INTERNATIONAL BACCALAUREATE
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IBO

M04/522/S(2)M+

MARKSCHEME

May 2004

MATHEMATICAL METHODS

Standard Level

Paper 2

22 pages

Paper 2 Markscheme

Instructions to Examiners

Note: Where there are 2 marks (*e.g.* M2, A2) for an answer do NOT split the marks unless otherwise instructed.

1 Method of marking

(a) All marking must be done using a **red** pen.

(b) Marks should be noted on candidates' scripts as in the markscheme:

- show the breakdown of individual marks using the abbreviations ***(M1)***, ***(A2)*** *etc*., unless a part is completely correct;
- write down each part mark total, indicated on the markscheme (for example, ***[3 marks]***) – it is suggested that this be written at the end of each part, and underlined;
- write down and circle the total for each question at the end of the question.

2 Abbreviations

The markscheme may make use of the following abbreviations:

(*M)* Marks awarded for **Method**

(A) Marks awarded for an **Answer** or for **Accuracy**

(N) Marks awarded for correct answers, if **no** working shown: they may not be all the marks for the question. Examiners should only award these marks for correct answers where there is no working.

(R) Marks awarded for clear **Reasoning**

(AG) **Answer Given** in the question and consequently marks are **not** awarded

Note: In general, it is not possible to award ***(M0)(A1)***.

Examiners should use **(d)** to indicate where discretion has been used. It should only be used for decisions on follow through and alternative methods. It must be accompanied by a brief note to explain the decision made

Follow through (ft) marks should be awarded where a correct method has been attempted but error(s) are made in subsequent working which is essentially correct.

- Penalize the error when it first occurs
- Accept the incorrect result as the appropriate quantity in all subsequent working
- If the question becomes much simpler then use discretion to award fewer marks
- Use **(d)** to indicate where discretion has been used. It should only be used for decisions on follow through and alternative methods. It must be accompanied by a brief note to explain the decision made.

3 Using the Markscheme

(a) This markscheme presents a particular way in which each question may be worked and how it should be marked. **Alternative methods** have not always been included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method in a manner which is consistent with the markscheme. Indicate the awarding of these marks by **(d)**.

Where alternative methods for complete questions or parts of questions are included, they are indicated by **METHOD 1**, **METHOD 2**, *etc*. Other alternative part solutions are indicated by **EITHER….OR**. It should be noted that ***G*** marks have been removed, and GDC solutions will not be indicated using the **OR** notation as on previous markschemes.

Candidates are expected to show working on this paper, and examiners should not award full marks for just the correct answer. Where it is appropriate to award marks for correct answers with no working, it will be shown on the markscheme using the ***N*** notation. All examiners will be expected to award marks accordingly in these situations.

(b) Unless the question specifies otherwise, accept **equivalent forms**. For example: $\frac{\sin\theta}{\cos\theta}$ for $\tan\theta$. On the markscheme, these equivalent numerical or algebraic forms will generally be written in brackets after the required answer. Paper setters will indicate the required answer, by allocating full marks at that point. Further working should be ignored, even if it is incorrect. For example: if candidates are asked to factorize a quadratic expression, and they do so correctly, they are awarded full marks. If they then continue and find the roots of the corresponding equation, do not penalize, even if those roots are incorrect, *i.e.* once the correct answer is seen, ignore further working.

(c) As this is an international examination, all **alternative forms of notation** should be accepted. For example: 1.7, $1\cdot 7$, 1,7; different forms of vector notation such as $\vec{u}$, $\bar{u}$, $\underline{u}$; $\tan^{-1} x$ for arctan *x*.

4 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy.

There are two types of accuracy error. Candidates should be penalized **once only IN THE PAPER** for an accuracy error **(AP)**.

Award the marks as usual then write –1**(AP)** against the answer and also on the **front** cover

Rounding errors: only applies to final answers not to intermediate steps.

Level of accuracy: when this is not specified in the question the general rule *unless otherwise stated in the question all numerical answers must be given exactly or to three significant figures* applies.

- If a final correct answer is incorrectly rounded, apply the **AP**
  OR
- If the level of accuracy is not specified in the question, apply the **AP** for answers not given to 3 significant figures. (Please note that this has changed from May 2003).

Note: If there is no working shown, and answers are given to the correct two significant figures, apply the **AP**. However, do **not** accept answers to one significant figure without working.

