IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
PROGRAMA DEL DIPLOMA DEL BI

MATHEMATICAL STUDIES
STANDARD LEVEL
PAPER 1

Candidate number

Thursday 6 May 2004 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Write your candidate number in the box above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.
- Write the make and model of your calculator in the appropriate box on your cover sheet *e.g.* Casio *fx-9750G*, Sharp EL-9600, Texas Instruments TI-85.

Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Working may be continued below the box, if necessary. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1.   The cumulative frequency table below shows the ages of 200 students at a college.

Age	Number of Students	Cumulative Frequency
17	3	3
18	72	75
19	62	137
20	31	m
21	12	180
22	9	189
23-25	5	194
> 25	6	n

(a)   What are the values of m and n?

(b)   How many students are younger than 20?

(c)   Find the value in years of the lower quartile.

Working:

Answers:

(a) _____

(b) _____

(c) _____

2.   Let $U = \left\{ -4, -\dfrac{2}{3}, 1, \pi, 13, 26.7, 69, 10^{33} \right\}$.

A is the set of all the integers in U.
B is the set of all the rational numbers in U.

(a)   List all the prime numbers contained in U.

(b)   List all the members of A.

(c)   List all the members of B.

(d)   List all the members of the set $A \cap B$.

Working:

Answers:

(a) _____

(b) _____

(c) _____

(d) _____

3.   Let p and q be the statements:

p : *Sarah eats lots of carrots*.
q : *Sarah can see well in the dark.*

Write the following statements in words.

(a) $p \Rightarrow q$.

(b) $\neg p \wedge q$.

(c)   Write the following statement in symbolic form.

   If Sarah cannot see well in the dark, then she does not eat lots of carrots.

(d)   Is the statement in part (c) the *inverse*, the *converse* or the *contrapositive* of the statement in part (a)?
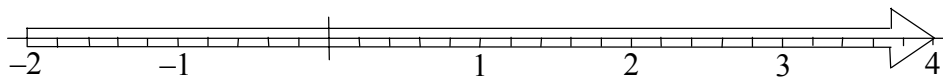
Working:

Answers:

(a) _____

_____

(b) _____

_____

(c) _____

(d) _____

4. (a) Solve the inequality $3x - 7 \le 2$.

(b) Solve the inequality $-x < -2$.

(c) The inequalities in (a) and (b) describe two regions on the number line. On the number line below, mark the **intersection** of those two regions.
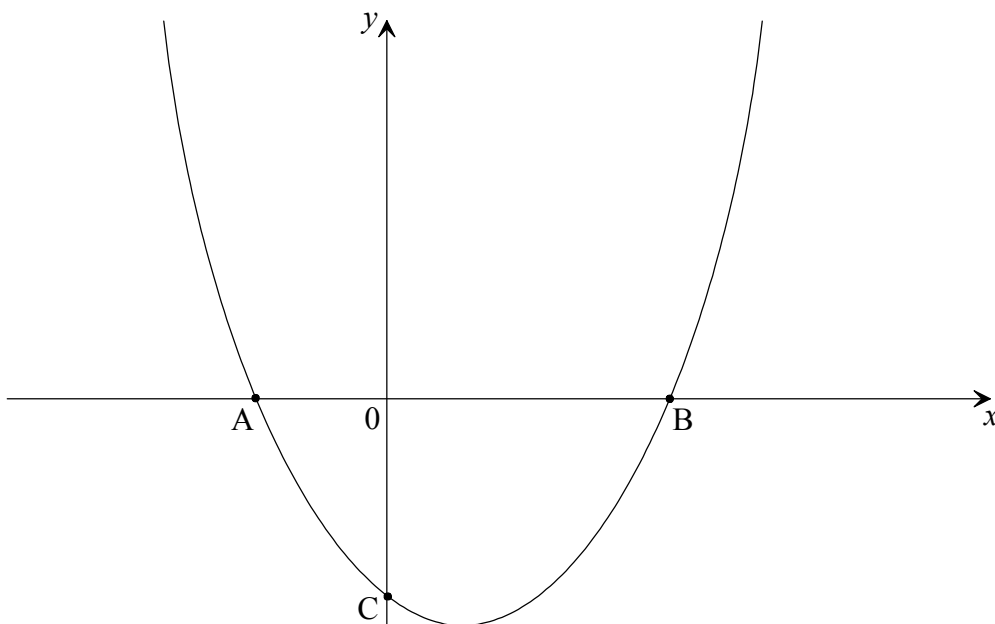

