

**MATHEMATICAL METHODS
STANDARD LEVEL
PAPER 2**

Friday 7 May 2004 (morning)

2 hours

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all five questions from Section A and one question from Section B.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.
- Write the make and model of your calculator in the appropriate box on your cover sheet
e.g. Casio *fx-9750G*, Sharp EL-9600, Texas Instruments TI-85.

Please start each question on a new page. You are advised to show all working, where possible. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

SECTION A

Answer all **five** questions from this section.

1. [Maximum mark: 15]

The points A and B have the position vectors $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ -1 \end{pmatrix}$ respectively.

(a) (i) Find the vector \vec{AB} .

(ii) Find $|\vec{AB}|$.

[4 marks]

The point D has position vector $\begin{pmatrix} d \\ 23 \end{pmatrix}$.

(b) Find the vector \vec{AD} in terms of d .

[2 marks]

The angle \hat{BAD} is 90° .

(c) (i) Show that $d = 7$.

(ii) Write down the position vector of the point D.

[3 marks]

The quadrilateral ABCD is a rectangle.

(d) Find the position vector of the point C.

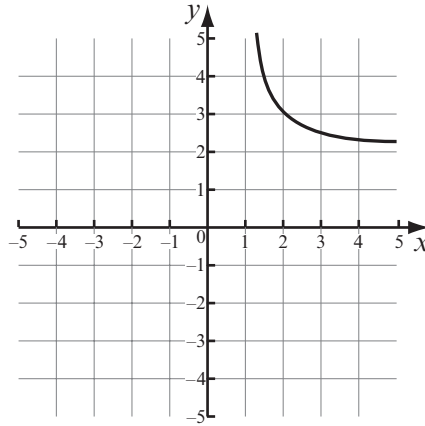
[4 marks]

(e) Find the area of the rectangle ABCD.

[2 marks]

2. [Maximum mark: 16]

- (a) Consider the function $f(x) = 2 + \frac{1}{x-1}$. The diagram below is a sketch of part of the graph of $y = f(x)$.



Copy and complete the sketch of $f(x)$.

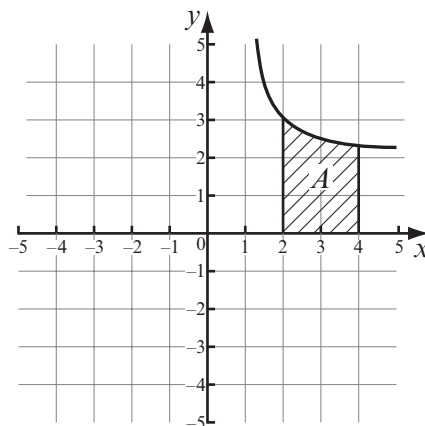
[2 marks]

- (b) (i) Write down the x -intercepts and y -intercepts of $f(x)$.
(ii) Write down the equations of the asymptotes of $f(x)$.
- (c) (i) Find $f'(x)$.
(ii) There are no maximum or minimum points on the graph of $f(x)$. Use your expression for $f'(x)$ to explain why.

[4 marks]

[3 marks]

The region enclosed by the graph of $f(x)$, the x -axis and the lines $x = 2$ and $x = 4$, is labelled A , as shown below.



- (d) (i) Find $\int f(x) dx$.
(ii) Write down an expression that represents the area labelled A .
(iii) Find the area of A .

[7 marks]

3. [Maximum mark: 13]

The depth y metres of water in a harbour is given by the equation

$$y = 10 + 4 \sin\left(\frac{t}{2}\right),$$

where t is the number of hours after midnight.