5 Graphic Display Calculators

Many candidates will be obtaining solutions directly from their calculators, often without showing any working. They have been advised that they must use mathematical notation, not calculator commands when explaining what they are doing. Incorrect answers without working will receive no marks. However, if there is written evidence of using a graphic display calculator correctly, method marks may be awarded. Where possible, examples will be provided to guide examiners in awarding these method marks.

Calculator penalties

Candidates are instructed to write the make and model of their calculator on the front cover. Please apply the following penalties where appropriate.

(a) Illegal calculators

If candidates note that they are using an illegal calculator, please report this on a PRF, and deduct 10% of their overall mark. Note this on the front cover. The most common examples are:

Texas Instruments: TI-89 (plus); TI-92 (plus); TI-Voyage 200
Casio: *fx9970*; *fx2.0* algebra; classpad
HP: 38–95 series

(b) Calculator box not filled in.

Please apply a calculator penalty ***(CP)*** of 1 mark if this information is not provided. Note this on the front cover.

Examples

1 Accuracy

A question leads to the answer 4.6789….

- 4.68 is the correct 3 s.f. answer.
- 4.7, 4.679 are to the wrong level of accuracy: both should be penalized the first time this type of error occurs.
- 4.67 is incorrectly rounded – penalize on the first occurrence.

Note: All these "incorrect" answers may be assumed to come from 4.6789…, even if that value is not seen, but previous correct working is shown. However, 4.60 is wrong, as is 4.5, 4.8, and these should be penalized as being incorrect answers, not as examples of accuracy errors.

2 Alternative solutions

Question

The points P, Q, R are three markers on level ground, joined by straight paths PQ, QR, PR as shown in the diagram. $\text{QR} = 9\text{ km}$, $\text{P}\hat{\text{Q}}\text{R} = 35^\circ$, $\text{P}\hat{\text{R}}\text{Q} = 25^\circ$.
(Note: in the original question, the first part was to find PR = 5.96)

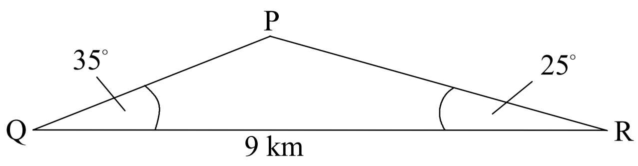
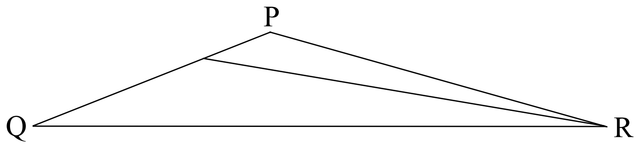


diagram not to scale

(a) Tom sets out to walk from Q to P at a steady speed of 8 km h^{-1}. At the same time, Alan sets out to jog from R to P at a steady speed of $a\text{ km h}^{-1}$. They reach P at the same time. Calculate the value of *a*. ***[7 marks]***

(b) The point S is on [PQ], such that $\text{RS} = 2\text{QS}$, as shown in the diagram.



Find the length QS. ***[6 marks]***

Markscheme

(a) **EITHER**

Sine rule to find PQ

$\text{PQ} = \dfrac{9\sin 25}{\sin 120}$ ***(M1)(A1)***

$\text{PQ} = 4.39\text{ km}$ ***(A1)***

OR

Cosine rule: $\text{PQ}^2 = 5.96^2 + 9^2 - (2)(5.96)(9)\cos 25$ ***(M1)(A1)***

$= 19.29$

$\text{PQ} = 4.39\text{ km}$ ***(A1)***

THEN

Time for Tom $= \dfrac{4.39}{8}$ ***(A1)***

Time for Alan $= \dfrac{5.96}{a}$ ***(A1)***

Then $\dfrac{4.39}{8} = \dfrac{5.96}{a}$ ***(M1)***

$a = 10.9$ ***(A1)*** ***(N5)***

[7 marks]

Note that the **THEN** part follows both **EITHER** and **OR** solutions, and this is shown by the alignment.

