MARKSCHEME

November 2004

MATHEMATICAL METHODS

Standard Level

Paper 2

21 pages

Instructions to Examiners

Note: **Where there are two marks (*e.g.* M2, A2) for an answer do not split the marks unless otherwise instructed.**

1 Method of marking

(a) All marking must be done using a **red** pen.

(b) Marks should be noted on candidates' scripts as in the markscheme:

- show the breakdown of individual marks using the abbreviations ***(M1)***, ***(A2)*** *etc*., unless a part is completely correct;
- write down each part mark total, indicated on the markscheme (for example, ***[3 marks]***) – it is suggested that this be written at the end of each part, and underlined;
- write down and circle the total for each question at the end of the question.

2 Abbreviations

The markscheme may make use of the following abbreviations:

(M) Marks awarded for **Method**

(A) Marks awarded for an **Answer** or for **Accuracy**

(N) Marks awarded for correct answers, if **no** working (or no relevant working) shown: they may not necessarily be all the marks for the question. Examiners should only award these marks for correct answers where there is no working.

(R) Marks awarded for clear **Reasoning**

(AG) **Answer Given** in the question and consequently marks are **not** awarded

Note: Unless otherwise stated, it is not possible to award ***(M0)(A1).***

Follow through (ft) marks should be awarded where a correct method has been attempted but error(s) are made in subsequent working which is essentially correct.

- Penalize the error when it first occurs
- Accept the incorrect result as the appropriate quantity in all subsequent working
- If the question becomes much simpler then use discretion to award fewer marks

Examiners should use **(d)** to indicate where discretion has been used. It should only be used for decisions on follow through and alternative methods. It must be accompanied by a brief note to explain the decision made

3 Using the Markscheme

(a) This markscheme presents a particular way in which each question may be worked and how it should be marked. **Alternative methods** have not always been included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method in a manner which is consistent with the markscheme. Indicate the awarding of these marks by (**d**).

Where alternative methods for complete questions or parts of questions are included, they are indicated by **METHOD 1**, **METHOD 2**, *etc*. Other alternative part solutions are indicated by **EITHER**….**OR.** It should be noted that ***G*** marks have been removed, and GDC solutions will not be indicated using the **OR** notation as on previous markschemes.

Candidates are expected to show working on this paper, and examiners should not award full marks for just the correct answer. Where it is appropriate to award marks for correct answers with no working (or no relevant working), it will be shown on the markscheme using the ***N*** notation. All examiners will be expected to award marks accordingly in these situations.

(b) Unless the question specifies otherwise, accept **equivalent forms**. For example: $\frac{\sin\theta}{\cos\theta}$ for $\tan\theta$. On the markscheme, these equivalent numerical or algebraic forms will generally be written in brackets after the required answer. Paper setters will indicate the required answer, by allocating full marks at that point. Further working should be ignored, even if it is incorrect. For example: if candidates are asked to factorize a quadratic expression, and they do so correctly, they are awarded full marks. If they then continue and find the roots of the corresponding equation, do not penalize, even if those roots are incorrect, *i.e.* once the correct answer is seen, ignore further working.

(c) As this is an international examination, all **alternative forms of notation** should be accepted. For example: 1.7 , 1·7, 1,7; different forms of vector notation such as $\vec{u}$, $\bar{u}$, $\underline{u}$; $\tan^{-1} x$ for arctan x.

4 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy.

There are two types of accuracy error. Candidates should be penalized **once only IN THE PAPER** for an accuracy error **(AP).**

Award the marks as usual then write –1(**AP**) against the answer and also on the **front** cover

Rounding errors: only applies to final answers not to intermediate steps.

Level of accuracy: when this is not specified in the question the general rule *unless otherwise stated in the question all numerical answers must be given exactly or to three significant figures* applies.

- If a final correct answer is incorrectly rounded, apply the **AP**
  OR
- If the level of accuracy is not specified in the question, apply the **AP** for answers not given to 3 significant figures. (Please note that this has changed from 2003).

Note: **If there is no working shown**, and answers are given to the correct two significant figures, apply the **AP**. However, do **not** accept answers to one significant figure without working.

5 Graphic Display Calculators

Many candidates will be obtaining solutions directly from their calculators, often without showing any working. They have been advised that they must use mathematical notation, not calculator commands when explaining what they are doing. Incorrect answers without working will receive no marks. However, if there is written evidence of using a graphic display calculator correctly, method marks may be awarded. Where possible, examples will be provided to guide examiners in awarding these method marks.

Examples

1. Accuracy

A question leads to the answer 4.6789….