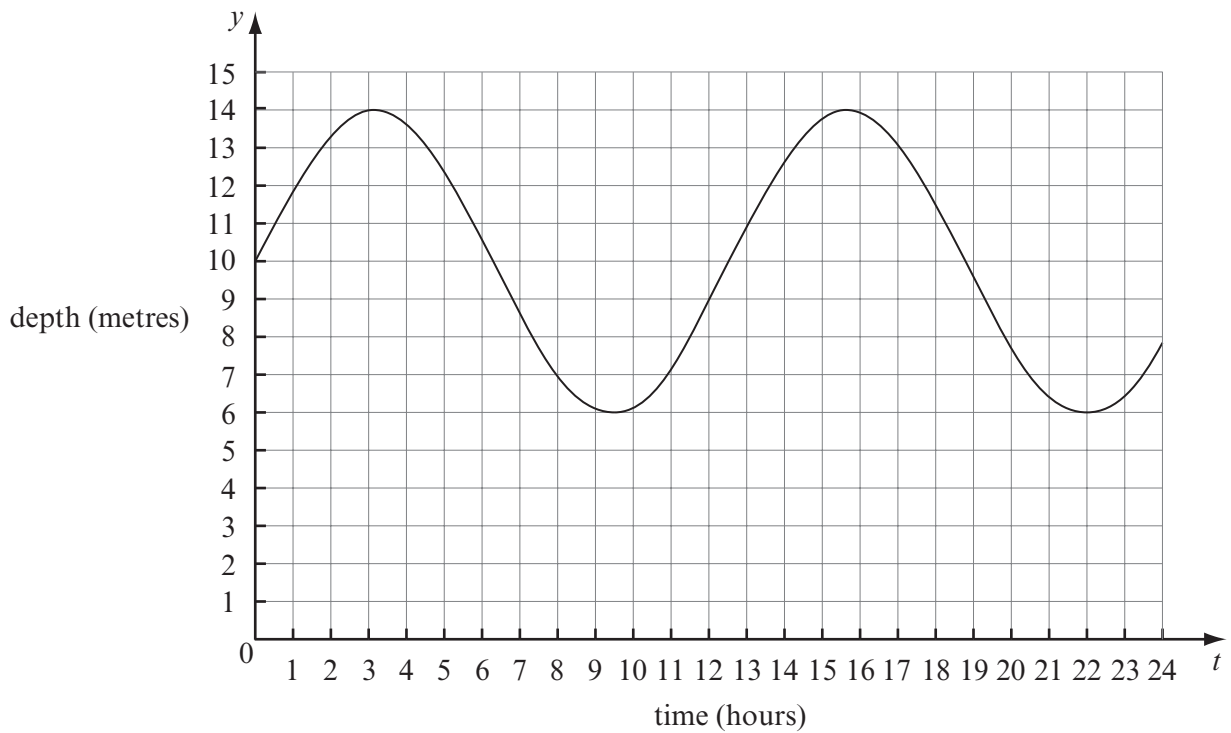
(a) Calculate the depth of the water

(i) when $t = 2$;

(ii) at 2100.

[3 marks]

The sketch below shows the depth y , of water, at time t , during one day (24 hours).



(This question continues on the following page)

(Question 3 continued)

- (b) (i) Write down the maximum depth of water in the harbour.
- (ii) Calculate the value of t when the water is first at its maximum depth during the day.

[3 marks]

The harbour gates are closed when the depth of the water is less than seven metres. An alarm rings when the gates are opened or closed.

- (c) (i) How many times does the alarm sound during the day?
- (ii) Find the value of t when the alarm sounds first.
- (iii) Use the graph to find the length of time during the day when the harbour gates are closed. Give your answer in hours, to the nearest hour.

[7 marks]

4. [Maximum mark: 11]

Dumisani is a student at IB World College.

The probability that he will be woken by his alarm clock is $\frac{7}{8}$.

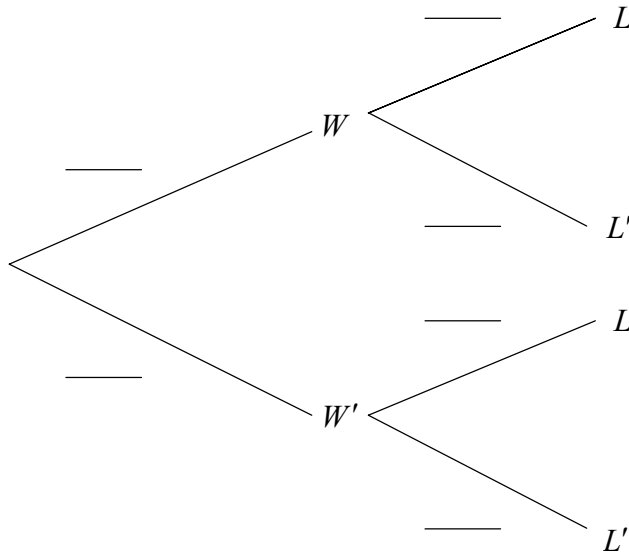
If he is woken by his alarm clock the probability he will be late for school is $\frac{1}{4}$.