(b) **METHOD 1**

$\text{RS}^2 = 4\text{QS}^2$ ***(A1)***

$4\text{QS}^2 = \text{QS}^2 + 81 - 18 \times \text{QS} \times \cos 35$ ***(M1)(A1)***

$3\text{QS}^2 + 14.74\text{QS} - 81 = 0$ (or $3x^2 + 14.74x - 81 = 0$) ***(A1)***

$\Rightarrow \text{QS} = -8.20$ or $\text{QS} = 3.29$ ***(A1)***

therefore QS = 3.29 ***(A1)*** ***(N2)***

METHOD 2

$\dfrac{\text{QS}}{\sin \text{S}\hat{\text{R}}\text{Q}} = \dfrac{2\text{QS}}{\sin 35}$ ***(M1)***

$\Rightarrow \sin \text{S}\hat{\text{R}}\text{Q} = \dfrac{1}{2} \sin 35$ ***(A1)***

$\text{S}\hat{\text{R}}\text{Q} = 16.7^\circ$ ***(A1)***

Therefore, $\text{Q}\hat{\text{S}}\text{R} = 180 - (35 + 16.7) = 128.3^\circ$ ***(A1)***

$\dfrac{9}{\sin 128.3} = \dfrac{\text{QS}}{\sin 16.7} \left(= \dfrac{\text{SR}}{\sin 35} \right)$ ***(M1)***

$\text{QS} = \dfrac{9 \sin 16.7}{\sin 128.3} \left(= \dfrac{9 \sin 35}{2 \sin 128.3} \right) = 3.29$ ***(A1)*** ***(N2)***

If candidates have shown no working award ***(N5)*** for the correct answer 10.9 in part (a) and ***(N2)*** for the correct answer 3.29 in part (b).

[6 marks]

3 Follow through

Question

Calculate the acute angle between the lines with equations

$$\boldsymbol{r} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} + s\begin{pmatrix} 4 \\ 3 \end{pmatrix} \text{ and } \boldsymbol{r} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

Markscheme

Angle between lines = angle between direction vectors. (May be implied.)	***(A1)***	
Direction vectors are $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$. (May be implied.)	***(A1)***	
$\begin{pmatrix} 4 \\ 3 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \left\lvert \begin{pmatrix} 4 \\ 3 \end{pmatrix} \right\rvert \left\lvert \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\rvert \cos\theta$	***(M1)***	
$4 \times 1 + 3 \times (-1) = \sqrt{(4^2 + 3^2)}\sqrt{\left(1^2 + (-1)^2\right)}\cos\theta$	***(A1)***	
$\cos\theta = \dfrac{1}{5\sqrt{2}}$ $(= 0.1414\ldots)$	***(A1)***	
$\theta = 81.9^\circ$ (1.43 radians)	***(A1)***	***(N3)***

Examples of solutions and marking

	Solutions	Marks allocated
1.	$\begin{pmatrix} 4 \\ 3 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \left\lvert \begin{pmatrix} 4 \\ 3 \end{pmatrix} \right\rvert \left\lvert \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\rvert \cos\theta$	***(A1)(A1)*** implied ***(M1)***
	$\cos\theta = \dfrac{7}{5\sqrt{2}}$	***(A0)(A1)***
	$\theta = 8.13^\circ$	***(A1)*ft**
		Total 5 marks
2.	$\cos\theta = \dfrac{\begin{pmatrix} 4 \\ -1 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 4 \end{pmatrix}}{\sqrt{17}\sqrt{20}}$	***(A0)(A0)*** wrong vectors implied ***(M1)*** for correct method, ***(A1)*ft**
	$= 0.2169$	***(A1)*ft**
	$\theta = 77.5^\circ$	***(A1)*ft**
		Total 4 marks
3.	$\theta = 81.9^\circ$	***(N3)***
		Total 3 marks

Note that this candidate has obtained the correct answer, but not shown any working. The way the markscheme is written means that the first 2 marks may be implied by subsequent correct working, but the other marks are only awarded if the relevant working is seen. Thus award the first 2 implied marks, plus the final mark for the correct answer.

END OF EXAMPLES

QUESTION 1

(a) (i) $\vec{AB} = \vec{OB} - \vec{OA} = \begin{pmatrix} -1 \\ -2 \end{pmatrix} - \begin{pmatrix} 4 \\ -3 \end{pmatrix}$ ***(M1)***

$= \begin{pmatrix} -5 \\ 1 \end{pmatrix}$ ***(A1)*** ***(N2)***

(ii) $|\vec{AB}| = \sqrt{25+1}$ ***(M1)***

$= \sqrt{26}$ $(= 5.10, 3\text{ s.f.})$ ***(A1)*** ***(N2)***

Note: An answer of 5.1 is subject to **AP**.