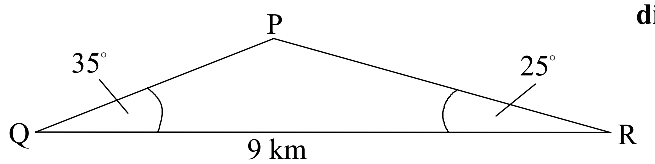
- 4.68 is the correct 3 s.f. answer.
- 4.7, 4.679 are to the wrong level of accuracy : both should be penalised the first time this type of error occurs.
- 4.67 is incorrectly rounded – penalise on the first occurrence.

Note: All these "incorrect" answers may be assumed to come from 4.6789..., even if that value is not seen, but previous correct working is shown. However, 4.60 is wrong, as is 4.5, 4.8, and these should be penalised as being incorrect answers, not as examples of accuracy errors.

2. Alternative solutions

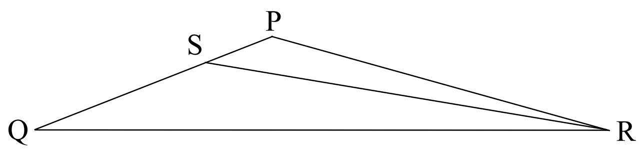
The points P, Q, R are three markers on level ground, joined by straight paths PQ, QR, PR as shown in the diagram. $QR = 9\text{ km}$, $P\hat{Q}R = 35^\circ$, $P\hat{R}Q = 25^\circ$.
(Note: in the original question, the first part was to find PR = 5.96)

diagram not to scale



(a) Tom sets out to walk from Q to P at a steady speed of 8 km h^{-1}. At the same time, Alan sets out to jog from R to P at a steady speed of $a\text{ km h}^{-1}$. They reach P at the same time. Calculate the value of *a*. ***[7 marks]***

(b) The point S is on [PQ], such that $RS = 2QS$, as shown in the diagram.



Find the length QS. ***[6 marks]***

MARKSCHEME

(a) **EITHER**

Sine rule to find PQ

$PQ = \frac{9 \sin 25}{\sin 120}$ *(M1)(A1)*

$PQ = 4.39$ km *(A1)*

OR

Cosine rule: $PQ^2 = 5.96^2 + 9^2 - (2)(5.96)(9) \cos 25$ *(M1)(A1)*

$= 19.29$

$PQ = 4.39$ km *(A1)*

THEN

Time for Tom $= \frac{4.39}{8}$ *(A1)*

Time for Alan $= \frac{5.96}{a}$ *(A1)*

Then $\frac{4.39}{8} = \frac{5.96}{a}$ *(M1)*

$a = 10.9$ *(A1)* *(N5)*

[7 marks]

Note that the **THEN** part follows both **EITHER** and **OR** solutions, and this is shown by the alignment.

(b) **METHOD 1**

$RS^2 = 4QS^2$ *(A1)*

$4QS^2 = QS^2 + 81 - 18 \times QS \times \cos 35$ *(M1)(A1)*

$3QS^2 + 14.74QS - 81 = 0$ (or $3x^2 + 14.74x - 81 = 0$) *(A1)*

$\Rightarrow QS = -8.20$ or $QS = 3.29$ *(A1)*

therefore $QS = 3.29$ *(A1)*

METHOD 2

$\frac{QS}{\sin S\hat{R}Q} = \frac{2QS}{\sin 35}$ *(M1)*

$\Rightarrow \sin S\hat{R}Q = \frac{1}{2} \sin 35$ *(A1)*

$S\hat{R}Q = 16.7^\circ$ *(A1)*

Therefore, $Q\hat{S}R = 180 - (35 + 16.7) = 128.3^\circ$ *(A1)*

$\frac{9}{\sin 128.3} = \frac{QS}{\sin 16.7} \left(= \frac{SR}{\sin 35} \right)$ *(M1)*

$QS = \frac{9 \sin 16.7}{\sin 128.3} \left(= \frac{9 \sin 35}{2 \sin 128.3} \right) = 3.29$ *(A1)* *(N2)*

If candidates have shown no working, award ***(N5)*** for the correct answer 10.9 in part (a), and ***(N2)*** for the correct answer 3.29 in part (b). ***[6 marks]***

3. Follow through

Question

Calculate the acute angle between the lines with equations

$$\boldsymbol{r} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} + s\begin{pmatrix} 4 \\ 3 \end{pmatrix} \quad \text{and} \quad \boldsymbol{r} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

Markscheme

Angle between lines = angle between direction vectors. (May be implied) ***(A1)***

Direction vectors are $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$. (May be implied) ***(A1)***