Working:

Answers:

(a) _____

(b) _____

Turn over

5. The graph of the function $y = x^2 - x - 2$ is drawn below.

(a) Write down the coordinates of the point C.

(b) Calculate the coordinates of the points A and B.

Working:

Answers:

(a) _____

(b) _____

6. Kurt wants to invest 2000 Euros in a savings account for his new grandson.

(a) Calculate the value of Kurt's investment based on a **simple interest rate** of 4 % *per annum*, after 18 years.

Inge tells Kurt about a better account which offers interest at a rate of 3.6 % *per annum*, **compounding monthly**.

(b) Giving your answer to the nearest Euro, calculate the value of Kurt's investment after 18 years if he follows Inge's advice.

Working:

Answers:

(a) _____

(b) _____

7. Two identical dice have sides numbered one to six. **The dice are weighted**. All the numbers except the four have equal probability of appearing on top. The four is three times as likely as each of the other numbers to appear on top.

The tree diagram below shows some of the probabilities.
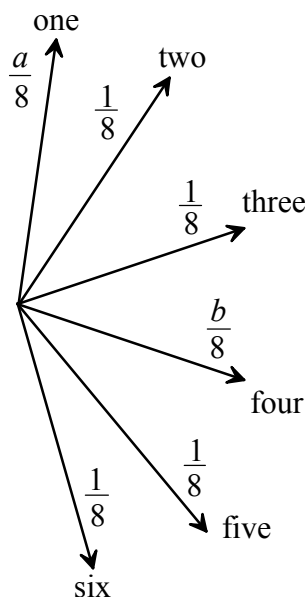
(a) Find the values of a and b in the diagram.

(b) Both dice are thrown. Calculate the probability that two fours appear on top.

(c) One of the dice is thrown once. The result is not a two or a three. What is the probability that it is a six?

Working:

Answers:

(a) _____

(b) _____

(c) _____

8.   The curve shown in the figure below is part of the graph of the function $f(x) = 2 + \sin(2x)$, where x is measured in degrees.
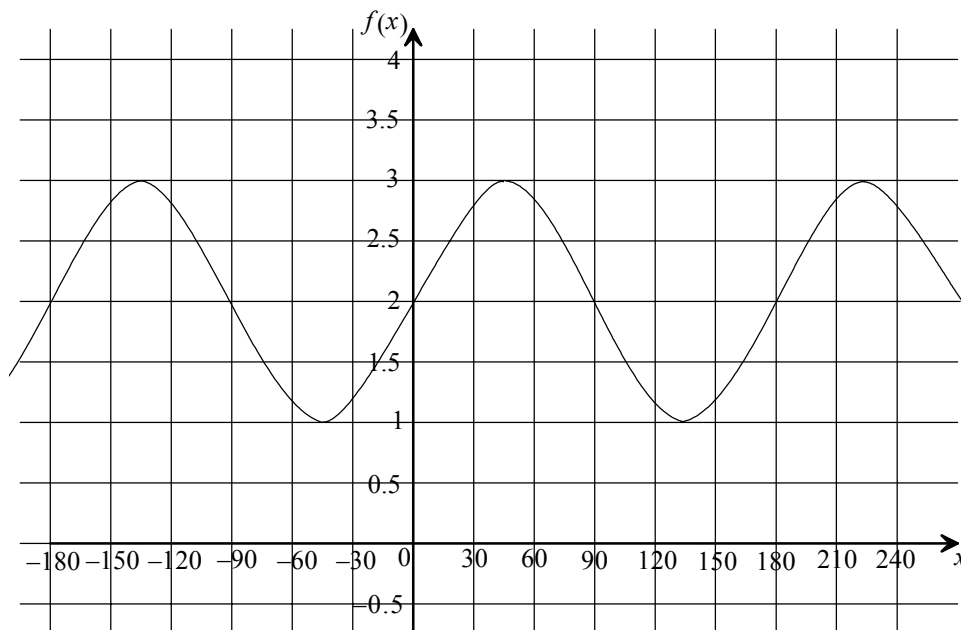

