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MARKSCHEME

May 2004

MATHEMATICAL STUDIES

Standard Level

Paper 2

15 pages

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Paper 2 Markscheme

Instructions to Examiners

1   Method of marking

(a)   All marking must be done using a **red** pen.

(b)   Marks should be noted on candidates' scripts as in the markscheme:

- show the breakdown of individual marks using the abbreviations *(M1)*, *(A2) etc*.

- write down each part mark total, indicated on the markscheme (for example, *[3 marks]*) – it is suggested that this be written at the end of each part, and underlined;

- write down and circle the total for each question at the end of the question.

2   Abbreviations

The markscheme may make use of the following abbreviations:

M   Marks awarded for **Method**

A   Marks awarded for an **Answer** or for **Accuracy**

G   Marks awarded for correct solutions, generally obtained from a **Graphic Display Calculator**, irrespective of working shown

R   Marks awarded for clear **Reasoning**

AG   **Answer Given** in the question and consequently marks are **not** awarded

3   Follow Through (ft) Marks

Errors made at any step of a solution can affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks should be awarded. The procedures for awarding these marks require that all examiners:

(i)   penalise an error when it **first occurs**;

(ii)   **accept the incorrect answer** as the appropriate value or quantity to be used in all subsequent working;

(iii)   award *M* marks for a correct method, and *A***(ft)** marks if the subsequent working contains no further errors.

Follow through procedures may be applied repeatedly throughout the same problem.

The following illustrates a use of the **follow through** procedure:

Markscheme		Candidate's Script		Marking	
$ 600 × 1.02	*M1*	Amount earned = $ 600 × 1.02		√	*M1*
= $ 612	*A1*	= $602		×	*A0*
$ (306 × 1.02) + (306 × 1.04)	*M1*	Amount = 301 × 1.02 + 301 × 1.04		√	*M1*
= $ 630.36	*A1*	= $ 620.06		√	*A1*(ft)

Note that the candidate made an arithmetical error at line 2; the candidate used a correct method at lines 3, 4; the candidate's working at lines 3, 4 is correct.

However, if a question is transformed by an error into a **different, much simpler question** then:

(i)   **fewer** marks should be awarded at the discretion of the Examiner;

(ii)   marks awarded should be followed by "**(d)**" (to indicate that these marks have been awarded at the **discretion** of the Examiner);

(iii)   a brief **note** should be written on the script explaining **how** these marks have been awarded.

4   Using the Markscheme

(a)   This markscheme presents a particular way in which each question may be worked and how it should be marked. **Alternative methods** have not always been included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method in a manner which is consistent with the markscheme.

In this case:

(i)   a mark should be awarded followed by "**(d)**" (to indicate that these marks have been awarded at the **discretion** of the Examiner);

(ii)   a brief **note** should be written on the script explaining **how** these marks have been awarded.

Where alternative methods for complete questions are included, they are indicated by **METHOD 1**, **METHOD 2**, *etc*.   Other alternative solutions, including graphic display calculator alternative solutions are indicated by **OR**. For example:

Mean = 7906/134   *(M1)*
     = 59   *(A1)*

OR

Mean = 59   *(G2)*

(b)   Unless the question specifies otherwise, accept **equivalent forms**. For example: $\dfrac{\sin\theta}{\cos\theta}$ for $\tan\theta$.

On the markscheme, these equivalent numerical or algebraic forms will generally be written in brackets after the required answer. Paper setters will indicate the required answer, by allocating full marks at that point. Further working should be ignored, even if it is incorrect. For example: if candidates are asked to factorize a quadratic expression, and they do so correctly, they are awarded full marks. If they then continue and find the roots of the corresponding equation, do not penalize, even if those roots are incorrect, *i.e*. once the correct answer is seen, ignore further working.

(c)   As this is an international examination, all **alternative forms of notation** should be accepted. For example: 1.7 , 1·7 , 1,7 ; different forms of vector notation such as $\vec{u}$, $\overline{u}$, $\underline{u}$; $\tan^{-1} x$ for arctan x.

5   Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy.