$\begin{pmatrix} 4 \\ 3 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \left|\begin{pmatrix} 4 \\ 3 \end{pmatrix}\right| \left|\begin{pmatrix} 1 \\ -1 \end{pmatrix}\right| \cos\theta$ ***(M1)***

$4 \times 1 + 3 \times (-1) = \sqrt{(4^2 + 3^2)}\sqrt{(1^2 + (-1)^2)} \cos\theta$ ***(A1)***

$\cos\theta = \dfrac{1}{5\sqrt{2}}$ (= 0.1414….) ***(A1)***

$\theta = 81.9^\circ$ (1.43 radians) ***(A1)*** ***(N3)***

Examples of solutions and marking

	Solutions	**Marks allocated**	
1.	$\begin{pmatrix} 4 \\ 3 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \left|\begin{pmatrix} 4 \\ 3 \end{pmatrix}\right| \left|\begin{pmatrix} 1 \\ -1 \end{pmatrix}\right| \cos\theta$	***(A1)(A1)*** implied ***(M1)***	
	$\cos\theta = \dfrac{7}{5\sqrt{2}}$	***(A0)(A1)***	
	$\theta = 8.13^\circ$	(***A1***)**ft**	Total 5 marks
2.	$\cos\theta = \dfrac{\begin{pmatrix} 4 \\ -1 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 4 \end{pmatrix}}{\sqrt{17}\sqrt{20}}$	***(A0)(A0)***) wrong vectors implied ***(M1)*** for correct method, ***(A1)*****ft**	
	$= 0.2169$	***(A1)*****ft**	
	$\theta = 77.5^\circ$	***(A1)*****ft**	Total 4 marks
3.	$\theta = 81.9^\circ$	***(N3)***	Total 3 marks

Note that this candidate has obtained the correct answer, but not shown any working. The way the markscheme is written means that the first 2 marks may be implied by subsequent correct working, but the other marks are only awarded if the relevant working is seen. Thus award the first 2 implied marks, plus the final mark for the correct answer.

END OF EXAMPLES

QUESTION 1

(a) $h = 3$ *(A1)*
$k = 2$ *(A1)*

[2 marks]

(b) $f(x) = -(x-3)^2 + 2$
$= -x^2 + 6x - 9 + 2$ (must be a correct expression) *(A1)*
$= -x^2 + 6x - 7$ *(AG)*

[1 mark]

(c) $f'(x) = -2x + 6$ *(A2)*

[2 marks]

(d) (i) tangent gradient $= -2$ *(A1)*
gradient of $L = \frac{1}{2}$ *(A1)* *(N2)*

(ii) **EITHER**

equation of L is $y = \frac{1}{2}x + c$ $\left(\text{accept } 1 = \frac{1}{2} \times 4 + c\right)$ *(M1)*
$c = -1$. *(A1)*
$y = \frac{1}{2}x - 1$

OR

$y - 1 = \frac{1}{2}(x - 4)$ *(A2)* *(N2)*

(iii) **EITHER**

$-x^2 + 6x - 7 = \frac{1}{2}x - 1$ *(M1)*
$2x^2 - 11x + 12 = 0$ (may be implied) *(A1)*
$(2x-3)(x-4) = 0$ (may be implied) *(A1)*
$x = 1.5$ (accept (1.5, 2.25)) *(A1)* *(N3)*

OR

$-x^2 + 6x - 7 = \frac{1}{2}x - 1$ (or a sketch) *(M1)*
$x = 1.5$ (accept (1.5, 2.25)) *(A3)* *(N3)*

[8 marks]

Total [13 marks]

QUESTION 2

(a) (i) $\vec{BC} = \vec{OC} - \vec{OB}$

$= -6\boldsymbol{i} - 2\boldsymbol{j}$ *(A1)(A1)* *(N2)*

(ii) $\vec{OD} = \vec{OA} + \vec{BC}$

$= -2\boldsymbol{i} + 0\boldsymbol{j}\ (= -2\boldsymbol{i})$ *(A1)(A1)* *(N2)*

[4 marks]