If he is not woken by his alarm clock the probability he will be late for school is $\frac{3}{5}$.

Let W be the event “Dumisani is woken by his alarm clock”.

Let L be the event “Dumisani is late for school”.

(a) Copy and complete the tree diagram below.



[4 marks]

(b) Calculate the probability that Dumisani will be late for school.

[3 marks]

(c) Given that Dumisani is late for school what is the probability that he was woken by his alarm clock?

[4 marks]

5. [Maximum mark: 15]

There were 1420 doctors working in a city on 1 January 1994. After n years the number of doctors, D , working in the city is given by

$$D = 1420 + 100n .$$

- (a) (i) How many doctors were there working in the city at the start of 2004?
- (ii) In what year were there first more than 2000 doctors working in the city?

[3 marks]

At the beginning of 1994 the city had a population of 1.2 million. After n years, the population, P , of the city is given by

$$P = 1\,200\,000 (1.025)^n .$$

- (b) (i) Find the population P at the beginning of 2004.
- (ii) Calculate the percentage growth in population between 1 January 1994 and 1 January 2004.
- (iii) In what year will the population first become greater than 2 million?
- (c) (i) What was the average number of people per doctor at the beginning of 1994?
- (ii) After how many **complete** years will the number of people per doctor first fall below 600?

[7 marks]

[5 marks]

SECTION B

Answer **one** question from this section.

Statistical Methods

6. [Maximum mark: 30]

- (i) A group of 75 people was asked the question “Are you in favour of banning the use of mobile phones while driving?” Their answers are shown in the following table.

	Yes	No	Don't know
Men	18	10	12
Women	8	11	16

A χ^2 test will be used to examine the claim that answers to the question are independent of gender.

The following table gives the expected frequencies for the above data.

	Yes	No	Don't know
Men	13.87	a	b
Women	12.13	c	d

- (a) (i) Show that $a = 11.2$.
- (ii) Calculate the value of d .
- (iii) Find χ^2 for this data. [6 marks]
- (b) Explain why it is correct to conclude, at the 10 % level of significance, that responses regarding the use of mobile phones while driving do not depend on gender. [3 marks]

(This question continues on the following page)

(Question 6 continued)

- (ii) A population is normally distributed with $\sigma = 8$. We wish to test the null hypothesis H_0 against the alternative hypothesis H_1 , where

$$H_0 : \mu = 82 ;$$

$$H_1 : \mu \neq 82 .$$

- (a) Which kind of test is appropriate, a one-tailed or a two-tailed test? *[1 mark]*
- (b) A sample of size 36 is drawn at random from the population and its mean is found to be 79.
- (i) Calculate the standardized value corresponding to 79.
- (ii) At the 5 % level of significance, would you accept or reject H_0 ? Give a reason for your answer. *[5 marks]*
- (c) Assume that the population mean μ is actually 80. Find the probability that a random sample of size 36 drawn from the population will have a mean of 79 or less. *[2 marks]*

(This question continues on the following page)

(Question 6 continued)

(iii) Fifteen books are selected at random from all the books in a bookstore. For each book, the number of pages (x) and the selling price (y) are determined.

(a) The correlation coefficient r is calculated.

(i) Write down the possible minimum and maximum values of r .

It is found that $r = 0.95$.

(ii) Sketch a possible scatter diagram to represent this information.

(iii) Which **two** of the following expressions describe the correlation between x and y ?

perfect, zero, linear, strong positive, strong negative, weak positive, weak negative.

[5 marks]

(b) For the fifteen books in the sample, the mean number of pages $\bar{x} = 500$ and the mean price $\bar{y} = 46$. Using the equation of the regression line of y on x , it was estimated that a book with 660 pages would sell for \$ 49.20.

(i) Find the equation of the regression line.

(ii) Hence, estimate the selling price of a 550 page book.

[4 marks]

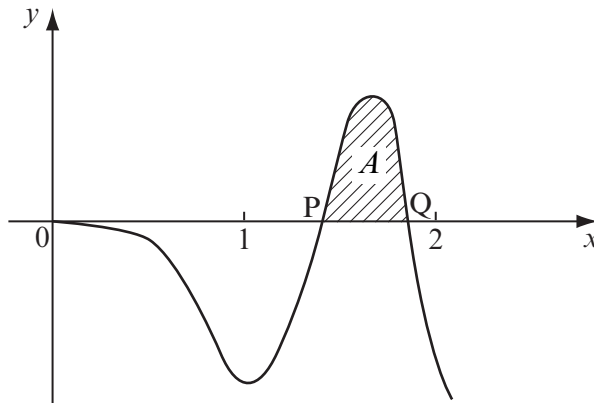
(c) Assume that the selling price of books in the bookstore is normally distributed with mean μ and standard deviation 8.5. Use the given sample of 15 books to calculate a 95 % confidence interval for μ .