[4 marks]

(b) $\vec{AD} = \vec{OD} - \vec{OA}$

$= \begin{pmatrix} d \\ 22 \end{pmatrix} - \begin{pmatrix} 4 \\ -3 \end{pmatrix}$

$= \begin{pmatrix} d-4 \\ 25 \end{pmatrix}$ ***(A1)(A1)***

[2 marks]

(c) (i) **EITHER**

$\mathrm{B\hat{A}D} = 90^\circ \Rightarrow \vec{AB} \bullet \vec{AD} = 0$ or mention of scalar (dot) product. ***(M1)***

$\Rightarrow \begin{pmatrix} -5 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} d-4 \\ 25 \end{pmatrix} = 0$

$-5d + 20 + 25 = 0$ ***(A1)***

$d = 9$ ***(AG)***

OR

$\left.\begin{array}{l} \text{Gradient of AB} = -\dfrac{1}{5} \\ \text{Gradient of AD} = \dfrac{25}{d-4} \end{array}\right\}$ ***(A1)***

$\left(\dfrac{25}{d-4}\right) \times \left(-\dfrac{1}{5}\right) = -1$ ***(A1)***

$\Rightarrow d = 9$ ***(AG)***

(ii) $\vec{OD} = \begin{pmatrix} 9 \\ 22 \end{pmatrix}$ (correct answer only) ***(A1)***

[3 marks]

continued...

Question 1 continued

(d) $\vec{AD} = \vec{BC}$ ***(M1)***

$\vec{BC} = \begin{pmatrix} 5 \\ 25 \end{pmatrix}$ ***(A1)***

$\vec{OC} = \vec{OB} + \vec{BC}$ ***(M1)***

$\vec{OC} = \begin{pmatrix} -1 \\ -2 \end{pmatrix} + \begin{pmatrix} 5 \\ 25 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ 23 \end{pmatrix}$ ***(A1)*** ***(N3)***

Note: Many other methods including scale drawing are acceptable.

[4 marks]

(e) $|\vec{AD}| \left(\text{or } |\vec{BC}| \right) = \sqrt{5^2 + 25^2} = \sqrt{650}$ ***(A1)***

Area $= \sqrt{26} \times \sqrt{650}$ $(= 5.099 \times 25.5)$

$= 130$ ***(A1)*** ***(N1)***

[2 marks]

Total [15 marks]

QUESTION 2

(a) $f(x) = \int f'(x)\,\mathrm{d}x$ ***(M1)***

Note: Award ***(M1)*** for evidence of integration. This evidence may be the integral sign **OR** the presence of $+c$ **OR** the presence of expressions such as $\frac{x^{1+1}}{1+1}$. No ***(A)*** marks can be awarded without this ***(M)*** mark.

$f(x) = \mathrm{e}^x + \frac{x^2}{2} - 5x + c$ ***(A1)(A1)(A1)(A1)***

$\mathrm{e} - 2 = \mathrm{e} + \frac{1}{2} - 5 + c$ ***(M1)***

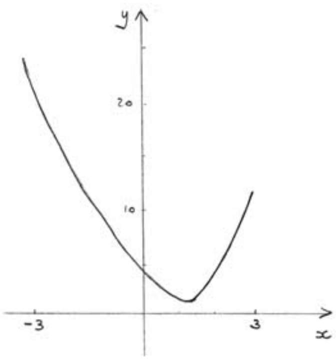
$c = 2.5$ ***(A1)*** ***(N0)***

$f(x) = \mathrm{e}^x + \frac{x^2}{2} - 5x + 2.5$ ***(AG)*** ***(N0)***

Note: An alternative method is to show that $(1, \mathrm{e} - 2)$ satisfies the given expression for $f(x)$ and that $f'(x) = \mathrm{e}^x + x - 5$.

[7 marks]

(b)



(A1)(A1)

Note: Award ***(A1)*** for the shape and ***(A1)*** for approximately the correct domain **and** range.

[2 marks]

(c) 0.514 ***(A2)***

Note: Award ***(A1)*** for (1.31, 0.514) and ***(A0)*** for 1.31 only.