(b) $\vec{BD} = \vec{OD} - \vec{OB}$

$= -3\boldsymbol{i} + 3\boldsymbol{j}$ *(A1)*

$\vec{AC} = \vec{OC} - \vec{OA}$

$= -9\boldsymbol{i} - 7\boldsymbol{j}$ *(A1)*

Let θ be the angle between $\vec{BD}$ and $\vec{AC}$

$\cos\theta = \left(\dfrac{(-3\boldsymbol{i} + 3\boldsymbol{j}) \bullet (-9\boldsymbol{i} - 7\boldsymbol{j})}{|(-3\boldsymbol{i} + 3\boldsymbol{j})||-9\boldsymbol{i} - 7\boldsymbol{j}|}\right)$ *(M1)*

numerator = + 27 − 21 (= 6) *(A1)*

denominator $= \sqrt{18}\sqrt{130}\ \left(= \sqrt{2340}\right)$ *(A1)*

therefore, $\cos\theta = \dfrac{6}{\sqrt{2340}}$

$\theta = 82.9^\circ\ (= 1.45 \text{ rad})$ *(A1)* *(N3)*

(accept the supplementary angle 97.1°, 1.69 or 1.70 rad)

Note: Award full marks for a correct trigonometric approach, provided the working is shown.

[6 marks]

(c) $\boldsymbol{r} = \boldsymbol{i} - 3\boldsymbol{j} + \boldsymbol{t}(2\boldsymbol{i} + 7\boldsymbol{j})\quad \left(= (1 + 2t)\boldsymbol{i} + (-3 + 7\boldsymbol{t})\boldsymbol{j}\right)$ *(A1)* *(N1)*

[1 mark]

(d) **EITHER**

$4\boldsymbol{i} + 2\boldsymbol{j} + s(\boldsymbol{i} + 4\boldsymbol{j}) = \boldsymbol{i} - 3\boldsymbol{j} + \boldsymbol{t}(2\boldsymbol{i} + 7\boldsymbol{j})$ (may be implied) *(M1)*

$\left.\begin{array}{r} 4 + s = 1 + 2t \\ 2 + 4s = -3 + 7t \end{array}\right\}$ *(A1)*

$t = 7$ and/or $s = 11$ *(A1)*

Position vector of P is $15\boldsymbol{i} + 46\boldsymbol{j}$ *(A1)* *(N2)*

OR

$7x - 2y = 13$ or equivalent *(A1)*

$4x - y = 14$ or equivalent *(A1)*

$x = 15$, $y = 46$ *(A1)*

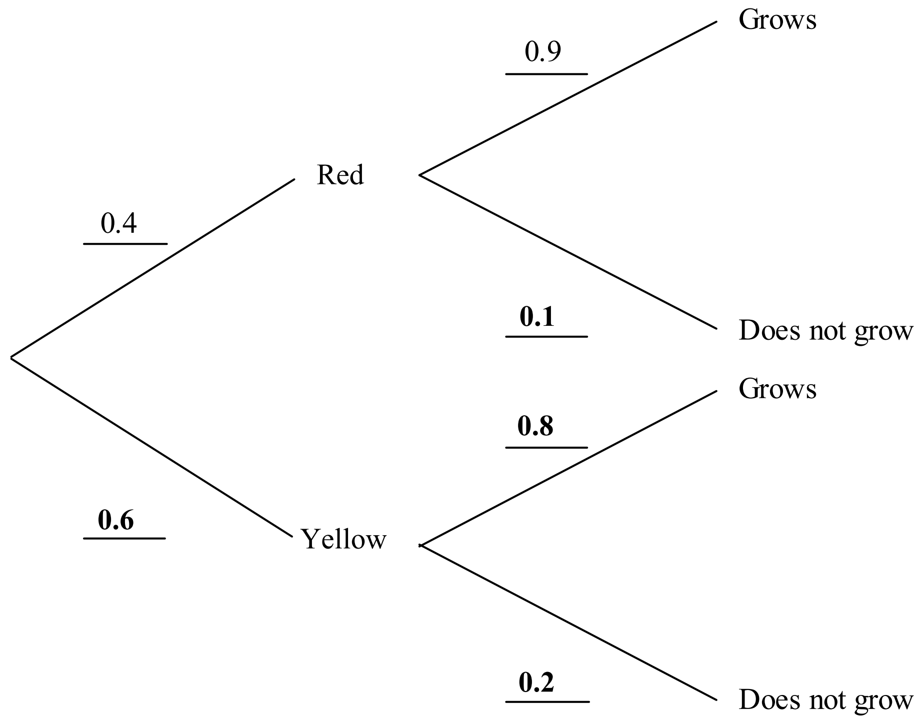
Position vector of P is $15\boldsymbol{i} + 46\boldsymbol{j}$ *(A1)* *(N2)*

[4 marks]

Total [15 marks]

QUESTION 3

(a)



(A3) *(N3)*

Note: Award *(A1)* for 0.6, *(A1)* for 0.1, *(A1)* for 0.8 and 0.2, all in correct places.