[4 marks]

Further Calculus

7. [Maximum mark: 30]

- (i) The diagram below shows part of the graph of $f(x) = x^2 \sin(x^3 + \pi)$ and the shaded region A .



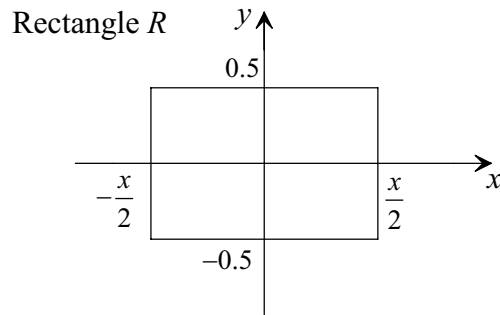
This graph crosses the x -axis at P and Q. The point P has coordinates $(\sqrt[3]{\pi}, 0)$.

- (a) Find the x -coordinate of Q. [2 marks]
- (b) Use the substitution $u = x^3 + \pi$ to find $\int f(x) dx$. [4 marks]
- (c) Hence, using your answer to (b), find the area of the region A . [3 marks]

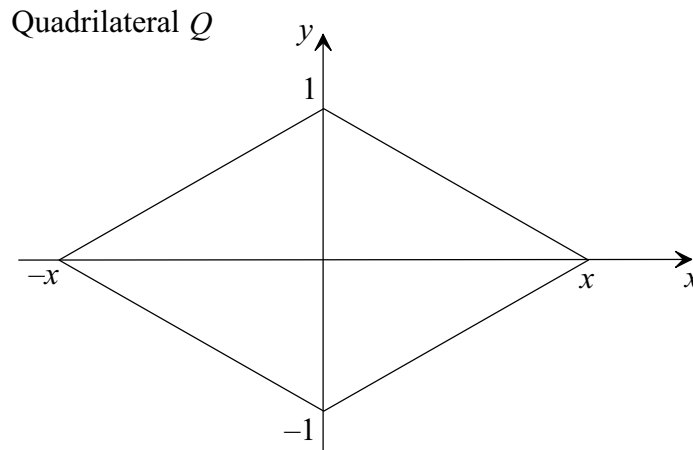
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(Question 7 continued)

(ii) The rectangle R and the quadrilateral Q are shown below.



R has vertices at $\left(\frac{x}{2}, 0.5\right), \left(-\frac{x}{2}, 0.5\right), \left(-\frac{x}{2}, -0.5\right), \left(\frac{x}{2}, -0.5\right)$.



Q has vertices at $(x, 0), (0, 1), (-x, 0), (0, -1)$, where $x > 0$.

(This question continues on the following page)

(Question 7 (ii) continued)

- (a) (i) Find the perimeter of R .
 (ii) Find the perimeter of Q . [2 marks]

(b) Let $g(x) = \frac{\text{perimeter of } R}{\text{perimeter of } Q}$, for $x > 0$, i.e. $g(x) = \frac{0.5(x+1)}{\sqrt{1+x^2}}$.

Show that $g'(x) = \frac{0.5(1-x)}{(1+x^2)^{\frac{3}{2}}}$. [3 marks]

- (c) Find the maximum value of the ratio $\frac{\text{perimeter of } R}{\text{perimeter of } Q}$. [5 marks]

(iii) The iterative formula below may be used to calculate $\sqrt[5]{5}$.

$$x_1 = 1;$$

$$x_{n+1} = 0.8x_n + \frac{1}{x_n^4}, n \geq 1.$$

- (a) Show that the above formula for x_{n+1} can be obtained by applying the Newton-Raphson method to the equation $x^5 - 5 = 0$. [3 marks]

- (b) You are to solve the equation $x^5 - 5 = 0$ using the Newton-Raphson method.