There are two types of accuracy error. Candidates should be penalized **once only IN THE PAPER** for an accuracy error **(AP)**.

Award the marks as usual then write –1**(AP)** against the answer and also on the **front** cover

Rounding errors: only applies to final answers not to intermediate steps.

Level of accuracy: when this is not specified in the question the general rule *unless otherwise stated in the question all numerical answers must be given exactly or to three significant figures* applies.

- If a final correct answer is incorrectly rounded, apply the **AP**
  OR
- If the level of accuracy is not specified in the question, apply the **AP** for answers not given to 3 significant figures. (Please note that this has changed from May 2003).

6   Graphic Display Calculators

Many candidates will be obtaining solutions directly from their calculators, often without showing any working. They have been advised that they must use mathematical notation, not calculator commands when explaining what they are doing. Incorrect answers without working will receive no marks. However, if there is written evidence of using a graphic display calculator correctly, method marks may be awarded. Where possible, examples will be provided to guide examiners in awarding these method marks.

Calculator penalties

Candidates are instructed to write the make and model of their calculator on the front cover. Please apply the following penalties where appropriate.

(i)   Illegal calculators

If candidates note that they are using an illegal calculator, please report this on a PRF, and deduct 10 % of their overall mark.. Note this on the front cover.

(ii)   Calculator box not filled in.

Please apply a calculator penalty (*CP*) of 1 mark if this information is not provided. Note this on the front cover.

QUESTION 1.

(a)

P(–2, 6) Q(6, 2) R(–8, a)
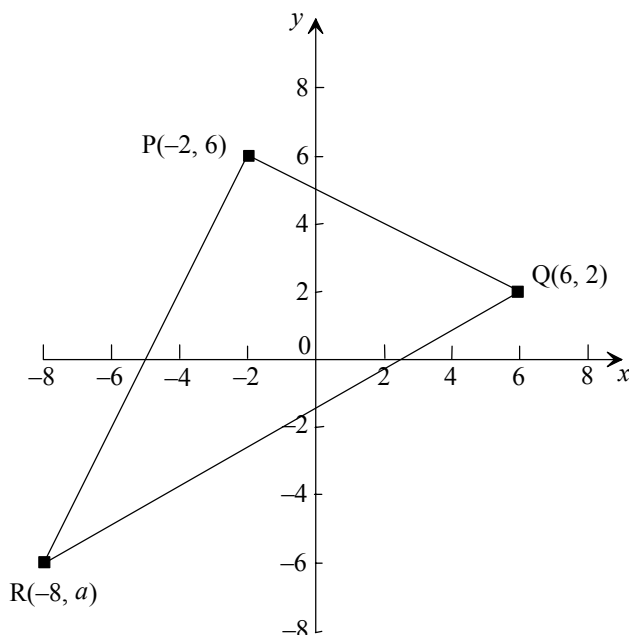
(A3)

Note: Award *(A1)* for correct and labelled scale, *(A1)(A1)* for points P and Q correctly plotted.

[3 marks]

(b) (i) $\overline{PQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(6+2)^2 + (2-6)^2}$ *(M1)*

$= \sqrt{80}$ (8.94 3 s.f.) *(A1)*

[2 marks]

(ii) $mPQ = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2-6}{6+2} = -\dfrac{4}{8} = -\dfrac{1}{2}$ *(M1)(A1)(A1)*

Note: Award *(A1)* for values, *(A1)* for sign.

[3 marks]

(iii) $mPR = \dfrac{2}{1}$ *(A1)*

[1 mark]

(iv) $\dfrac{a-6}{-8+2} = \dfrac{2}{1} \Rightarrow a-6 = -12$, therefore $a = -6$ *(M1)(A1)* or *(G2)*

[2 marks]

(c) Area of triangle PQR $= \left(\dfrac{\overline{PQ} \times \overline{PR}}{2} \right)$

$= \left(\dfrac{\sqrt{180} \times \sqrt{80}}{2} \right)$ (**ft** from (b)(i)) *(M1)*

$= 60$ *(A1)*

[2 marks]

Total [13 marks]

QUESTION 2

(i)   (a) $u_1 = 59$ $u_2 = 55$ *(A1)(A1)*

[2 marks]

        (b) $63 - 4n = -13$ $-4n = -76$ $n = 19$ *(M1)(A1)*
                                                                        or (G2)

[2 marks]

        (c) $63 - 4k + 63 - 4(k+1) = 34$
                                $-8k = -88$ $k = 11$ *(M1)(M1)(A1)*

> **Note:**   Award *(M1)* for the terms 15 and 19.