[3 marks]

(b) (i) 0.4×0.9 *(A1)*

$= 0.36$ *(A1)* *(N2)*

(ii) $0.36+0.6\times0.8 \quad (=0.36+0.48)$ *(A1)*

$= 0.84$ *(A1)* *(N1)*

(iii) $\dfrac{\text{P}(\text{red} \cap \text{grows})}{\text{P}(\text{grows})}$ (may be implied) *(M1)*

$= \dfrac{0.36}{0.84}$ *(A1)*

$= 0.429 \left(\dfrac{3}{7}\right)$ *(A1)* *(N2)*

[7 marks]

Total [10 marks]

QUESTION 4

(a) (i) $\$11\,400,\ \$11\,800$ *(A1)*

(ii) Total salary $= \frac{10}{2}(2 \times 11\,000 + 9 \times 400)$ *(A1)*

$= \$128\,000$ *(A1)* *(N2)*

[3 marks]

(b) (i) $\$10\,700,\ \$11\,449$ *(A1)(A1)*

(ii) 10^{th} year salary $= 10\,000(1.07)^9$ *(A1)*

$= \$18\,384.59$ or $\$18\,400$ or $\$18\,385$ (no ***ft***) *(A1)* *(N2)*

[4 marks]

(c) **EITHER**

Scheme A $S_A = \frac{n}{2}\left(2 \times 11\,000 + (n-1)400\right)$ *(A1)*

Scheme B $S_B = \frac{10\,000(1.07^n - 1)}{1.07 - 1}$ *(A1)*

Solving $S_B > S_A$ (accept $S_B = S_A$, giving $n = 6.33$) (may be implied) *(M1)*

Minimum value of n is 7 years. *(A1)* *(N2)*

OR

Using trial and error *(M1)*

	Arturo	Bill
6 years	\$ 72 000	\$ 71532.91
7 years	\$ 85 400	\$ 86 540.21

(A1)(A1)

Note: Award *(A1)* for **bot**h values for 6 years, and *(A1)* for **both** values for 7 years.

Therefore, minimum number of years is 7. *(A1)* *(N2)*

[4 marks]

Total [11 marks]

QUESTION 5

(i) (a) (i) $f'(x) = -6\sin 2x$ ***(A1)(A1)***

(ii) **EITHER**

$f'(x) = -12\sin x\cos x = 0$

$\Rightarrow \sin x = 0$ or $\cos x = 0$ ***(M1)***

OR

$\sin 2x = 0$,

for $0 \le 2x \le 2\pi$ ***(M1)***

THEN

$x = 0, \frac{\pi}{2}, \pi$ ***(A1)(A1)(A1)*** ***(N4)***

Note: Deduct one mark for extra solutions.

[6 marks]

(b) (i) translation ***(A1)***

in the y-direction of -1 (or equivalent). ***(A1)***

(ii) 1.11 (1.10 from TRACE is subject to **AP**) ***(A2)***

Note: Award ***(A1)(A0)*** for 1.11 and 2.03.

[4 marks]

continued…

Question 5 continued

(ii) (a) (i) $a = 1 - \pi$ $\left(\text{accept } (1-\pi, 0)\right)$ ***(A1)***

(ii) $b = 1 + \pi$ $\left(\text{accept } (1+\pi, 0)\right)$ ***(A1)***

Note: Award ***(A1) (A0)*** for -2.14 and 4.14.

[2 marks]

(b) (i) $\int_{-2.14}^{1} h(x)\,dx - \int_{1}^{2} h(x)\,dx$ ***(M1)(A1)(A1)***

OR

$\int_{-2.14}^{1} h(x)\,dx + \left|\int_{1}^{2} h(x)\,dx\right|$ ***(M1)(A1)(A1)***

OR

$\int_{-2.14}^{1} h(x)\,dx + \int_{2}^{1} h(x)\,dx$ ***(M1)(A1)(A1)***

Notes: Award ***(M1)*** for splitting into two parts, ***(A1)*** for all correct limits, ***(A1)*** for correct combination. Accept $(x-2)\sin(x-1)$ for $h(x)$. Award ***(A1)(A0)(A0)*** for the special case $\int_{-2.14}^{2}(x-2)\sin(x-1)\,dx$.

(ii) $5.141... - (-0.1585...)$
$= 5.30$ ***(A2)***

Note: Award ***(A1)*****ft** for 4.98 obtained from the special case.

[5 marks]

(c) (i) $y = 0.973$ ***(A1)***

(ii) $-0.240 < k < 0.973$ ***(A3)***

Note: Award ***(A2)*** for $-0.240 \le k \le 0.973$.

[4 marks]

Total [21 marks]

QUESTION 6

Note: Do not penalize accuracy in this question.

