ ***(AG)***

[2 marks]

(b) (i) That gender and grade obtained are independent. (There is no connection between gender and grade obtained.) ***(A1)***

(ii) $(3-1)(2-1) = 2$ ***(A1)***

(iii) $\chi^2 = 5.991$ ***(A1)***

(iv) Calculated $\chi^2 = 39.957$

Therefore, reject the Null hypothesis. Gender and grade obtained are dependent (or there is a connection between gender and grade). ***(R1)***

[4 marks]

(ii) (a) **a** = 5.1 m (510 cm) ***(A1)***

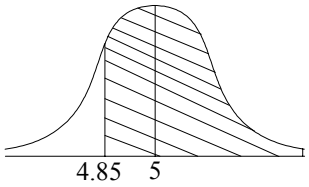
b = 5.2 m (520 cm) ***(A1)***

c = 4.7 m (470 cm) ***(A1)***

[3 marks]

(b) $z = \dfrac{4.85 - 5}{0.1}$ ***(M1)***

$= -1.5$ ***(A1)***



$\Phi(z > -1.5) = 0.9332$

Therefore, probability = 0.933 (93.3 %) (Accept 0.9332) ***(A1)***

OR

0.933 (93.3 %) (Accept 0.9332) ***(G3)***

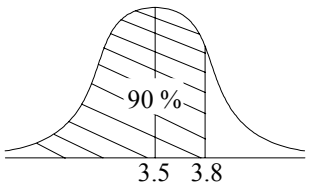
[3 marks]

(c) $\Phi = 0.9 \Rightarrow z = 1.282$ ***(A1)***

$1.282 = \dfrac{3.8 - 3.5}{\sigma}$ ***(M1)***

$1.282\sigma = 0.3$ ***(A1)***

$\sigma = 0.234$ ***(A1)***



OR

$\sigma = 0.234$ (If candidate has used GDC) ***(G4)***

[4 marks]

continued…

Question 7 continued

(iii) (a) (i) ① ***(A1)***

(ii) ③ ***(A1)***

[2 marks]

(b) (i) ① 0.04 ***(A1)***
② -0.20 ***(A1)***
③ -0.85 ***(A1)***

(ii) 1.60 A product-moment correlation coefficient cannot be greater than 1. ***(R1)***
0.90 There is no diagram with a strong positive correlation. ***(R1)***

[5 marks]

(c) (i) Product-moment correlation $= \dfrac{55.00}{6.08 \times 10.50}$ ***(A1)***

$= 0.8615$

$= 0.862$ ***(A1)***

(ii) $\overline{t} = \dfrac{124}{20} = 6.2 \quad \overline{w} = \dfrac{250}{20} = 12.5$ (both correct) ***(A1)***

$w - \overline{w} = \dfrac{S_{tw}}{S_t^2}\left(t - \overline{t}\right)$

$(w - 12.5) = \dfrac{55.00}{(6.08)^2}(t - 6.2)$ ***(M1)(A1)***

$w - 12.5 = 1.4878(t - 6.2)$

$w = 1.49t + 3.28$ ***(A1)(A1)***

Note: **ft** from candidate's mean values

[7 marks]

Total [30 marks]

QUESTION 8

(i) (a) $B \to D, G \to L$ (or $G \to K$ and $K \to L$) (both correct) (accept C, H, L) ***(A1)***

[1 mark]

(b) $A \to B, D \to G$ (both correct) (accept A, E, F) ***(A1)***

[1 mark]

(c) D ***(A1)***

[1 mark]

(d) B or G (accept either) ***(A1)***

[1 mark]

(e) Point of inflexion ***(A1)***

[1 mark]

(ii) (a) $g'(x) = 4x^3 + 9x^2 + 4x + 1$ ***(A3)***

Note: Award ***(A3)*** for all five terms correctly differentiated, ***(A2)*** for four terms, ***(A1)*** for three terms, ***(A0)*** for two or less terms correctly differentiated.

[3 marks]

(b) $g'(1) = 4(1)^3 + 9(1)^2 + 4(1) + 1$ ***(M1)***

$= 4 + 9 + 4 + 1$

$= 18$ ***(A1)***

OR

18 ***(G2)***

[2 marks]

(iii) (a) $y = ax^2 + bx + 6$

$\frac{dy}{dx} = 2ax + b$ ***(A1)(A1)***

[2 marks]

(b) Gradient = 2 when $x = 6$.

Therefore, $2 = 2a \times 6 + b$ ***(M1)***

$2 = 12a + b$ ***(A1)***

[2 marks]

(c) $y = -15$ when $x = 3$.

Therefore, $-15 = 9a + 3b + 6$

or $-21 = 9a + 3b$ or $-7 = 3a + b$ ***(M1)(A1)***

[2 marks]

continued…

Question 8 continued

(iv) (a) $x-15$ ***(A1)***

[1 mark]

(b) Profit $=(x-15)(100000-4000x)$ ***(M1)***

$=100000x-4000x^2-1500000+60000x$ ***(A2)***

Note: Award ***(A1)*** for one error, ***(A0)*** for 2 or more errors.

$=160000x-4000x^2-1500000$ ***(AG)***

[3 marks]

(c) (i) $\frac{\mathrm{d}P}{\mathrm{d}x}=160000-8000x$ ***(A1)(A1)***

(ii) $0=160000-8000x$ ***(M1)***

$x=\frac{160000}{8000}$

$x=20$ ***(A1)***

[4 marks]

(d) Books sold $=100000-4000\times 20$ ***(M1)***

$=20000$ ***(A1)***

OR

Books $=20000$ ***(A2)***

[2 marks]

(v) $a=2t-8$

$a=\frac{\mathrm{d}v}{\mathrm{d}t}$ ***(M1)***

OR

$v=\int a\mathrm{d}t$ ***(M1)***

$v=\frac{2t^2}{2}-8t+c$ ***(A1)***

$v=20$ when $t=0$

$20=c$ ***(A1)***

$v=t^2-8t+20$ ***(A1)***

[4 marks]

Total [30 marks]