

MARKSCHEME

May 2004

MATHEMATICAL STUDIES

Standard Level

Paper 1

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Paper 1 Markscheme

Instructions to Examiners

Note: The number of marks for each question has been increased to 8. Where there are 2 marks (e.g. M2, A2) for an answer do NOT split the marks unless otherwise instructed.

1 Method of Marking

- (a) All marking must be done using a **red** pen.
- (b) In this paper, the maximum mark is awarded for a **correct answer**, irrespective of the method used. Thus, if the correct answer appears in the answer box, award the maximum mark and move onto the next question; in this case there is no need to check the method.
- (c) If an **answer is wrong**, then marks should be awarded for the method according to the markscheme. (A correct answer incorrectly transferred to the answer box is awarded the maximum mark.)

2 Abbreviations

The markscheme may make use of the following abbreviations:

- M** Marks awarded for **Method**
- A** Marks awarded for an **Answer** or for **Accuracy**
- G** Marks awarded for correct solutions, generally obtained from a **Graphic Display Calculator**, irrespective of working shown
- C** Marks awarded for **Correct** answers (irrespective of working shown)
- R** Marks awarded for clear **Reasoning**

3 Follow Through (ft) Marks

Errors made at any step of a solution can affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks should be awarded. The procedures for awarding these marks require that all examiners:

- (i) penalize the error when it **first occurs**;
- (ii) **accept the incorrect answer** as the appropriate value or quantity to be used in all subsequent working;
- (iii) award **M** marks for a correct method and **A(ft)** marks if the subsequent working contains no further errors.

Follow through procedures may be applied repeatedly throughout the same problem.

The following illustrates a use of the **follow through** procedure.

Markscheme		Candidate's Script	Marking	
$\$ 600 \times 1.02$	<i>MI</i>	Amount earned = $\$ 600 \times 1.02$	✓	<i>MI</i>
= $\$ 612$	<i>AI</i>	= $\$ 602$	×	<i>A0</i>
$\$ (306 \times 1.02) + (306 \times 1.04)$	<i>MI</i>	Amount = $301 \times 1.02 + 301 \times 1.04$	✓	<i>MI</i>
= $\$ 630.36$	<i>AI</i>	= $\$ 620.06$	✓	<i>AI(ft)</i>

Note that the candidate made an arithmetical error at line 2; the candidate used a correct method at lines 3, 4; the candidate's working at lines 3, 4 is correct.

However, if a question is transformed by an error into a **different, much simpler question** then:

- (i) **fewer** marks should be awarded at the discretion of the Examiner;
- (ii) marks awarded should be followed by “(d)” (to indicate that these marks have been awarded at the **discretion** of the Examiner);
- (iii) a brief **note** should be written on the script explaining **how** these marks have been awarded.

4 Using the Markscheme

- (a) This markscheme presents a particular way in which each question may be worked and how it should be marked. **Alternative methods** have not always been included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method in a manner which is consistent with the markscheme.

In this case:

- (i) a mark should be awarded followed by “(d)” (to indicate that the marks have been awarded at the **discretion** of the Examiner);
- (ii) a brief **note** should be written on the script explaining **how** these marks have been awarded.

Where alternative methods for complete questions are included, they are indicated by **METHOD 1**, **METHOD 2**, *etc.* Other alternative solutions, including graphic display calculator alternative solutions are indicated by **OR**. For example:

$$\begin{aligned} \text{Mean} &= 7906/134 && \text{(M1)} \\ &= 59 && \text{(A1)} \end{aligned}$$

OR

$$\text{Mean} = 59 \qquad \qquad \qquad \text{(G2)}$$

- (b) Unless the question specifies otherwise, accept **equivalent forms**. For example: $\frac{\sin \theta}{\cos \theta}$ for $\tan \theta$.

On the markscheme, these equivalent numerical or algebraic forms will be written in brackets after the required answer. Paper setters will indicate the required answer, by allocating full marks at that point. Further working should be ignored, even if it is incorrect. For example: if candidates are asked to factorize a quadratic expression, and they do so correctly, they are awarded full marks. If they then continue and find the roots of the corresponding equation, do not penalize, even if those roots are incorrect *i.e.*, once the correct answer is seen, ignore further working.