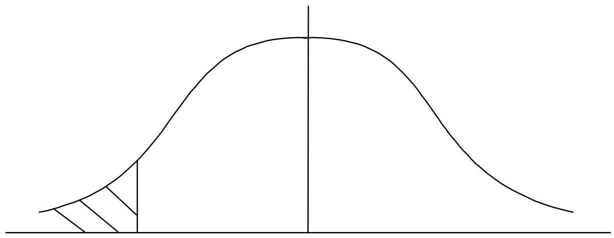
(i) (a) (i) $a = -1$ ***(A1)***
$b = 0.5$ ***(A1)***

(ii) (a) 0.841 ***(A2)***

(b) $0.6915 - 0.1587$ (or $0.8413 - 0.3085$) ***(M1)***
$= 0.533$ (3 s.f.) ***(A1)*** ***(N2)***

[6 marks]

(b) (i) Sketch of normal curve ***(A1)(A1)***



Note: Award ***(A1)*** for a line to the left of the mean, ***(A1)*** for area to the left of the line shaded.

(ii) $c = 0.647$ ***(A2)***

[4 marks]

(ii) (a) $\text{SE} = \frac{2.5}{\sqrt{25}}$ $(= 0.5)$ ***(A1)***

$z = \frac{297.2 - 298}{0.5} = -1.6$ ***(A1)***

Probability $= 0.0548$ ***(A1)*** ***(N3)***

[3 marks]

(b) (i) H_0 ***(A1)***
Retain the null hypothesis since $0.05 < 0.0548$ ***(R2)***

(ii) For a 99 % C.I., $z = 2.58$ ***(A1)***
Error is $\pm 2.58(0.5) = 1.29$ ***(A1)***
C.I. is 297.2 ± 1.29 or (295.91, 298.49) or (296, 298) ***(A1)(A1)*** ***(N4)***

[7 marks]

continued…

Question 6 continued

(iii) (a) The employment situation is independent of gender. ***(R2)***

[2 marks]

(b) $r = 14.93\ (= 14.9)$ ***(A1)*** ***(N1)***

$t = \frac{40}{75} \times \frac{26}{75} \times 75$ ***(M1)***

$= 13.866....\ (= 13.9)$ ***(A1)*** ***(N2)***

[3 marks]

(c) $\chi^2 = 6.27$ (Accept 6.28, or 6.36 if 3 sf values are used) ***(A2)***

[2 marks]

(d) Employment situation is **not** independent of gender. ***(A2)***

EITHER

The critical value at 5 % level is 5.991 and $6.27 > 5.991$ ***(R1)***

Note: The value of 5.991 must be seen in the reason.

OR

$\text{P}(\chi^2 > 6.27) = 0.0435$ and $0.0435 < 0.05$. ***(R1)***

Note: The value of 0.0435 must be seen in the reason.

[3 marks]

Total [30 marks]

QUESTION 7

(i) (a) $y = 0$ ***(A1)***

> **Note:** Award ***(A0)*** for 0, ***(A0)*** for $y \neq 0$.

[1 mark]

(b) $f'(x) = \frac{-2x}{(1+x^2)^2}$ ***(A1)(A1)(A1)***

> **Note:** Award ***(A1)*** for negative, ***(A1)*** for $2x$, ***(A1)*** for denominator.

[3 marks]

(c) $\frac{6x^2 - 2}{(1+x^2)^3} = 0$ (or sketch of $f'(x)$ showing the maximum) ***(M1)***

$6x^2 - 2 = 0$ ***(A1)***

$x = \pm\sqrt{\frac{1}{3}}$ ***(A1)***

$x = \frac{-1}{\sqrt{3}} (= -0.577)$ ***(A1)*** ***(N4)***

> **Note:** Award ***(N2)*** for writing $f''(x) = 0$ and going straight to $x = 0.577$ (the positive value).

[4 marks]

(d) (i) $\int_{-0.5}^{0.5} \frac{1}{1+x^2}\,dx \left(= 2\int_{0}^{0.5} \frac{1}{1+x^2}\,dx = 2\int_{-0.5}^{0} \frac{1}{1+x^2}\,dx \right)$ ***(A1)(A1)***

> **Notes:** Award ***(A1)*** for both limits correct, ***(A1)*** for correct function.
> Award ***(A1)(A0)*** for $\int_{0}^{0.5} \frac{1}{1+x^2}\,dx$.