[3 marks]

(ii)   (a) $u_n = 2(0.9)^7 = 0.957 \text{ m}$ *(M1)(A1)*

> **Note:**   Award *(M1)* for substitution into formula, list or suitable diagram.

[2 marks]

        (b) $S_n = \dfrac{2\left(1 - (0.9)^5\right)}{1 - (0.9)} = 8.19 \text{ m}$ *(M1)(M1)(A1)*

> **Note:**   Award *(M1)* for substitution into formula, list or suitable diagram.

         Total distance travelled $= 2 \times 8.19 = 16.4 \text{ m}$.   *(A1)*

[4 marks]

Total [13 marks]

QUESTION 3

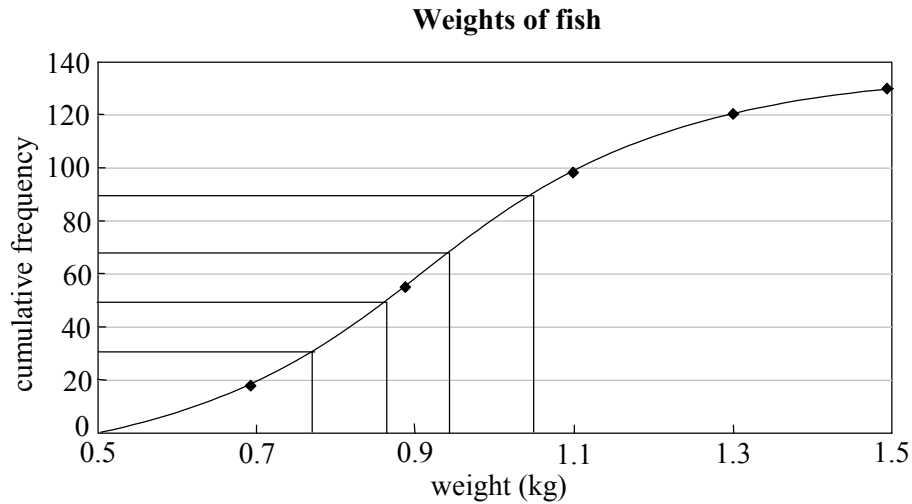(a)   (i) $c = 97$ *(A1)*

[1 mark]

(ii)   (not drawn to scale)

Weights of fish

Note: Award *(A1)* for correct scales and axes labelled, *(A2)* for 5 correct points,
(A1) for 3 to 4 correct, *(A1)* for the curve.

[4 marks]

(iii)   median is 0.95 kg   line drawn correctly on diagram   *(M1)(AG)*

[1 mark]

(b)   (i)   90th percentile … $130 \times 0.9 = 117$ *(M1)*
The zoo buys 13 fish (± 2).   *(A1)*
or (G2)

[2 marks]

(ii)   First quartile = 32/33 fish.   *(M1)*
Maximum weight $= 0.79$ kg (± 0.03).   *(M1)(A1)*

[3 marks]

(c)   (i)   maximum : $0.95 \times 1.10 = 1.045$ kg (1.05 to 3 s.f.)   *(A1)*
minimum : $0.95 \times 0.90 = 0.855$ kg   *(A1)*

[2 marks]

(ii)   number of fish bought by restaurant $= 88 - 46 = 42 (\pm 4)$ *(M1)(M1)(A1)*

[3 marks]

Total [16 marks]

QUESTION 4

(a) (i) $P(\text{chocolate}, \text{chocolate}) = \dfrac{3}{10} \times \dfrac{3}{10} = \dfrac{9}{100} = (0.09)$ **(M1)(A1)**