(a)   Find the range of $f(x)$.

(b)   Find the amplitude of $f(x)$.

(c)   Find the period of $f(x)$.

(d)   If the function is changed to $f(x) = 2 + \sin(4x)$ what is the effect on the period, compared to the period of the original function?

Working:

Answers:

(a) _____

(b) _____

(c) _____

(d) _____

9. A student has drawn the two straight line graphs L_1 and L_2 and marked in the angle between them as a right angle, as shown below. The student has drawn one of the lines incorrectly.
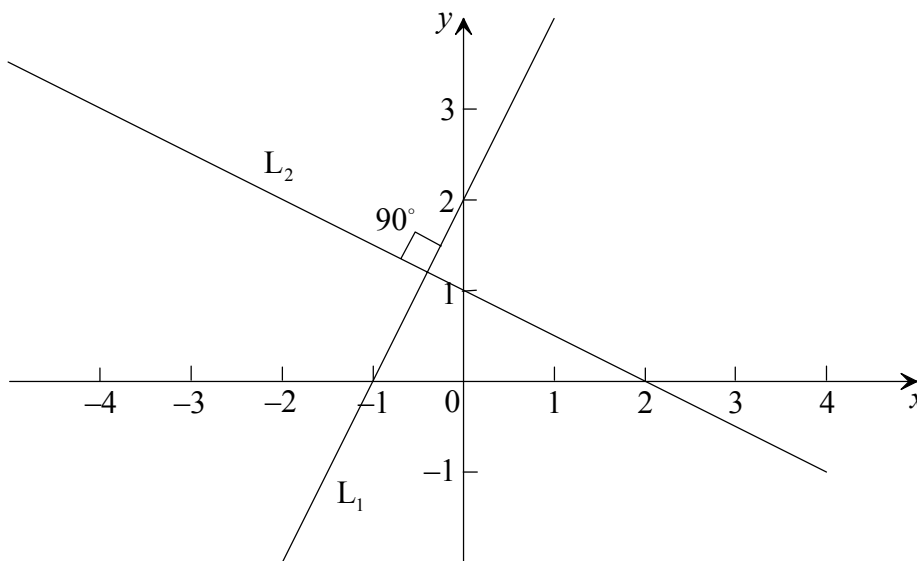
Consider L_1 with equation $y = 2x + 2$ and L_2 with equation $y = -\dfrac{1}{4}x + 1$.

(a) Write down the gradients of L_1 and L_2 **using the given equations**.

(b) Which of the two lines has the student drawn incorrectly?

(c) How can you tell from the answer to part (a) that the angle between L_1 and L_2 should not be $90°$?

(d) Draw the correct version of the incorrectly drawn line on the diagram.

Working:

Answers:

(a) _____

(b) _____

(c) _____

10. Arthur needs to calculate a value from a trigonometric formula. He uses his calculator to find the value of r given by $r = \dfrac{1}{\sin(86°) - \sin(85°)}$.

(a) Calculate the value of r, correct to three significant figures.

(b) Arthur makes the mistake of rounding both of the sines to three significant figures **before** taking their difference. Calculate the value of r found by Arthur. Call this value r_A.