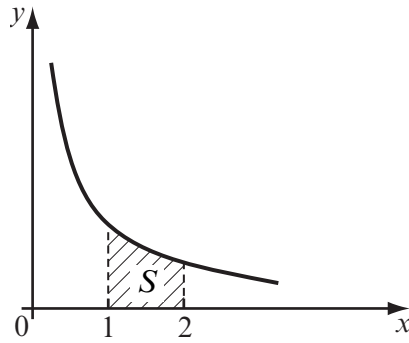
- (i) Starting with $x_1 = 1$, calculate successive approximations x_2, x_3, \dots for the root of the equation until the error is less than 0.0001.

- (ii) Hence, write down the root, correct to **five** significant figures. [3 marks]

(This question continues on the following page)

(Question 7 continued)

- (iv) The diagram below shows a sketch of the curve $y = \frac{1}{x}$. The shaded region S is between $x = 1$ and $x = 2$.



Two students, Carl and Joanne, estimated the area of S . Carl's result was 0.696, while Joanne's was 0.692. Only one of the students used the trapezium rule.

- (a) Find a value for the area of S to **five** significant figures. [2 marks]
- (b) State, with reasons, which student used the trapezium rule. [3 marks]

Further Geometry

8. [Maximum mark: 30]

(i) (a) Write down the matrices representing the following transformations.

(i) H , shear of scale factor 2 in the direction of the x -axis.

(ii) S , stretch of scale factor 2 in the direction of the y -axis.

(iii) R , reflection in the x -axis. [3 marks]

(b) Give a full geometric description for each one of the transformations represented by H^{-1} , S^{-1} , R^{-1} . [3 marks]

(c) The transformation M is R , followed by S , followed by H .

(i) Express the matrix M as a product of R , S and H .

This gives $M = \begin{pmatrix} 1 & -4 \\ 0 & -2 \end{pmatrix}$.

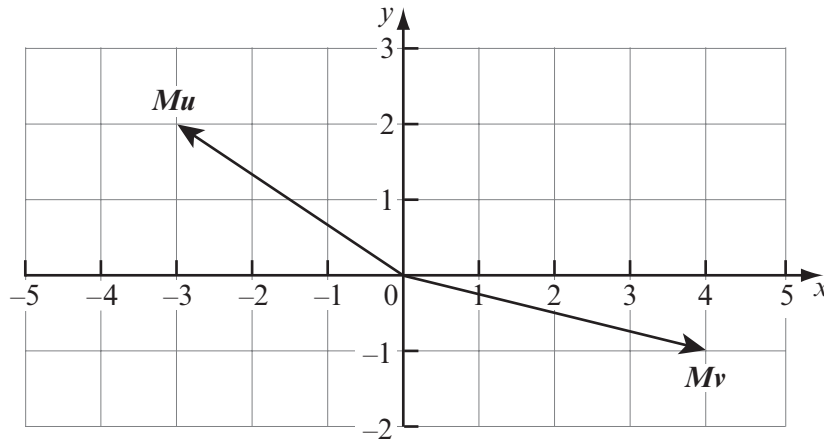
(ii) Find all points that are invariant under M .

(iii) Find the image of the line $y = -x + 2$ under M . [8 marks]

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(Question 8 (i) continued)

- (d) The vectors \mathbf{u} and \mathbf{v} are transformed by \mathbf{M} . Their images \mathbf{Mu} and \mathbf{Mv} are shown below.



- (i) Write down the column vectors \mathbf{Mu} and \mathbf{Mv} .
- (ii) Let $\mathbf{w} = 3\mathbf{u} - 2\mathbf{v}$. Its image under \mathbf{M} is $\begin{pmatrix} a \\ b \end{pmatrix}$. Find a and b . [4 marks]
- (ii) The line L has equation $y = \frac{\sqrt{3}}{3}x + 2$. The transformation \mathbf{T} is the reflection in L and it may be expressed as

$$\mathbf{T} \begin{pmatrix} x \\ y \end{pmatrix} = \mathbf{F} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} h \\ k \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix}.$$

- (a) (i) Find the matrix \mathbf{F} .
- (ii) Find the vector $\begin{pmatrix} h \\ k \end{pmatrix}$. [8 marks]
- (b) (i) Show that the image of the origin $(0, 0)$ under \mathbf{T} is $(-\sqrt{3}, 3)$.
- (ii) Hence calculate the perpendicular distance d from the origin to L . [4 marks]