[2 marks]

(ii) $P(\text{one is plain}) = P(\text{chocolate}, \text{plain}) + P(\text{plain}, \text{chocolate})$ **(M1)**

$= \left(\dfrac{3}{10} \times \dfrac{7}{10}\right) + \left(\dfrac{7}{10} \times \dfrac{3}{10}\right) = \dfrac{42}{100} = (0.42)$ **(M1)(A1)**

[3 marks]

(b) (i) $a = 8, b = 9$ **(A1)(A1)**

[2 marks]

(ii) $P(\text{chocolate}, \text{chocolate}) = 0$ **(A1)**

[1 mark]

(iii) $P(\text{at least one chocolate}) = P(\text{one is chocolate})$ **(M1)**

$= \left(\dfrac{9 \times 1}{10 \times 9}\right) + \left(\dfrac{1 \times 9}{10 \times 9}\right) = \dfrac{18}{90} = 0.2$ **(M1)(A1)**

[3 marks]

(c) $P(\text{tin}, \text{chocolate})$

$\left(\dfrac{1}{2} \times \dfrac{3}{10}\right) + \left(\dfrac{1}{2} \times \dfrac{1}{10}\right)$ **(M1)(M1)(M1)**

$= \dfrac{4}{20} = (0.2)$ **(A1)**

[4 marks]

Total [15 marks]

QUESTION 5

(a) The inequality is $25x + 50y \le 3600$ which is determined from the given information for 'nuts'. It reduces to the given inequality. *(R1)(R1)*

[2 marks]

(b) $(0, 0), (0, 72), (80, 0), (48, 48)$ *(A4)*

[4 marks]

(c)

x , y	Profit
0, 0	$ 0.00
0, 72	$ 79.20
80, 0	$ 60.00
48, 48	$ 88.80

(M1)

maximum profit when 48 of each cake is made is $ 88.80. *(A1) or (G2)*

[2 marks]

(d) (i) $60 \times 50 \text{ g} = 3000 \text{ g (3 kg)}$ each of nuts and fruit. *(A1)*

[1 mark]

(ii) $\dfrac{3\,600 - 3\,000}{25} = 24$ Triple Cakes *(M1)(A1)*

[2 marks]

(iii) new profit $= (60 \times \$1.10) + (24 \times 0.75)$ *(M1)*
$= \$84.00$ *(A1)*

[2 marks]

Total [13 marks]

QUESTION 6

(i) (a)

$$\text{matches} \begin{pmatrix} 3 & 5 & 2 \\ 2 & x & 1 \\ 2 & 5 & 1 \\ 3 & 2x & 4 \end{pmatrix} \quad \overset{\text{shots}}{}$$

(M1)(A1)

[2 marks]

(b)
$$\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

(A1)

[1 mark]

(c)
$$\begin{pmatrix} 3 & 5 & 2 \\ 2 & x & 1 \\ 2 & 5 & 1 \\ 3 & 2x & 4 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 9+10+2 \\ 6+2x+1 \\ 6+10+1 \\ 9+4x+4 \end{pmatrix} = \begin{pmatrix} 21 \\ 7+2x \\ 17 \\ 13+4x \end{pmatrix}$$

(A2)

[2 marks]

(d) $58 + 6x = 82$, therefore $x = \dfrac{82-58}{6} = 4$.

(M1)(A1)
or (G2)

[2 marks]

(ii) (a) $\det A = 2y - 3$

(A1)

[1 mark]

(b) $2y - 3 = 0$, inverse does not exist when $y = 1.5$.