(ii) $h = \frac{1}{4}$ ***(A1)***

$A = \frac{1}{2} \times \frac{1}{4}\left[f(-0.5) + 2f(-0.25) + 2f(0) + 2f(0.25) + f(0.5)\right]$ ***(A1)***

$= \frac{1}{8}\left[0.8 + 2(0.941) + 2 + 2(0.941) + 0.8\right]$

$= 0.921$ ***(A1)*** ***(N2)***

(iii) A simple diagram can be used to explain this in which a straight line is drawn connecting two points on a curve which is concave downwards. Alternatively, simply stating that the curve is concave downwards in the interval would be sufficient. Award ***(R0)*** if diagram uses rectangles instead of trapezia. ***(R1)***

[6 marks]

continued…

Question 7 continued

(ii) (a) $x = 4$ ***(A1)***

g'' changes sign at $x = 4$ or concavity changes ***(R1)***

Note: $g''(4) = 0$ is not sufficient for ***(R1)***.

[2 marks]

(b) $x = 2$ ***(A1)***

EITHER

g' goes from negative to positive ***(R1)***

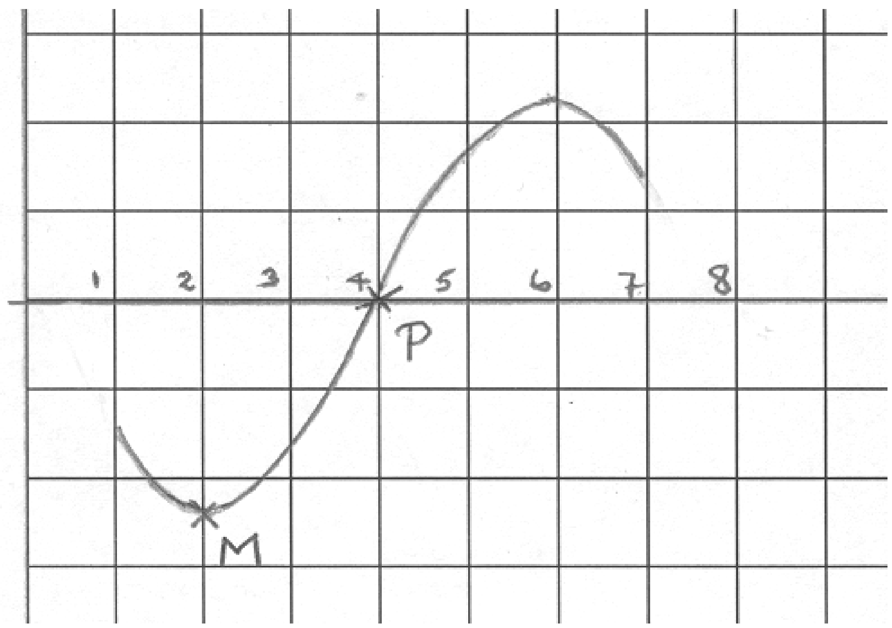
OR

$g'(2) = 0$ **and** $g''(2)$ is positive ***(R1)***

Note: Award ***(R1)*** for **both** conditions.

[2 marks]

(c)



(A2)(A1)(A1)

Notes: Award ***(A2)*** for a suitable cubic curve through (4, 0), ***(A1)*** for M at $x = 2$, ***(A1)*** for P at (4, 0).

[4 marks]

continued…

Question 7 continued

(iii) (a) **EITHER**

$u = \tan x$; $\mathrm{d}u = \dfrac{\mathrm{d}x}{\cos^2 x}$ ***(M1)***

$\int (u+1)\,\mathrm{d}u = \dfrac{u^2}{2} + u + c \left(\text{or } \dfrac{(u+1)^2}{2} + c \right)$ ***(A1)***

$= \dfrac{\tan^2 x}{2} + \tan x + c \left(\text{or } \dfrac{(\tan x + 1)^2}{2} + c \right)$ ***(A1)(A1)*** ***(N4)***

Note: The final ***(A1)*** is for the 'c'.

OR

$u = \tan x + 1$; $\mathrm{d}u = \dfrac{\mathrm{d}x}{\cos^2 x}$ ***(M1)***

$\int u\,\mathrm{d}u = \dfrac{u^2}{2} + c$ ***(A1)***

$= \dfrac{(\tan x + 1)^2}{2} + c$ ***(A1)(A1)*** ***(N4)***

Note: The final ***(A1)*** is for the 'c'.