- (c) As this is an international examination, all **alternative forms of notation** should be accepted. For example: 1.7, 1·7, 1,7 ; different forms of vector notation such as \vec{u} , \overline{u} , \underline{u} ; $\tan^{-1} x$ for $\arctan x$.

5 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy.

There are two types of accuracy error. Candidates should be penalized once only **IN THE PAPER** for an accuracy error (**AP**). Award the marks as usual then write **-1(AP)** against the answer and also on the **front cover**.

Rounding errors: only applies to final answers not to intermediate steps.

Level of accuracy: when this is not specified in the question the general rule *unless otherwise stated in the question all numerical answers must be given exactly or to three significant figures* applies.

- If a final correct answer is incorrectly rounded, apply the **AP**
- OR**
- If the level of accuracy is not specified in the question, apply the **AP** for answers not given to 3 significant figures. (Please note that this has changed from May 2003).

Incorrect answers are wrong, and the accuracy penalty should not be applied to incorrect answers.

6 Graphic Display Calculators

Many candidates will be obtaining solutions directly from their calculators, often without showing any working. They have been advised that they must use mathematical notation, not calculator commands when explaining what they are doing. Incorrect answers without working will receive no marks. However, if there is written evidence of using a graphic display calculator correctly, method marks may be awarded. Where possible, examples will be provided to guide examiners in awarding these method marks

Calculator penalties

Candidates are instructed to write the make and model of their calculator on the front cover. Please apply the following penalties where appropriate.

- (i) Illegal calculators

If candidates note that they are using an illegal calculator, please report this on a PRF, and deduct 10 % of their overall mark. Note this on the front cover.

- (ii) Calculator box not filled in.

Please apply a calculator penalty (**CP**) of **[1 mark]** if this information is not provided. Note this on the front cover.

QUESTION 1

- (a) $m = 137 + 31 = 168$, (M1)(A1)
 $n = 194 + 6 = 200$ or just $n = 200$ (M1)(A1) (C4)
- (b) 137 students are aged below 20. (A2) (C2)
- (c) 25 % of 200 is 50. (M1)
 50th student is 18 years old. (A1) (C2)

QUESTION 2

- (a) The only prime number in U is 13. (A2) (C2)
- Note:** Award (A1) for $\{1, 13\}$ and (A0) for any other answer.
- (b) $A = \{-4, 1, 13, 69, 10^{33}\}$ (A2) (C2)
- (c) $B = \left\{-4, -\frac{2}{3}, 1, 13, 26.7, 69, 10^{33}\right\}$ (A2) (C2)
- (d) $A \cap B = \{-4, 1, 13, 69, 10^{33}\} (= A)$ (A2) (C2)

Note: In (b) and (d) allow (A1) for correct membership with **at most** one missing or one incorrect entry. A list with no set brackets is acceptable.
 In (c) allow at most one missing entry for (A1) but if π is present award (A0).

QUESTION 3

- (a) If Sarah eats lots of carrots, then she can see well in the dark. (A1)(A1) (C2)
- (b) Sarah does not eat lots of carrots and she can see well in the dark. (A1)(A1) (C2)
- Note:** Award (A1)(A0) if “and” is replaced with “or” but everything else is correct.
- (c) $\neg q \Rightarrow \neg p$ (A1)(A1)(A1) (C3)
- (d) It is the contrapositive of $p \Rightarrow q$. (A1) (C1)

QUESTION 4

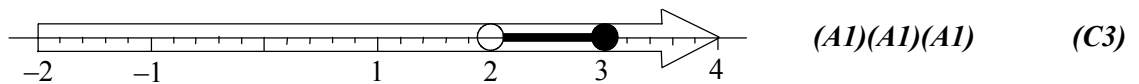
- (a) Solve $3x - 7 \leq 2$ *(M1)*
 Rearrange to get $3x \leq 7 + 2 = 9$ *(M1)*
 $x \leq \frac{9}{3}$ *(A1)*
 $x \leq 3$ *(C3)*

Note: Award *(M2)(A0)* or *(M1)(A0)* for $x = 3$, depending on where = is first introduced.