(M1)(A1)

[2 marks]

(c) $2\begin{pmatrix} 2 & 3 \\ 1 & y \end{pmatrix} + 3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \dfrac{1}{2}\begin{pmatrix} 14 & 12 \\ 4 & 11 \end{pmatrix}$

(M1)

$\begin{pmatrix} 7 & 6 \\ 2 & 2y+3 \end{pmatrix} = \begin{pmatrix} 7 & 6 \\ 2 & 5.5 \end{pmatrix}$

(M1)

$2y + 3 = 5.5 \quad y = 1.25$

(A1)

[3 marks]

continued…

Question 6 continued

(iii) (a) (i) $M \rightarrow N \rightarrow O \rightarrow Q \rightarrow S \rightarrow U \rightarrow V$ *(A2)*

       (ii)   total cost $= \$152$ million   *(A1)*

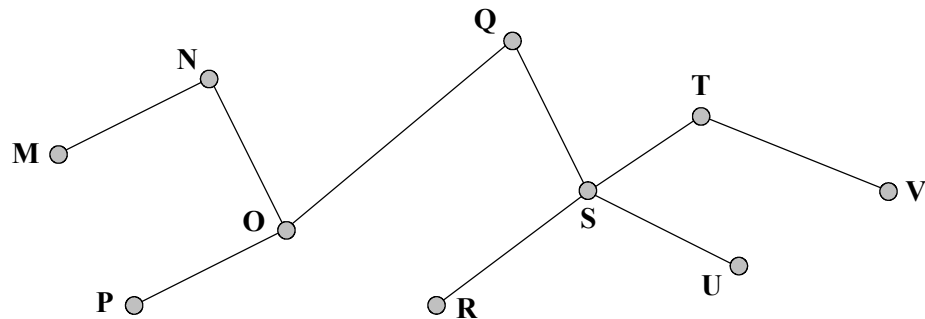   *[3 marks]*

       (b) (i) $M \rightarrow N \rightarrow O \rightarrow R \rightarrow S \rightarrow U \rightarrow V$ *(A1)*

       (ii)   additional cost $= \$1$ million   *(M1)(A1)*
                                                                                  or (G2)

                                                                                  [3 marks]

       (c)   spanning tree (accept any suitable alternative)

                                                                                  (M2)

                                                                                  [2 marks]

       (d)   Mark according to candidates tree.
              In the above example four vertices are even degree (**N,Q,S,T**).   *(A1)*

                                                                                  [1 mark]

(iv) (a)   Jack should choose row 3…   *(A1)*

                                                                                  [1 mark]

       (b)   Jill will choose column 1 and win 5.   *(A2)*

                                                                                  [2 marks]

       (c) (i)   row minimums $\begin{pmatrix} -5 \\ 0 \\ -5 \end{pmatrix}$, Jack should play row 2.   *(A2)*

       (ii)   row 2 / column 3 … Result is zero.   *(A2)*

                                                                                  [4 marks]

       (d)   A fair game is one which has no loser. The zero result means that neither Jack
              nor Jill loses.   *(R1)*
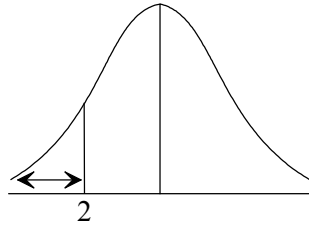
                                                                                  [1 mark]

                                                                                  Total [30 marks]

QUESTION 7

(i)   (a)   (i)   proportion of cats within 1 standard deviation = 68.3 %   **(A1)**

           (ii)   number of cats $= 0.683 \times 150 = 102$ **(M1)(A1)**
                                                  or (G2)
                                                  [3 marks]

       (b)   (i)

                                                    (A1)

           (ii) $z = \dfrac{2 - 3.42}{0.82} = -1.73$ **(M1)(A1)**

           (iii)   percentage of cats $< 2 \text{ kg} = 0.0417 \times 100 = 4.17 \%$ **(M1)(A1)**
                                                  or (G2)
                                                  [5 marks]

       (c) $z_2 = \dfrac{2 - 3.42}{0.82} = -1.73 \rightarrow 0.958\,(0.042)$ **(M1)**

           $z_{4.8} = \dfrac{4.8 - 3.42}{0.82} = 1.68 \rightarrow 0.954$ **(M1)**

           $P(2 < x < 4.8) = 0.954 - 0.042 = 91.2 \%$ **(A1)**
                                                  [3 marks]