[2 marks]

(d) $\int_0^2 f(x)\,\mathrm{d}x \left(= \int_0^2 \left(\mathrm{e}^x + \frac{1}{2}x^2 - 5x + 2.5 \right) \mathrm{d}x \right)$ ***(A1)(A1)***

Area $= 2.72$. ***(A1)*** ***(N3)***

[3 marks]

Total [14 marks]

QUESTION 3

(a) $h = 24 - 14\sin 0$ ***(M1)***
$= 24$ (cm) ***(A1)*** ***(N2)***

[2 marks]

(b) 38 (cm) ***(A1)***

[1 mark]

(c) 2.36 (secs) $\left(\text{or } \dfrac{3\pi}{4}\right)$ (Accept 135 (working in degrees)) ***(A1)***

[1 mark]

(d) Use of period of $\sin bt = \dfrac{2\pi}{b}$ ***(M1)***

$\dfrac{2\pi}{2} = \pi$ (Accept 180) ***(A1)*** ***(N2)***

[2 marks]

(e) $a = 14$ ***(A1)***
$b = 24$ ***(A1)***

[2 marks]

(f) $h = 24 + 14\sin 2t$ ***(A2)***

OR

$h = 24 - 14\sin(2t + \pi)$ ***(A2)***

OR

$h = 24 - 14\cos\left(2t + \dfrac{\pi}{2}\right)$ ***(A2)***

[2 marks]

Total [10 marks]

QUESTION 4

(i) (a) AA AB AC BA BB BC CA CB CC *(A1)*

[1 mark]

(b) (i) $\frac{3}{9}$ *(A1)*

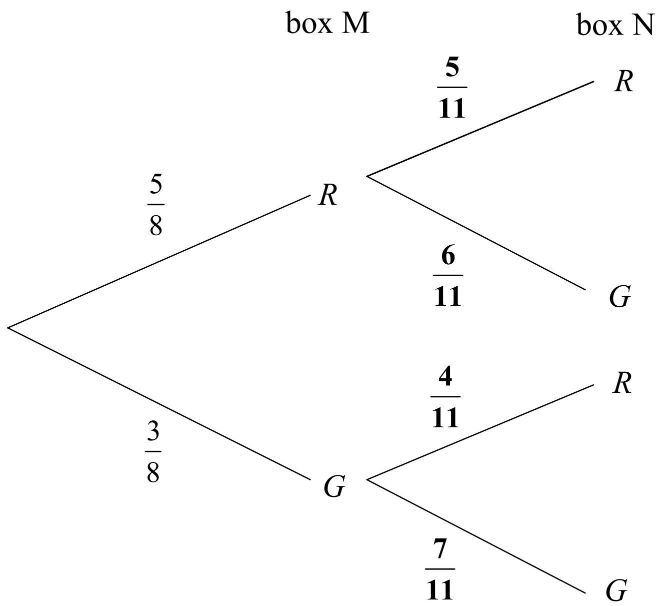
(ii) $\frac{5}{9}$ *(A1)*

(iii) $\frac{1}{9}$ *(A1)*

(iv) $\frac{7}{9}$ *(A1)*

[4 marks]

(ii) (a)



(A1)(A1)(A1)(A1)

[4 marks]

continued...

Question 4 continued

(b) P(green from box N)

$$= \frac{5}{8} \times \frac{6}{11} + \frac{3}{8} \times \frac{7}{11}$$ ***(A1)(A1)***

$$= \frac{51}{88} \ (0.580)$$ ***(A1)*** ***(N2)***

[3 marks]

(c) $$\mathrm{P}(\text{red from M} \mid \text{green from box N}) = \frac{\mathrm{P}(G \cap R)}{\mathrm{P}(G)}$$

$$\mathrm{P}(G \cap R) = \frac{30}{88}$$ ***(A1)***

$$\mathrm{P}(G) = \frac{51}{88}$$ ***(A1)***

$$\mathrm{P}(\text{red from M} \mid \text{green from box N}) = \frac{\frac{30}{88}}{\frac{51}{88}}$$ ***(M1)***

$$= \frac{30}{51} \left(= \frac{10}{17}, 0.588 \right)$$ ***(A1)*** ***(N3)***

[4 marks]

Total [16 marks]

QUESTION 5

(a) (i) 2420 ***(A1)***

(ii) $1420 + 100n > 2000$ ***(M1)***

$n > 5.8$

1999 (accept 6th year or $n = 6$) ***(A1)*** ***(N1)***

Note: Award ***(A0)*** for 2000, or after 6 years, or $n = 6$, 2000.

[3 marks]

(b) (i) $1\,200\,000(1.025)^{10} = 1\,536\,101$ $\left(\text{accept } 1\,540\,000 \text{ or } 1.54 \text{ (million)}\right)$ ***(A1)***

(ii) $\dfrac{1\,536\,101 - 1\,200\,000}{1\,200\,000} \times 100$ ***(M1)***

28.0 % (accept 28.3 % from $1\,540\,000$) ***(A1)*** ***(N2)***

(iii) $1\,200\,000\,(1.025)^n > 2\,000\,000$ (accept an equation) ***(M1)***

$n \log 1.025 > \log\left(\dfrac{2}{1.2}\right) \Rightarrow n > 20.69$ ***(M1)(A1)***

2014 (accept 21st year or $n = 21$) ***(A1)*** ***(N3)***

Note: Award ***(A0)*** for 2015, after 21 years, or $n = 21$, so 2015.