- (b) To solve $-x < -2$, multiply by -1 , *(M1)*
 obtaining $x > 2$. *(A1)* *(C2)*

Note: Award *(M1)(A0)* for $x = 2$, $x < 2$, $x \geq 2$ or $x \leq 2$ if working is shown but with no working award *(M0)(A0)*.

- (c) The figure should look like the following figure:



Note: Correct section of line distinguished in an appropriate way *(A1)*, open dot at 2 *(A1)*, closed dot at 3 *(A1)*.

QUESTION 5

- (a) Put $x = 0$ to find $y = -2$. *(M1)*
 Coordinates are $(0, -2)$. *(A1)* *(C2)*

Note: Award *(M1)(A0)* for -2 if working is shown. If not, award *(M0)(A0)*.

- (b) Factorise fully, $y = (x - 2)(x + 1)$. *(A1)(A1)*
 $y = 0$ when $x = -1, 2$. *(A1)(A1)*
 Coordinates are $A(-1, 0)$, $B(2, 0)$. *(A1)(A1)* *(C6)*

Note: Award *(C2)* for each correct x value if no method shown and full coordinates not given. If the quadratic formula is used correctly award *(M1)(A1)(A1)(A1)(A1)(A1)*. If the formula is incorrect award only the last *(A1)(A1)* as ft.

QUESTION 6

- (a) $I = 0.04 \times 2000 \times 18 = 1440$ Euros *(M1)(A1)*
 Total amount = $I + 2000 = 3440$ Euros. *(M1)(A1)* *(C4)*

- (b) $2000 \left(1 + \frac{0.036}{12} \right)^{18 \times 12}$ *(M1)(A1)*
 $= 3819.72$ *(A1)*
 $= 3820$ Euros, to nearest Euro. *(A1)* *(C4)*

QUESTION 7

(a) $a = 1, b = 3$ (allow $\frac{1}{8}, \frac{3}{8}$). (A1)(A1) (C2)

(b) $P(\text{two fours}) = \frac{3}{8} \times \frac{3}{8}$ (M1)
 $= \frac{9}{64}$ (or 0.141 to 3 s.f.) (A1) (C2)

(c) $P(\text{not two or three}) = \frac{6}{8}, \left(\text{or } \frac{3}{4} \text{ or } 0.75 \right)$ (A2)
 so $P(\text{six} \mid \text{not two or three}) = \frac{\frac{1}{8}}{\frac{6}{8}} = \frac{1}{6}$ (or 0.167) (M1)(A1) (C4)

QUESTION 8

(a) The range is $[1, 3]$ or $1 \leq y \leq 3$ (A1)(A1)(A1) (C3)

Note: Award (A1) for both closed interval brackets or for both correct inequality signs with y or $f(x)$, and (A1) for each correct end value. Award (A0) if the domain is given.

(b) The amplitude is 1. (A1) (C1)

(c) The period is $\frac{2 \times 180}{2} = 180^\circ$. (M1)(A1) (C2)

Note: Or award (R1)(A1) for just observing the period on the graph.

(d) Halve the period, 90° (marks for either). (A2) (C2)

Note: Award (M1)(A1) for a correct graph sketch as long as the new period is indicated.

QUESTION 9

- (a) L_1 has gradient 2 and L_2 has gradient $-\frac{1}{4}$. (A1)(A1) (C2)

Note: Award (A0)(A1) if the order of the gradients is reversed or both signs are wrong or both are reciprocals of the correct answer.

- (b) L_2 is drawn incorrectly. (A2) (C2)

- (c) The product of the gradients is $2 \times -\frac{1}{4} = -\frac{1}{2} \neq -1$. (M1)(A1) (C2)

Note: Award (M1) for looking at product of gradients, (A1) for comparing something to -1.

- (d) The drawing should show a straight line passing through x and y intercepts at (4, 0) and (0, 1) respectively. (A1)(A1) (C2)

Note: Award (A1) for each intercept. If these are wrong but gradient is $-\frac{1}{4}$ then (A1). If correct line is very poorly drawn then (A1).