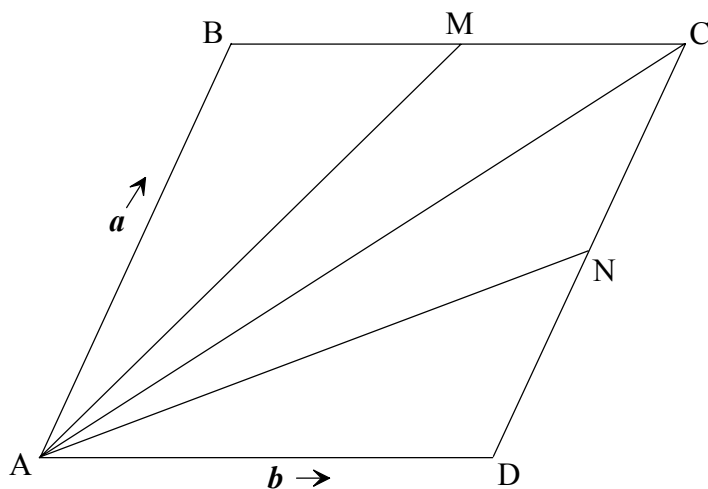(c) Calculate the percentage error E in Arthur's calculation, given by the formula

$$E = \frac{100(r - r_A)}{r}.$$

Working:

Answers:

(a) _____

(b) _____

(c) _____

Turn over

11. The figure below is a parallelogram with two of its sides given by the vectors *a* and *b* as shown. The midpoint of side BC is M and the midpoint of side DC is N.
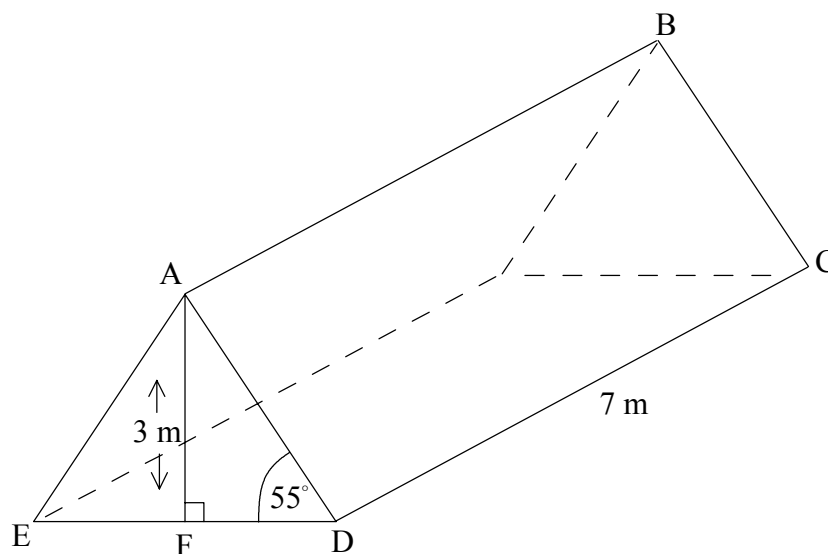

(a) Write down an expression for the diagonal $\overrightarrow{AC}$ in terms of *a* and *b*.

(b) Given that $\overrightarrow{AM} = a + \dfrac{1}{2}b$ write down an expression for the vector $\overrightarrow{AN}$ in terms of *a* and *b*.

(c) If $\overrightarrow{AM} + \overrightarrow{AN} = k\,\overrightarrow{AC}, (k \in \mathbb{Q})$, find *k*.

Working:

Answers:

(a) _____

(b) _____

(c) _____

12. The following diagram shows a sloping roof. The surface ABCD is a rectangle. The angle ADE is $55°$. The vertical height, AF, of the roof is 3 m and the length DC is 7 m.

(a) Calculate AD.