[7 marks]

(c) (i) $\dfrac{1\,200\,000}{1420} = 845$ ***(A1)***

(ii) $\dfrac{1\,200\,000\,(1.025)^n}{1420 + 100n} < 600$ ***(M1)(M1)***

$\Rightarrow n > 14.197$

15 years ***(A2)*** ***(N2)***

[5 marks]

Total [15 marks]

QUESTION 6

(i) (a) $\mathrm{P}\left(Z < -\frac{4}{2.7}\right) \quad (= \mathrm{P}(Z < -1.481))$ ***(M1)***

$= 1 - \mathrm{P}(Z < 1.481) \quad (1 - 0.931)$ ***(M1)***

$= 0.0692$ (accept 0.0693 and 0.0694) ***(A1)*** ***(N3)***

[3 marks]

(b) $Z = -1.28(16)$ ***(M1)***

Volume $= 379 - 1.28 \times 3.6$ ***(A1)***

$= 374$ ***(A1)*** ***(N3)***

[3 marks]

(c) $Z = -\frac{9}{d}$ ***(A1)***

$Z = -2.3263$ ***(A1)***

$-\frac{9}{d} = -2.3263$ ***(A1)***

$d = 3.87 \pm 0.01$ ***(A1)*** ***(N2)***

[4 marks]

(ii) (a) $y = 9.76x + 166$ ***(A3)***

[3 marks]

(b) (i) unit cost (or cost of producing one box) ***(R1)***

(ii) fixed costs ***(R1)***

Note: Award ***(R0)(R0)*** for strictly geometric interpretations.

[2 marks]

(c) (i) $9.76 \times 55 + 166$ ***(M1)***

= \$ 703 (Accept \$ 702.80 ± 0.20) ***(A1)*** ***(N2)***

(ii) $13.20x > 9.76x + 166$ (or equivalent) ***(M1)***

$3.44x > 166$ ***(A1)***

$x > 48.3$ ***(A1)***

49 boxes ***(A1)*** ***(N2)***

[6 marks]

continued…

Question 6 continued

(iii) (a) $\text{SE} = \dfrac{8}{\sqrt{30}} (= 1.46)$ ***(A1)***

$Z > \dfrac{2}{1.46}$ (or equivalent) ***(M1)***

$\text{P}(C > 72) = 0.0855$ ***(A1)*** ***(N3)***

[3 marks]

(b) $\text{SE} = \dfrac{8}{\sqrt{n}}$ ***(A1)***

$-\dfrac{2\sqrt{n}}{8} < Z < \dfrac{2\sqrt{n}}{8}$ ***(M1)***

$\text{P}\left(\dfrac{-2\sqrt{n}}{8} < Z < \dfrac{2\sqrt{n}}{8}\right) = 0.95$ ***(M1)***

$\dfrac{2\sqrt{n}}{8} > 1.96$ ***(A1)***

$n > 61.46$ ***(A1)***

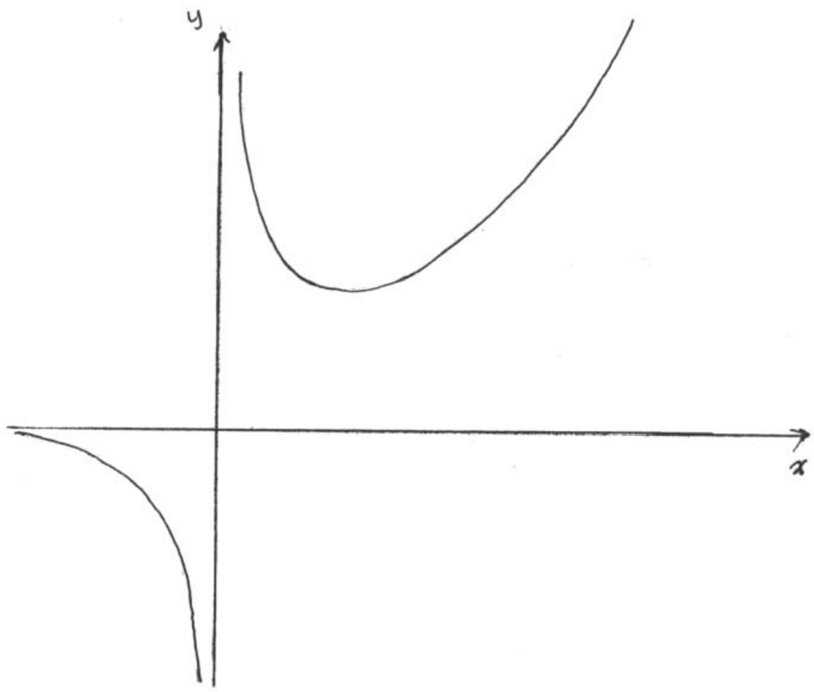
sample size at least 62 ***(A1)*** ***(N3)***