       (d) $2.5 \% \Rightarrow P(z) = 0.975 \rightarrow z = 1.96$ **(M1)**

           $w = zs + \bar{X} = (1.96)(0.82) + 3.42 = 5.03 \text{ kg}$ **(M1)(A1)**
                                                    [3 marks]

(ii)   (a)   (i) $a = 29.45, \qquad b = -5.55, \qquad c = 30.8025, \qquad d = 1.046$ **(A4)**

           (ii) $\chi^2_{\text{calc}} = 1.012 + 0.946 + 1.118 + 1.046 = 4.12$ **(A1)**

           (iii) degrees of freedom $= (2-1)(2-1) = 1$ **(A1)**

           (iv) $\chi^2_{\text{crit}} = 3.84$ **(A1)**
                                                    [7 marks]

       (b)   Do not accept H_0. The weight of a puppy is related to the weight of the
             parent.
             $\chi^2_{\text{calc}} > \chi^2_{\text{crit}}$ **(A1)(R1)**
                                                    [2 marks]

Question 7 continued

(iii) (a) $r = \dfrac{0.442}{0.850 \times 0.689} = 0.755$ *(M1)(M1)(AG)*

[2 marks]

(b) (i) $y - \bar{y} = \dfrac{Sxy}{(Sx)^2}(x - \bar{x});\ y - 3.46 = \dfrac{0.442}{(0.85)^2}(x - 3.78)$ *(M1)*

$y = 0.612x + 1.15$ *(A1)(A1)*

(ii)   weight of rabbit $= 0.612 \times 3.71 + 1.15 = 3.42 \text{ kg}$ *(M1)(A1)*

[5 marks]

Total [30 marks]

QUESTION 8

(i) (a) $h(2) = 24(2) - 2.4(2)^2 = 48 - 9.6 = 38.4$ cm *(M1)(A1)*

[2 marks]

(b) (i) $\dfrac{dh}{dw} = 24 - 4.8w$ *(A1)(A1)*

(ii) $24 - 4.8k = 7.2, \ k = \dfrac{24 - 7.2}{4.8} = 3.5$ weeks *(M1)(M1)(A1)*

(iii) maximum height when $24 - 4.8w = 0$

$$w = \frac{24}{4.8} = 5 \text{ weeks}$$ *(M1)(A1)*

$$\text{height } = 24(5) - 2.4(5)^2 = 60 \text{ cm}$$ *(M1)(A1)*

[9 marks]

(c) 70 days = 10 weeks *(A1)*
$h(10) = 24(10) - 2.4(10)^2 = 0$ *(M1)(A1)*
(height of zero indicates that the daffodil is lying on the ground)

[3 marks]

(ii) (a) (i)

Time (sec)	0	0.5	1.0	1.5	2.0	2.5
Distance (m)	**2**	1.125	1.0	0.875	**0**	–2.375

(A2)

(ii) negative value indicates that the train is left of the station. *(R1)*

[3 marks]

(b) (i) $v(t) = -3t^2 + 6t - 3$ *(A1)*

(ii) $v(0) = -3 \text{ ms}^{-1}$ *(M1)(A1)*

(iii) $-3(1)^2 + 6(1) - 3 = 0 \rightarrow$ train has zero velocity at $t = 1$ *(M1)*
or $-3(t-1)(t-1) = 0$, therefore $t = 1$ second *(A1)(AG)*

(iv) passes under bridge when $t = 1.6$ seconds
$$v(1.6) = -3 \times 1.6^2 + 6 \times 1.6 - 3$$ *(M1)*
$$= -1.08 \text{ ms}^{-1}$$ *(A1)*
or (G2)

[7 marks]

(iii) (a) $\int u(t) \times dt = t^2 + 1.5t + c$ *(A1)(A1)*

$(1)^2 + 1.5(1) + c = 0.5$ *(M1)*

$$c = -2$$

$R(t) = t^2 + 1.5t - 2$ *(A1)*

[4 marks]

(b) $R(1.6) = (1.6)^2 + 1.5(1.6) - 2 = 2.96$ m *(M1)(A1)*

[2 marks]
Total [30 marks]