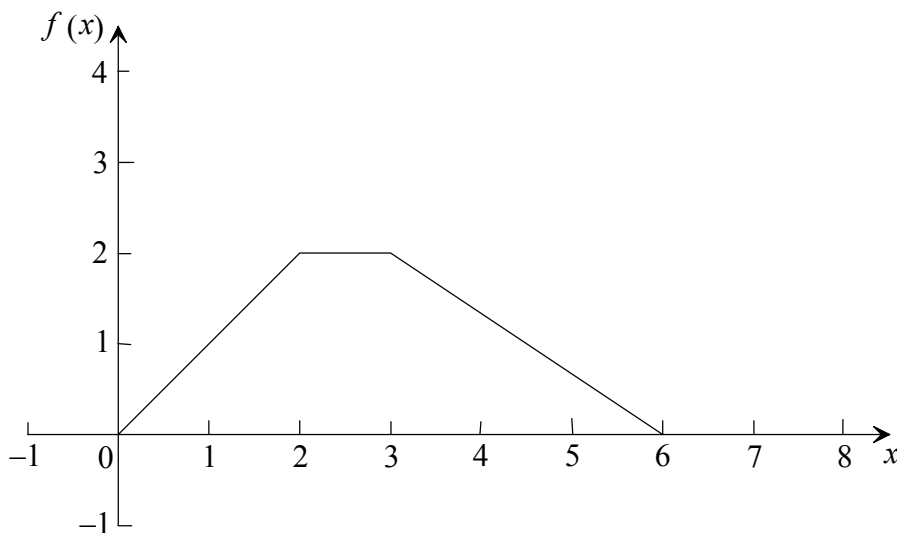(b) Calculate the length of the diagonal DB.

Working:

Answers:

(a) _____

(b) _____

13.   The sketch below shows the graph of a function $y = f(x)$ for x between $x = 0$ and $x = 6$.

The function $f(x)$ is defined as follows $f(x) = \begin{cases} x & \text{if } 0 \le x \le 2, \\ a & \text{if } 2 < x \le 3, \\ mx + c & \text{if } 3 < x \le 6 \end{cases}$

where a, c and m are constants.

(a)   State the value of a.

(b)   Find **two** equations for the constants m and c.

(c)   Hence or otherwise calculate the values of m and c.

Working:

Answers:

(a) _____

(b) _____

(c) _____

14. The following diagram shows the side view of a tent. The side of the tent AC is 6 m high. The ground AB slopes upwards from the bottom of the tent at point A, at an angle of 5° from the horizontal. The tent is attached to the ground by a rope at point B, a distance of 8 m from its base.
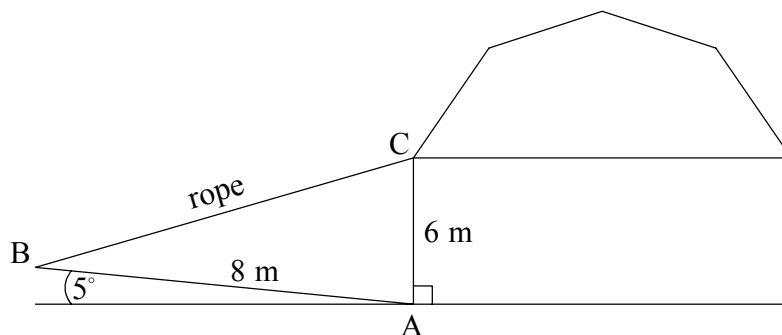
(a)   Calculate the angle BAC.

(b)   Calculate the length of the rope, BC.

(c)   Calculate the angle CBA that the rope makes with the sloping ground.

Working:

Answers:

(a) _____

(b) _____

(c) _____

15. A group of students has measured the heights of 90 trees. The class calculate the mean height to be $\bar{x} = 12.4$ m with standard deviation $s = 5.35$ m. One student notices that two of the measurements, 44.5 m and 43.2 m, are much too big and must be wrong.

(a) How many standard deviations away from the mean of 12.4 is the value 44.5?

The incorrect measurements of 44.5 m and 43.2 m must be removed from the data.

(b) Calculate the new value of $\bar{x}$ after removing the two unwanted values.

Working:

Answers:

(a) _____

(b) _____