[6 marks]

Total [30 marks]

QUESTION 7

(i) (a) ***(A2)***



Note: Award ***(A1)*** for correct shape of left branch and ***(A1)*** for correct shape of right branch.

[2 marks]

(b) (i) **EITHER**

$$g'(x) = \frac{xe^x - e^x(1)}{x^2}$$ ***(M1)(A1)*** ***(N2)***

OR

$$g(x) = e^x \cdot x^{-1}$$

$$g'(x) = \frac{-e^x}{x^2} + \frac{e^x}{x}$$ ***(M1)(A1)*** ***(N2)***

(ii) For minimum point $\frac{e^x(x-1)}{x^2} = 0$ (or $g'(x) = 0$) (may be implied) ***(M1)***

$e^x(x-1) = 0$ ***(M1)***

$x = 1, y = e$ (accept (1, e)) ***(A1)(A1)*** ***(N3)***

Note: Penalize one mark for having $y = 2.72$ instead of e.

[6 marks]

(c) For setting $g''(x) = 0$ (may be implied) ***(M1)***

$\Rightarrow e^x\left(x^2 - 2x + 2\right) = 0$ ***(A1)***

$e^x \neq 0$ ***(A1)***

For saying there is no real solution to the quadratic equation. ***(R1)***

[4 marks]

continued...

Question 7 continued

(ii) (a) $f'(x) = \cos x + 2$ ***(A1)***

$x_{n+1} = x_n - \dfrac{\sin x_n + 2x_n - 1}{\cos x_n + 2}$ ***(A1)***

solution $x = 0.3354180$ ***(A1)*** ***(N1)***

Note:	Award the second ***(A1)*** for evidence of using Newton-Raphson either by showing the general form as above or by appearance of one or more of $x_1 = 0.27510977$, $x_2 = 0.3352394$, $x_3 = 0.3354180$.

[3 marks]

(b) (i) $x_1 = 0.0792645$ ***(A1)***

$x_2 = 0.4604092$ ***(A1)***

(ii) convergence guaranteed if $\left|\dfrac{dy}{dx}\right| < 1$ ***(R1)***

$\dfrac{dy}{dx} = -\dfrac{\cos x}{2}$ ***(A1)***

$\left|-\dfrac{\cos x}{2}\right| \le \dfrac{1}{2}$ (or since $|\cos x| < 1 \Rightarrow \left|\dfrac{dy}{dx}\right| < 1$) ***(R1)***

[5 marks]

(iii) (a) $\dfrac{1}{2}\left(0 + 108 + 2 \times 9\sqrt{2}\right)$ ***(M1)(A1)***

Note:	Award ***(M1)*** for evidence of $h = 1$ used, award ***(A1)*** for correct terms in expression. Since the trapezium rule is explicitly required, evidence of its use is necessary for method marks.

$= 66.7$ ***(A2)*** ***(N2)***

[4 marks]

(b) (i) $\dfrac{du}{dx} = 3x^2 dx$ ***(A1)***

$\left(\int 9x^2\sqrt{x^3+1}\,dx = \right)3\int \sqrt{u}\,du$ ***(A1)***

$= 3\left(\dfrac{2}{3}\right)u^{\frac{3}{2}} + c$ ***(A1)***

$= 2(x^3+1)^{\frac{3}{2}} + c$ ***(AG)***

(ii) $\left[2\left(x^3+1\right)^{\frac{3}{2}}\right]_0^k = 594$ (may be implied) ***(A1)***

$2\left(k^3+1\right)^{\frac{3}{2}} - 2 = 594$ ***(A1)***

$k = 3.52$ ***(A1)*** ***(N1)***

[6 marks]