[4 marks]

(b) $\int_0^{\frac{\pi}{4}} h'(x)\,\mathrm{d}x = [h(x)]_0^{\frac{\pi}{4}}$ (may be implied) ***(M2)***

$= h\left(\dfrac{\pi}{4}\right) - h(0)$ (any equivalent evaluation) ***(A1)***

$= 4 - 1$ (must be seen) ***(A1)***

$= 3$ ***(AG)*** ***(N0)***

Note: If candidates find $h'(x)$ correctly and then use gdc integration, award ***(A2)*** for

$$h'(x) = \frac{\frac{1}{\cos^2 x}\cos^2 x - (\tan x + 1)(-2\cos x \sin x)}{\cos^4 x} = \left(\frac{1 + 2\cos x \sin x(\tan x + 1)}{\cos^4 x} \right),$$

and then ***(A0)(A0)***.

[4 marks]

Total [30 marks]

QUESTION 8

(i) (a) (i) ***S*** is a shear *(A1)*
with y-axis $(x = 0)$ the invariant line *(A1)*
and scale factor –2. *(A1)*

> **Note:** Accept alternative descriptions of magnitude of shear.

(ii) 1 (or area is invariant) *(A1)*

[4 marks]

(b) (i) $y = 0$ (accept x-axis) *(A1)*

(ii) $\frac{1}{4}$ *(A1)*

(iii) 48 *(A1)*

(iv) 48 *(A1)*

[4 marks]

(c) (i) $\begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} 4 \\ 4 \end{pmatrix} = \begin{pmatrix} 16 \\ 16 \end{pmatrix}$ *(M1)*

$\Rightarrow$ L is $(16, 16)$ *(A1)* *(N2)*

(ii) $\begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} -2 \\ 4 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$ *(M1)*

$\Rightarrow$ M is $(-2, 4)$ *(A1)* *(N2)*

> **Note:** Accept answers in column form.

[4 marks]

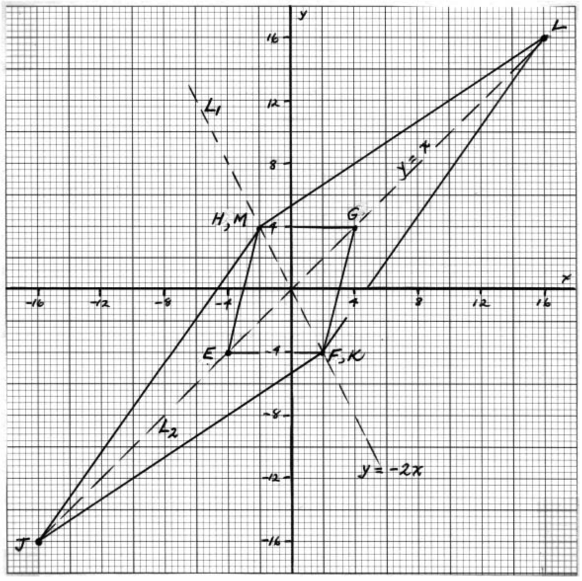
(d) Area scale factor $= \det \boldsymbol{T} = [3(2) - 1(2)] = 4$ *(M1)*
Area of JKLM $= 4(48) = 192$ *(A1)* *(N2)*

[2 marks]

continued…

Question 8 (ii) continued

(e), (f), (g)



(A3)

> **Note:** Award ***(A1)*** for correctly scaled diagram, ***(A1)*** for EFGH plotted correctly and ***(A1)*** for JKLM plotted correctly.

[3 marks]

(f) (i) H ***(A1)***

(ii) See graph for L_1. ***(A1)***

(iii) $y = -2x$ ***(A1)***

[3 marks]

(g) See graph for L_2 ($y = x$ not required) ***(A1)***

[1 mark]

(h) $\boldsymbol{V} = \boldsymbol{T}^{-1}$ ***(M1)***

$= \frac{1}{4}\begin{pmatrix} 2 & -1 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} 0.5 & -0.25 \\ -0.5 & 0.75 \end{pmatrix}$ or equivalent. ***(A1)*** ***(N2)***

[2 marks]

continued…

Question 8 continued

(ii) (a) Reflection matrix has form $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$ ***(M1)***

$\cos 2\theta = \cos^2\theta - \sin^2\theta = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$ ***(A1)***

$\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right) = \left(\frac{24}{25}\right)$ ***(A1)***

[3 marks]

(b) $\frac{1}{25}\begin{pmatrix} 7 & 24 \\ 24 & -7 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -6 \\ 8 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$ ***(M1)***

$\Rightarrow 7x + 24y + -6(25) = 25x$

$24x - 7y + 8(25) = 25y$ ***(A1)***

$\Rightarrow 3x - 4y + 25 = 0$ (or equivalent) ***(A1)*** ***(N3)***

[3 marks]

(c) Any line perpendicular to the line of reflection will be an invariant line. ***(A1)***

[1 mark]

Total [30 marks]