QUESTION 10

- (a) $r = \frac{1}{\sin(86^\circ) - \sin(85^\circ)}$
 (= 730.2723312) = 730 to 3 s.f. (A2) (C2)

- (b) $r_A = \frac{1}{0.998 - 0.996}$ (M1)(A1)
 $= \frac{1}{0.002} = 500$. (A1) (C3)

- (c) $E = \frac{100(730 - 500)}{730}$ (M1)(A1)
 $= 31.5\%$ (to 3 s.f.) (A1) (C3)

QUESTION 11

- (a) $\vec{AC} = \vec{AB} + \vec{BC}$ (M1)
 Hence $\vec{AC} = \mathbf{a} + \mathbf{b}$. (A1) (C2)
- (b) By symmetry $\vec{AN} = \mathbf{b} + \frac{1}{2}\mathbf{a}$ (M1)(A1) (C2)
- (c) $\vec{AM} + \vec{AN} = \mathbf{a} + \frac{1}{2}\mathbf{b} + \mathbf{b} + \frac{1}{2}\mathbf{a}$ (M1)
 $= \frac{3}{2}\mathbf{a} + \frac{3}{2}\mathbf{b}$ (or $\frac{3}{2}(\mathbf{a} + \mathbf{b})$) $= \frac{3}{2}\vec{AC}$ (M1)(A1)
 so $k = \frac{3}{2}$. (A1) (C4)

QUESTION 12

- (a) $\sin(55^\circ) = \frac{3}{AD}$ (M1)(A1)
 $AD = \frac{3}{\sin(55^\circ)}$ (M1)
 $AD = 3.66232 = 3.66$ m to 3 s.f. (units not required). (A1) (C4)
- (b) $DB^2 = AD^2 + DC^2 = 3.66232^2 + 7^2$ (M1)(A1)
 $DB^2 = 62.4126$ hence $DB = 7.90$ m (units not required). (A1)(A1) (C4)

Note: Use of 3.66² makes no difference to final answer.
 Award at most (M0)(A0)(A0)(A1)ft for an incorrect cosine rule formula.
 Award at most (M1)(A0)(A0)(A1)ft for incorrect substitution into correct cosine rule formula.

QUESTION 13

- (a) $a = 2$ (A1) (C1)
- (b) Substitute (6, 0) and (3, 2) (M1)
 $6m + c = 0$ (A1)
 $3m + c = 2$ (A1)

Note: May award (M1) for any single reasonable attempt to substitute a point.

OR

- $m = \frac{0-2}{6-3} = -\frac{2}{3}$ (A1)
 with either of $6m + c = 0$ (M1)
 $3m + c = 2$ (A1) (C3)

Question 13 continued

- (c) Subtract equations $3m = -2$ (M1)
 $m = -\frac{2}{3}$ (A1)
- OR**
- use $m = -\frac{2}{3}$ (M2)
then $-2 + c = 2$ or $-4 + c = 0$ (A1)
Hence $c = 4$ (A1) (C4)

QUESTION 14

- (a) Angle $A = 90 - 5 = 85^\circ$. (M1)(A1) (C2)
- (b) $BC^2 = 6^2 + 8^2 - 2 \times 8 \times 6 \cos(85^\circ)$ (M1)(A1)
so $BC = \sqrt{91.6330487} = 9.57$ (3 s.f.) (A1) (C3)
- (c) $\frac{BC}{\sin(A)} = \frac{AC}{\sin(B)}$ (M1)
 $\sin(B) = \frac{6 \sin(85^\circ)}{9.572515275} = 0.6244093654$ (A1)
Angle $B = \sin^{-1}(0.6244093654) = 38.6^\circ$ (A1) (C3)

Note: Allow 38.7° if obtained using 9.57.

QUESTION 15

- (a) $\frac{44.5 - 12.4}{5.35} = 6$ (M1)(A1) (C2)
- (b) $90 \times 12.4 = 1116$ (M1)(A1)
 $1116 - 44.5 - 43.2 = 1028.3$ (M1)(A1)
 $\frac{1028.3}{88} = 11.7$ (M1)(A1) (C6)

Note: Award (M0)(A0) then ft for 88×12.4 .
Award (M0)(A0) for $\frac{1028.3}{90}$.