Total [30 marks]

QUESTION 8

(i) (a) (i) $\boldsymbol{H} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ *(A1)*

(ii) $\boldsymbol{S} = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ *(A1)*

(iii) $\boldsymbol{R} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ *(A1)*

[3 marks]

(b) $\boldsymbol{H}^{-1} =$ shear of scale factor -2 in the direction of the x-axis *(A1)*

$\boldsymbol{S}^{-1} =$ stretch of scale factor $\frac{1}{2}$ in the direction of the y-axis *(A1)*

$\boldsymbol{R}^{-1} =$ reflection in the x-axis *(A1)*

Notes:	All components of the description are needed to receive marks. Award no marks if the inverse matrix is given.

[3 marks]

(c) (i) $\boldsymbol{M} = \boldsymbol{HSR}$ *(A1)*

(ii) $\begin{pmatrix} 1 & -4 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$ *(M1)*

$$\left.\begin{aligned} x - 4y &= x \\ -2y &= y \end{aligned}\right\}$$ *(A1)*

all points $(x, 0)$ are invariant under $\boldsymbol{M}$. *(A2)* *(N3)*

(iii) **EITHER**

$\begin{pmatrix} 1 & -4 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} t \\ -t+2 \end{pmatrix} = \begin{pmatrix} 5t-8 \\ 2t-4 \end{pmatrix}$ *(A1)*

OR

$x = 5t - 8$
$y = 2t - 4$ *(A1)*

THEN

Image $y = \frac{2}{5}x - \frac{4}{5}$ *(A1)(A1)* *(N2)*

Notes:	One alternative method is to find two points on $y = -x + 2$, find their images and then find the line between them. Another valid alternative method is to express x and y in terms of x' and y' using $\boldsymbol{M}^{-1}$.

[8 marks]

continued…

Question 8 (i) continued

(d) (i) $\boldsymbol{Mu} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$, $\boldsymbol{Mv} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$ ***(A1)***

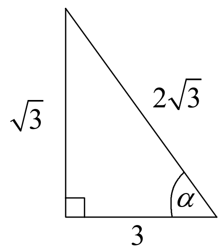
(ii) $\boldsymbol{Mw} = 3\boldsymbol{Mu} - 2\boldsymbol{Mv}$ $\left(\text{or } = 3\begin{pmatrix} -3 \\ 2 \end{pmatrix} - 2\begin{pmatrix} 4 \\ -1 \end{pmatrix}\right)$ ***(M1)***

$a = -17$; $b = 8$ ***(A1)(A1)*** ***(N2)***

Note: $\boldsymbol{u}$ and $\boldsymbol{v}$ may be found using $\boldsymbol{M}^{-1}$.

[4 marks]

(ii) (a) (i) **EITHER**



(M1)

OR

$\tan\alpha = \frac{\sqrt{3}}{3}$ ***(M1)***

THEN

$\sin\alpha = \frac{1}{2}$, $\cos\alpha = \frac{\sqrt{3}}{2}$ ***(A1)***

$$\boldsymbol{F} = \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$$ ***(A1)(A1)*** ***(N3)***

(ii) $(0, 2)$ (or any point on L) is invariant under $\boldsymbol{T}$. ***(M1)***

$$\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}\begin{pmatrix} 0 \\ 2 \end{pmatrix} + \begin{pmatrix} h \\ k \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$ ***(A1)***

$h = -\sqrt{3}$, $k = 3$ ***(A1)(A1)***

$\left(\text{vector} \begin{pmatrix} -\sqrt{3} \\ 3 \end{pmatrix}\right)$ ***(N3)***

[8 marks]

continued…

Question 8 (ii) continued

(b) (i) **EITHER**

$$\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}\begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -\sqrt{3} \\ 3 \end{pmatrix} = \begin{pmatrix} -\sqrt{3} \\ 3 \end{pmatrix}$$ ***(R1)***

OR

$$\boldsymbol{T}\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} h \\ k \end{pmatrix} = \begin{pmatrix} -\sqrt{3} \\ 3 \end{pmatrix}$$ ***(R1)***

(ii) $d = \frac{1}{2}$ distance from $(0, 0)$ to $\boldsymbol{T}\ (0, 0)$. ***(M1)***

$d = \frac{1}{2}\sqrt{(-\sqrt{3})^2 + 3^2}$ ***(A1)***

$d = \sqrt{3}$ ***(A1)*** ***(N2)***

[4 marks]

Total [30 marks]